Module 3B4 2012 Crib

1. (a) Specific magnetic loading is the average airgap flux density over one pole. Its value is limited to around 0.5 T by the need to avoid excessive saturation of the magnetic circuit and excessive iron losses.

Specific electrical loading is the total effective current per unit length averaged around the airgap. Its value is limited by the need to avoid excessive conductor losses, with a typical value of 30,000 Am^{-1}. Note that there is far more variation in specific electric loading values than magnetic loading values because improved cooling methods enable greater specific electric loading to be achieved (albeit causing reduced efficiency).

From the definition of $\overline{B}$: $\overline{B} = \frac{1}{(\pi/p)} \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \sqrt{2}B_{rms} \cos p\theta \, d\theta = \sqrt{2}B_{rms} \frac{p}{\pi} [\sin p\theta]_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} = \frac{2\sqrt{2}B_{rms}}{\pi}$

(b) (i) N = 60f/p and so 300 = 60×50/p giving p = 10.

P = Tω so 200000 = T × 300 × 2π/60 giving T = 6.37 kNm

S = 200/0.8 = 250 kVA = (π/√2) × π $(d/2)^2$ × 2d × (2π × 50/10) × 0.4 × 25000 giving d = 0.611 m and so l = 1.22 m.

(ii) For a 5:1 gearbox the motor speed will now be 1500 rpm, and so a 4 pole motor is required, and so p = 2. For the same output power, the torque will be reduced by a factor of 5, and so the new torque is 6.37/5 = 1.27 kNm.

The angular speed of the motor is increased by as factor of 5, and so for the same machine rating, the machine volume will be reduced by a factor of 5, and so d is reduced by the factor $\sqrt[3]{5}$ giving d=0.357 m and l = 0.715 m.

The only way to increase the power density of a motor given fixed specific magnetic and electrical loading values is to increase its speed. Thus, a gearbox is often used in low-speed drive applications to enable smaller and hence cheaper motors to be used. This is borne out by the above calculations.

(c) 4 pole in 48 slots means p = 2, m = 48/6p = 48/12 = 4. β = 360/48 = 7.5^0 and short-pitching by two stator slots means that α = 2β = 15^0. Thus the winding factor k_w may be found as:

k_w = sin(mpβ/2)/(msin(pβ/2))cos(pα/2) = sin(4×2×7.5/2)/(4sin(2×7.5/2) × cos(2 × 15/2) = 0.925

Assume $E_{ph} = V_{ph}$ = 3300/√3 and using $E_{rms} = l(\omega/p)dN_{ph}k_wB_{rms}$ with B_{rms} = (π/2√2)×0.4 = 0.444
Putting in the numbers gives N_{ph} = 115.7.

Double layer winding means that the number of coils is the same as the number of slots ie 48 and so there are 48/3 = 16 coils per phase giving the number of turns per coil, N_{coil} = 115.7/16 = 7.23. This should be rounded up to the nearest integer to reduce the magnetic loading slightly ie N_{coil} = 8 giving Nph = 16×8 = 128.

(d) Peak tooth flux occurs when a tooth is opposite the peak airgap flux density. Assume all the flux over one slot pitch flows up the tooth, giving:

$$\hat{B}(w_s + w_t)l = \hat{B}_t w_t l$$

Rearranging : $\dfrac{\hat{B}}{\hat{B}_t} = \dfrac{w_t}{w_s + w_t}$

Note that $\hat{B} = \dfrac{\pi}{2}\overline{B}$ giving $\hat{B} = \dfrac{\pi}{2} \times 0.4 = 0.628$ T.

The slot pitch measured at the airgap is $\pi d/48 = 23.4$ mm $= w_s + w_t$ and so

$$\frac{1.8}{0.628} = \frac{w_t}{23.4}$$

giving $w_t = 8.15$ mm and hence $w_s = 23.4 - 8.15 = 15.25$ mm.

