ENGINEERING TRIPOS PART IIA 2012

Module 3B5 SEMICONDUCTOR ENGINEERING

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a The wavefunction provides a means of quantum-mechanically describing a particle. It is a function of time and spatial coordinates, and it contains everything that we can know about the particle, and is a valid solution to the Schrödinger equation. The modulus squared of the wavefunction gives the probability of the particle existing at a particular point in space at a particular moment in time. Performing other operations on the wavefunction allow other physical properties to be determined, such as momentum or total energy.

b i. We are given that

$$\psi = A\exp[j(kx - \omega t)]$$

Now,

$$\frac{\partial(\psi)}{\partial x} = Ajk\exp[j(kx - \omega t) = jk\psi$$

However, from de Broglie's postulate,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

Hence

$$\frac{\partial\psi}{\partial x} = \frac{jp}{\hbar}.\psi$$

$$\therefore p\psi = -j\hbar\frac{\partial\psi}{\partial x}$$

∴ Similarly,

$$\frac{\partial}{\partial t}(\psi) = -A j\omega \exp\{j(\omega kx - \omega t)\} = -j\omega\psi$$

However, from the Einstein postulate,

$$E = h\nu = \hbar\omega$$

Hence,

$$\frac{\partial\psi}{\partial t} = -\frac{jE}{\hbar}\psi$$

$$\therefore E\psi = j\hbar\frac{\partial\psi}{\partial t}$$

c The wavefunction must be a single-valued function of space and time so that the modulus squared is also single-valued. This is the probability, which must be single-valued - there cannot be two probabilities of the particle existing at a particular point in space and time. The wavefunction must be continuous space coordinates so that $\partial\psi/\partial x$ is finite, leading to a finite momentum. The first derivative in space must also be continuous so that $\partial^2\psi/\partial x^2$ is finite, as $-\hbar^2/2m\ \partial^2\psi/\partial x^2$ is the kinetic energy, which must also be finite.

d The probability of finding the particle outside the infinitely-deep potential well is zero, so ψ outside the well must also be zero. Therefore, for ψ to be continuous, it must be zero at the edges of the well. This leads to a set of solutions to the wavefunction where the width of the well must be an integer number of half-wavelengths (rather like waves in an organ pipe). Each solution will have a particular energy. (The eagle-eyed will note that this does not allow for $d\psi/dx$ to be continuous, but we reconcile this by saying that the infinitely-deep well is unphysical anyway!

a Electrons obey the Pauli exclusion principle (which applies to all fermions) which states that in a multi-electron system, there can never be more than one electron in the same state. Boltzmann statistics apply when multiple particles can exist in the same state. Particles will tend to exist in the lowest ~~energy~~ state, but may be excited to higher energy states, and this will follow an exponential decay. However, when Pauli applies, ~~the~~ particles occupy the lowest available energy state. Therefore, higher energy states are forced to be occupied. The Fermi function describes the probability of occupation of states. The Fermi energy is the energy at which there is a 50% probability that a state is occupied (or is the energy of the highest occupied state at 0K). For low probabilities of occupation ($E \gg E_F$) the Fermi function approximates to a Boltzmann distribution.

b At 0K, the Fermi function becomes

$$f(E) = \begin{cases} 0 & \text{for} \quad E > E_F \\ 1 & \text{for} \quad E \leq E_F \end{cases}$$

As the number of states which will be occupied in a particular volume of material is nV, then

$$nV = \int_0^{\infty} g(E) f(E)\, dE$$

$$nV = \int_0^{E_F} g(E)\, dE$$

Substituting for $g(E)$ gives

$$nV = \int_0^{E_F} \frac{V}{2\pi^2\hbar^3}(2m)^{3/2} E^{1/2}\, dE$$

$$n = \frac{(2m)^{3/2}}{2\pi^2\hbar^3}\int_0^{E_F} E^{1/2}\, dE$$

$$= \frac{(2m)^{3/2}}{2\pi^2\hbar^3}\cdot\left[\tfrac{2}{3}E^{3/2}\right]_0^{E_F}$$

$$n = \frac{(2m)^{3/2}}{3\pi^2\hbar^3}\cdot E_F^{3/2}$$

$$E_F^{3/2} = \frac{3\pi^2\hbar^3 n}{(2m)^{3/2}}$$

$$\therefore E_F = \frac{(3\pi^2 n)^{2/3}\cdot\hbar^2}{2m}$$

c For magnesium in Group II, each atom contributes 2 free electrons, so

$$n = 2.392\times10^{29}$$
$$n = 7.84\times10^{28}\ m^{-3}$$

Hence,

$$E_F = \frac{(3\pi^2\, 7.84\times10^{28})^{2/3}\,(1.055\times10^{-34})^2}{2.9.109\times10^{-31}}$$