The rated current is given by $3V_{ph}I_{ph} = 250000$ and so $I_{ph(max)} = 43.7$ A. Thus the cross-sectional area of one turn assuming a current density of 5 Amm^{-2} is $43.7/5 = 8.75$ mm^2. There are 16 turns of wire in one slot (double-layered winding with 8 turns per coil) and so the conductor area in one slot is $16 \times 8.75 = 140$ mm^2. The slot area has to take account of the 70% fill factor, giving $A_{slot} = 140/0.7 = 200$ mm^2.

From the previous part, $w_s = 15.25$ mm at the airgap surface. Note that the slot width will increase with depth into the stator coreback, because the tooth width remains fixed. However, this may be ignored providing the slot depth is significantly smaller than the airgap radius.

Thus, the slot depth = 200/15.25 = 13.1 mm. This is substantially less than the airgap radius (178.5 mm) which justifies the assumption of treating the slot as being rectangular.

2(a) VVVF control of induction motor drives enables efficient operation ie steep torque-speed curve over a wide range of operating speeds. Combined with vector control, the stator voltage can be controlled to maintain a 90^0 angle between the rotor current and the motor flux, thereby maximizing the torque per unit current, and hence efficiency. Operation beyond the base speed of the motor may also be obtained with this type of scheme by field-weakening. This can be useful if a load requires a smaller torque at a higher speed than base speed.

(i) With rotor resistance control, the peak torque of the motor is fixed but the slip at which it occurs increases with increasing rotor resistance. Speed control is only possible up to the base speed of the motor. This method of speed control is only applicable to wound-rotor induction motors, and results in large rotor losses and hence loss of drive efficiency.

(ii) With voltage control, speed control up to base speed only is possible. Whilst being applicable to all types of induction motor, this method results in substantial loss of peak torque with reducing voltage. Thus, it is only useful for certain classes of load eg pump/fan loads where the torque varies with the square of the speed, so that a low torque at low speeds is acceptable.

(b) $T = 3I_2^2R_2/s\omega_s$ and $\omega_s = \omega/p$ giving $T = 3pI_2^2R_2/s\omega$

$I_2^2 = V_1^2/((R_2/s)^2 + \omega^2L_2^2) = s^2V_1^2/(R_2^2 + s^2\omega^2L_2^2)$ assuming that the voltage across the rotor branch, E, is equal to the stator applied voltage, V_1. Substituting into the expression for torque and replacing V_1 with $k\omega$ gives:

$T = 3pk^2s\omega R_2/(R_2^2 + s^2\omega^2L_2^2)$

For small slip, $s\omega L_2 << R_2$ and so the denominator may be simplified to R_2^2 giving:

$T = 3pk^2s\omega/R_2$

Small slip corresponds to the steep part of the torque-speed curve. For typical induction motors, the small slip approximation is typically valid up to 1.5×full load torque. After this, the effect of L_2 becomes significant and should be accounted for in the analysis.

(c) (i) Ignore $R_1 + jX_1$ and jX_2. This is justified because the motor will be operating at a small value of slip.

$I_1 = 415/jX_m + I_2 = -j4.61 + I_2$

Since $R_2/s >> X_2$, I_2 is approximately in phase with V_1 and so at the stator current limit of 20 A:

$I_2 = \sqrt{(20^2 - 4.61^2)} = 19.46$ A

$k = V_b/\omega_b = 415/(2\pi\times50) = 1.32$

$I_2 = V_1/(R_2/s) = V_1s\omega/(\omega R_2) = s\omega k/R_2 = 19.46$ so $s\omega = 19.46 \times 1.4/1.32 = 20.6$ and this is fixed for rated torque and rated rotor (and hence stator) current.

$T = 3pk^2s\omega/R_2 = 3 \times 3 \times 1.32^2 \times 20.6/1.4 = 231$ Nm

V_1 is at its maximum when f = 50 Hz and delivering rated torque. The slip at this frequency is given by $2\pi \times 50s = 20.6$ so $s = 0.0656$.

Maximum speed = $(1 - s) \times 60f/p = (1 - 0.0656) \times 1000 = 934$ rpm.

(ii) Maximum unloaded speed = $(100/50) \times 1000 = 2000$ rpm

For the maximum speed at which the drive can deliver 80% rated torque the motor is clearly in the field weakening region, so $V/\omega = k$ no longer applies, and torque is given by:

$T = 3pV_b^2 s/(\omega R_2)$ and this is a maximum at $s = s_b = I_{2max}R_2/V_b = 19.46 \times 1.4/415 = 0.0656$ (as before).

Thus for 80% of rated torque: $0.8 \times 231 = 3 \times 3 \times 415^2 \times 0.0656/(\omega \times 1.4)$ giving $\omega = 393$ rads^{-1} and so $f = 62.6$ Hz. Therefore $N_r = (1 - 0.0656) \times 60 \times 62.6/3 = 1169$ rpm.

(iii) $f = 2$ Hz so voltage across magnetizing reactance for rated flux is $(2/50) \times 415 = 16.6$ V. Slip is found from:

$T = 3pk^2 s\omega/R_2$ with $\omega = 2\pi \times 2 = 12.56$ rads^{-1} and T set to 10% of rated torque ie 23.1 Nm.

$23.1 = 3 \times 3 \times 1.32^2 s \times 12.56/1.4$ giving $s = 0.164$ and $I_2 = 16.6s/1.4 = 1.95$ A. Thus, total stator current is $(1.95 - j4.61)$ A and this will produce a voltage drop across R_1 (no need to include X_1 since its value will be negligible compared to R_1 at $f = 2$ Hz) of:

$V_{R1} = 2.2 \times (1.95 - j4.61) = 4.29 - j10.1$

and so $V_1 = 16.6 + 4.29 - j10.1 = 20.89 - j10.1$ and the magnitude of V_1 is 23.2 V. Voltage boost is the difference between V_1 and the voltage required to produce rated flux ie the voltage across the magnetizing reactance, which is 16.6 V. Therefore:

$V_{boost} = 23.2 - 16.6 = 6.6$ V.

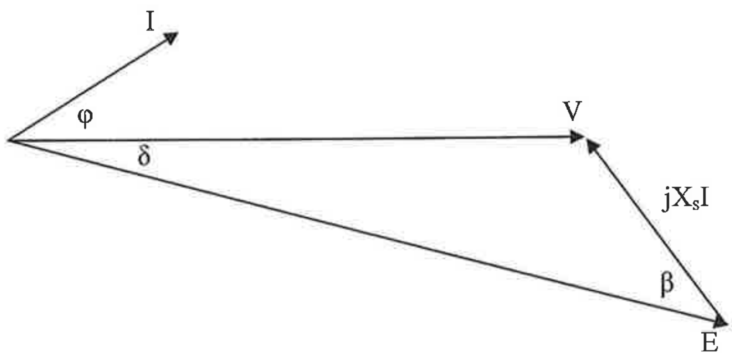
3 (a) The three phase permanent magnet synchronous motor (PMSM) is similar in construction to a field wound synchronous motor except that the rotor is furnished with radially magnetized permanent magnets so that adjacent magnets are of opposite polarity. The stator is of the usual construction and wound with a balanced three-phase winding designed to produce the same number of poles as the rotor.

The iron of the rotor is roughly cylindrical, although flats are sometimes milled onto the surface to facilitate the mounting of the permanent magnets. Since the stator bore is also cylindrical, there is no minimum energy point and hence no reluctance torque.

Because of the presence of the permanent magnets, with relative permeability close to 1, the effective airgap length is large (it is equal to the magnet radial length plus the actual airgap length). A large airgap implies low winding inductances, hence the small value of synchronous reactance

In order to produce torque efficiently the machine should be operated at the design value of specific magnetic loading. As speed increases the induced emf due to the rotor increases in proportion to the rotor speed, as does the synchronous reactance. Therefore, operation at constant magnetic loading means that the applied voltage must increase in proportion to speed. However, at low speeds the resistive voltage drop must be overcome and so the applied voltage must be somewhat larger (voltage boost)

(b)



Start from the databook expression $T = 3VE\sin\delta/\omega_s X_s$

$V\sin\delta = X_s I\sin\beta$ giving $T = 3EI\sin\beta/\omega_s = 3k\omega I\sin\beta/(\omega/p) = 3kpI\sin\beta$

Therefore $k' = 3kp$

If the applied voltage is controlled so that the torque angle β is always maintained at 90^0 then the expression for the motor torque becomes $T = k'I$, which is identical to the torque expression for a dc machine. To obtain maximum torque per amp means controlling the motor so that the torque angle is fixed at 90^0.