$$= 1.07\times10^{-18}\ J$$

$$E_F = 6.69\,eV$$

d For a free electron

$$E = \frac{\hbar^2 |k|^2}{2m}$$

$$\therefore |k| = \frac{(2mE)^{1/2}}{\hbar}$$

$$= \frac{(2 \times 9.109 \times 10^{-31} \; 1.07 \times 10^{-18})^{1/2} \; 2\pi}{6.626 \times 10^{-34}}$$

$$|k| = 1.324 \times 10^{10} \; m^{-1}$$

$$\therefore \lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{2\pi}{1.324 \times 10^{10}}$$

$$\underline{\lambda = 4.75 \times 10^{-10} \; m}$$

$$p = \hbar k = mv$$

$$\therefore v = \frac{\hbar k}{m}$$

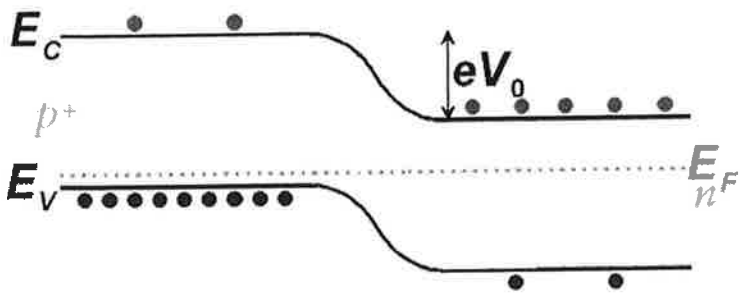
$$= \frac{6.626 \times 10^{-34} \times 1.324 \times 10^{10}}{2\pi . 9.109 \times 10^{-31}}$$

$$\underline{v = 1.53 \times 10^{6} \; ms^{-1}}$$

The wavelength is of a similar order to the interatomic spacing. The velocity is orders of magnitude greater than drift velocities.

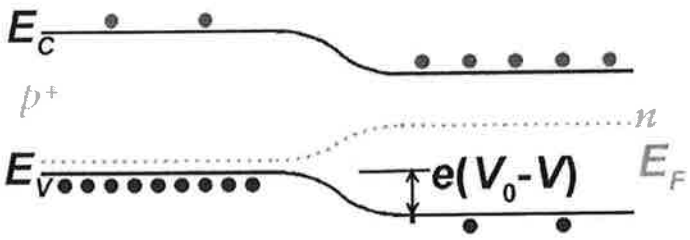
Question 3:

(a) Band diagram for unbiased p^+n junction:



V_0 is built-in potential.

Band diagram for forward biased p^+n junction:



V is externally applied bias.

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(b) $\frac{\partial(\Delta p)}{\partial t} =$ total rate of change in excess hole concentration

$-\frac{\Delta p}{\tau_h} =$ net rate of change of excess holes due to difference in recombination and generation

. $D_h \frac{\partial^2(\Delta p)}{\partial x^2} =$ rate of change due to diffusion of excess holes

In the stated form, the contribution of drift is neglected, which is valid for regions of negligible electrical field.

[15%]

(c) Assumptions:
1) low injection case: the injected minority carrier concentration is much less than the majority carrier concentration
2) no recombination takes place in the depletion region
3) there are negligible fields outside the depletion region.

Under steady state condition and with assumption 3, the continuity equation of (b) becomes

$$D_h \frac{d^2 p}{dx^2} = \frac{p - p_{n0}}{\tau_h}.$$

This differential equation has the general solution

$$p - p_{n0} = A\exp\left(\frac{-x}{L_h}\right) + B\exp\left(\frac{x}{L_h}\right).$$

where $L_h = \sqrt{(D_h \times \tau_h)}$ is the diffusion length of holes.

The minority carrier concentration can only decay away from the depletion edge, hence $B = 0$ and

$$p - p_{n0} = A\exp\left(\frac{-x}{L_h}\right).$$

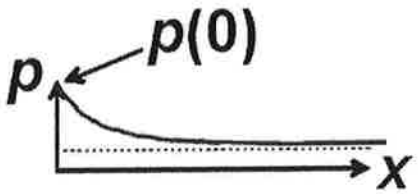
The constant A can be found by using the given boundary condition at $x = 0$,

$$p = p(0) = p_{n0}\exp\left(\frac{eV}{kT}\right),$$

this gives the solution

$$\underline{p - p_{n0} = p_{n0}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_h}\right).}$$

Sketch (dashed line corresponds to p_{n0}):



[40%]

(d) The ideality factor typically is larger than 1 due to recombination within the depletion zone, which is excluded in assumptions for derivation of ideal Shockley equation. Further, generation in depletion region causes increase in reverse bias current, the resistance of regions outside the depletion layer limits current under forward bias and a current will also flow under a large reverse bias due to breakdown (Zener or avalanche breakdown). [15%]

(e) The position of the Fermi level in the p-Si can be calculated via:

$$p = N_V \exp\left(\frac{-E_F}{kT}\right)$$

assuming $p=N_A$, ie full dopant ionisation,

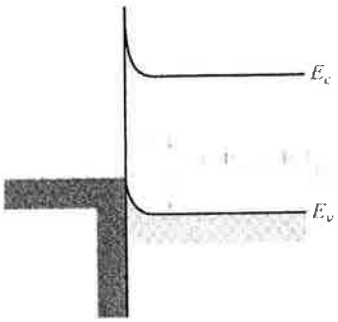
$$E_F = kT \ln\left(\frac{N_V}{N_A}\right) = 0.12eV$$

Hence the work function of the p-Si is

$$e\Phi_{pSi} = e\chi + E_{gap} - E_F = 4.05eV + 1.1eV - 0.12eV = 5.03eV$$

For an ohmic contact the work function of the metal should be larger than that.