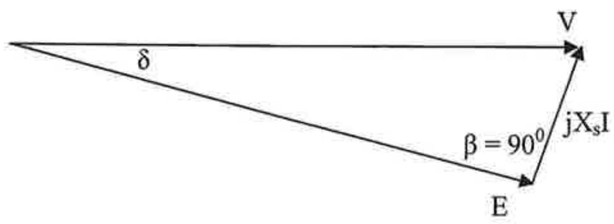
(c) (i) Using $T = k'I\sin\beta$ and noting that $\beta = 90^0$, maximum torque corresponds to maximum current, so I = 50 A.

$T_{max} = 3 \times 0.3 \times 4 \times 50 = 180$ Nm

$N_{max} = 60f_{max}/p = 60 \times 80/4 = 1200$ rpm (note that it won't be possible to deliver maximum torque at this speed since inverter voltage is limited)

(ii) 50% of T_{max} means that I = 25 A. Speed of 600 rpm means f = 40 Hz (from N = 60f/p) and therefore $\omega = 2\pi f = 503$ rads^{-1}.

$E = k\omega = 0.3 \times 503 = 151$ V, $Xs = \omega Ls = 503 \times 0.005 = 2.52\ \Omega$. This is over ten times the phase resistance, so the phase resistance can be ignored when calculating the applied voltage. Since $\beta = 90^0$, the phasor diagram below applies.

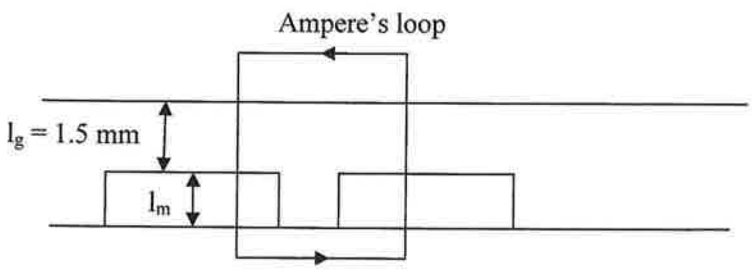


$V = (E^2 + (X_sI)^2)^{1/2} = 164$ V phase $= \sqrt{3} \times 164 = 283$ V line so well within the 415 V rating of the inverter.

$\sin\delta = X_sI/V = 62.9/164$ giving $\delta = 22.6^0$

Power loss $= 3I^2R = 3 \times 252 \times 0.2 = 375$ W

(d)



From N44H characteristic, B_{rem} = 1.34 T and H_{co} = -1040 kAm^{-1}. So for B = 0.65 T, H is approximately $0.5H_{co}$ = -520 kAm^{-1}. Assuming zero mmf drop in the rotor and stator iron and applying Ampere's Law around the path shown in the figure above:

$2(H_m l_m + H_g l_g) = 0$

$-520 \times 10^3\ l_m + B_g l_g/\mu_0 = 0$ giving $l_m = 1.49$ mm

4 (a) Using the maths databook, the Fourier series for a square wave centered on $\theta = 0$ and of unit amplitude is

$$f(\theta) = \frac{4}{\pi} \sum_{n\ odd} \frac{\cos n\theta}{n}$$

In this case, the amplitude is NI/2, and only the fundamental (n = 1) is of interested giving

$$F(\theta) = \frac{2NI}{\pi} \cos\theta$$

Putting I(t) into this expression gives:

$$F(\theta,t) = \frac{2N\hat{I}\cos\omega t}{\pi}\cos\theta = \frac{N\hat{I}}{\pi}\left[\cos(\omega t - \theta) + \cos(\omega t + \theta)\right]$$

Hence there are two mmf waves, one rotating in the positive θ direction and one rotating in the negative θ direction. Note that both are of equal amplitude.

(b) The rotor will 'see' the two mmf waves, and if the rotor speed is ω_r, then defining slip with respect to the rotating mmf wave which is in the same direction of rotation as the rotor as s_f we have:

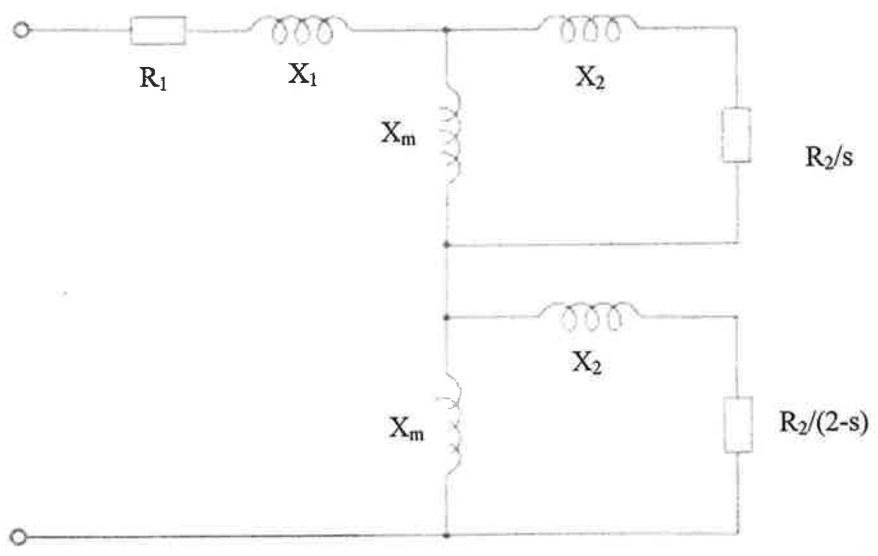
$s = s_f = (\omega_s - \omega_r)/\omega_s$

Thus, the backwards slip is

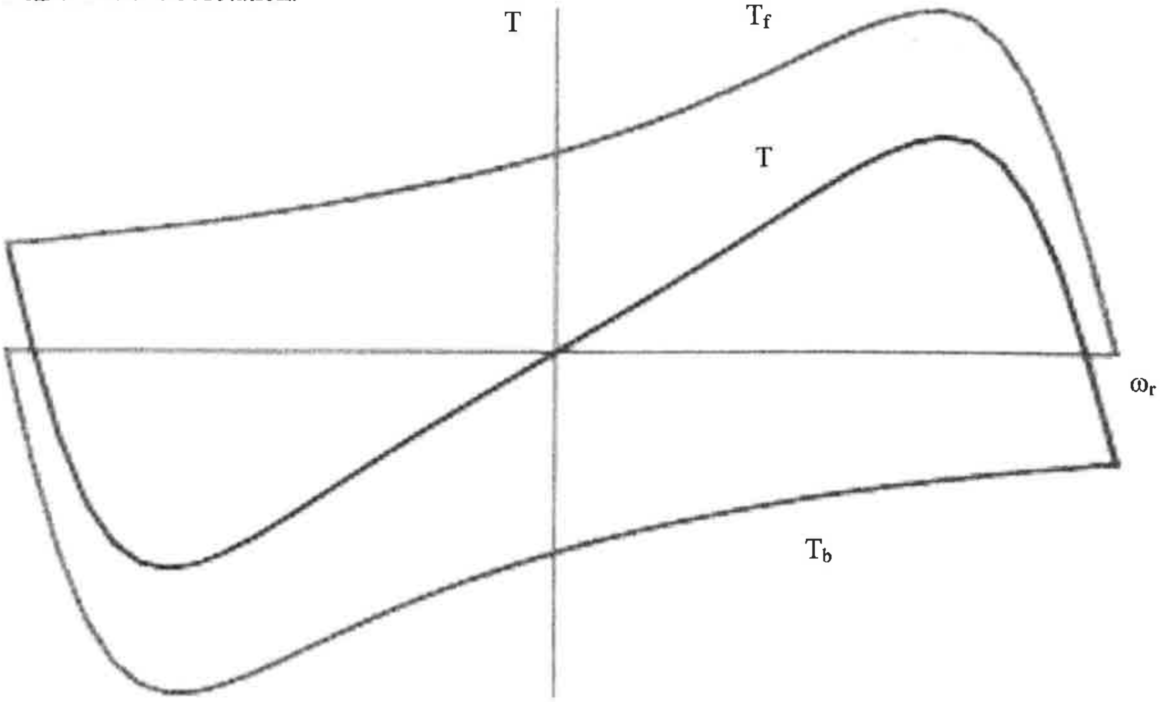
$s_b = (-\omega_s - \omega_r)/-\omega_s$