Band diagram:



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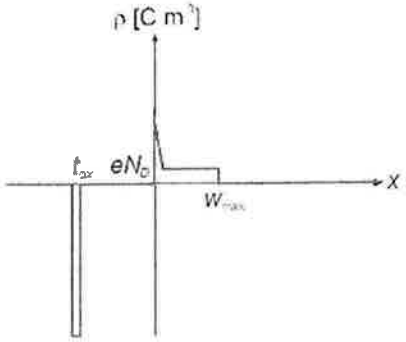
Question 4:

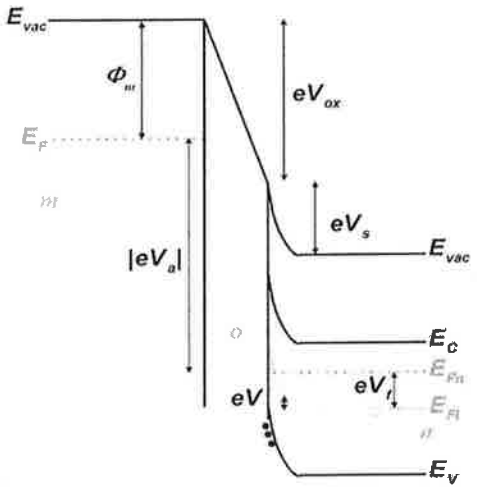
(a) (i) For a strong enough negative bias on the metal (with respect to the n-type Si) the band bending in the n-type Si will cause the Fermi level to lie below midgap at the oxide interface. This corresponds to a p-type interface layer and is referred to as inversion.

At strong inversion the Fermi level at the interface is as close to the valence band as it is to the conduction band in the bulk (assuming that $N_C = N_V$).

(ii) Distribution of charge at strong inversion:



(iii) Band diagram at strong inversion:



[20%]

(b) At strong inversion: p(interface)=n(bulk)=N_D (assuming all dopants are ionized)

Hence $$\sigma_{inv} = p_{\text{int}erface} \times e \times \mu_{FE} = 3.2 \frac{1}{\Omega cm}$$ [10%]

(c) Surface potential at onset of strong inversion:

$$V_s(inv) = \frac{2(E_F - E_i)}{e} = 2V_f \text{ (see band diagram)}$$

in general: $$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

assume all dopants are ionised: $N_D = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

thus $E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$

and $V_s(\text{inv}) = 2V_f = \frac{2kT}{e} \ln\left(\frac{N_D}{n_i}\right) = 0.8V$

Charge per unit area in the depletion region at strong inversion is

$$Q_d = eN_D w_{\max} = 1.6 \times 10^{-3} \frac{C}{m^2}$$

where w_{max} is the depletion width at strong inversion.

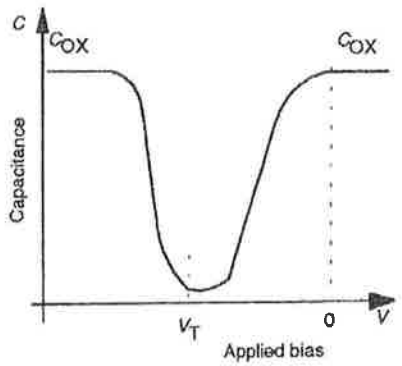
The threshold voltage V_T is dropped across both the oxide and the depletion region, and so if the oxide capacitance per unit area is C_{ox}, with

$$C_{Ox} = \frac{\varepsilon_0 \varepsilon_r}{d_{Ox}} = \frac{3.9 \times 8.85 \times 10^{-12}}{10 \times 10^{-9}} \frac{F}{m^2} = 3.5 \times 10^{-3} \frac{F}{m^2}$$

then $V_T = -\frac{Q_d}{C_{ox}} - V_s = -1.26V$

[35%]

(d) C-V characteristics:



High frequencies (more rapid variation than the time required for inversion to occur) lead to deep depletion. Hence, the capacitance for |voltages| >$|V_T|$ (V_T negative here) does not increase again and remains below C_{min} .

[25%]

(e) The threshold voltage will shift by the flat band voltage

$$V_{FB} = \frac{\phi_m - \phi_{sc}}{e} = -1V$$

[10%]