$= 2 - s$

Thus the equivalent circuit is as shown below



The torque-speed curve is obtained by superimposing the torque-speed curves of two three-phase induction motors, one forwards rotating, the other backwards rotating, but both producing torque in their directions of rotation.



(c) 6 pole at 50 Hz means a synchronous speed of 1000 rpm. Thus, 900 rpm corresponds to a forward slip of (1000 – 900)/1000 = 0.1.

The synchronous speed is $2\pi f/p = 104.7$ rads^{-1}.

The total impedance of the motor is:

$Z = 3 + 4/0.1 + 4/(2 - 0.1) + j(5 + 5 + 5) = (45.1 + j15)\ \Omega$

Thus the input current $I = 240/(45.1^2 + 15^2)^{1/2} = 5.04$ A

The forwards torque $T_f = I^2R_2/(s\omega_s) = 9.74$ Nm

The backwards torque $T_b = I^2R_2/((2\text{-}s)\omega_s) = 0.511$ Nm giving a total torque of $T_f - T_b = 9.23$ Nm

Stator loss = $I^2R_1 = 5.04^2 \times 3 = 76.2$ W

Rotor loss = $I^2R_2 \times 2 = 203$ W

$P_{out} = T\omega_r = 9.23 \times 0.9 \times 104.7 = 870$ W

$P_{in} = P_{out} + P_{loss} = 870 + 203 + 76.2 = 1149$ W

$\eta = P_{out}/P_{in} = 870/1149 = 75.7$ %

(d) The starter winding effectively turns the single-phase motor into a two-phase motor, and with appropriate design can be made so that in combination with the main winding, only a forwards-rotating mmf wave is produced. In that case, the motor will behave like a 3 phase induction motor, and will produce a substantial starting torque.

Assuming that the starter winding is displaced by 90^0 wrt the main winding, and that the starter winding current lags the main winding current by 90^0, then the mmf produced by the starter winding will be:

$$F_s(\theta,t) = \frac{2N_s\hat{I}_s \sin\omega t}{\pi}\sin\theta = \frac{N_s\hat{I}_s}{\pi}\left[\cos(\omega t-\theta)-\cos(\omega t+\theta)\right]$$

Thus, if the peak ampere-turns of the starter winding are made identical to that of the main winding then adding together the two mmfs will result in the backwards-rotating wave being exactly cancelled.

T Flack, May 30th 2012

3B4 Electric Drive Systems 2012 Short Answers

1 (b) (i) p = 10, T = 6.37 kNm, d = 0.611 m, l = 1.22 m (ii) p = 2, T = 1.27 kNm, d = 0.357 m, l = 0.715 m (c) k_w = 0.925, N_{ph} = 128 (d) Tooth width = 8.15 mm, slot width = 15.25 mm, slot depth = 13.1 mm

2 (c) (i) Rated torque = 231 Nm, speed = 934 rpm (ii) Maximum unloaded speed = 2000 rpm, maximum speed at 80% rated torque = 1169 rpm (iii) Voltage boost = 6.6 V.

3 (c) Maximum torque = 180 Nm, speed = 1200 rpm (ii) Inverter frequency = 40 Hz, voltage = 283 V line, load angle = 22.6^0, power loss = 375 W (d) Magnet thickness = 1.49 mm

4 (c) Slip = 0.1, input current = 5.04 A, torque = 9.23 Nm, stator loss = 76.2 W, rotor loss = 203 W, output power = 870 W, efficiency = 75.7%