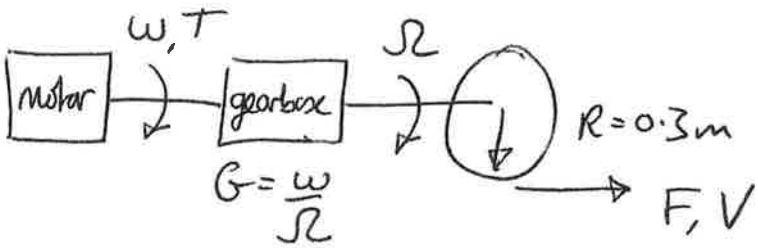
3C8 Machine Design - 2012

Solutions - by D J Cole

1\. a)



$V = \Omega R \quad \therefore \quad V = \dfrac{\omega R}{G}$

conservation of power $\quad F.V = \omega . T$

$$\therefore \quad F = \frac{\omega T}{V} = \frac{G T}{R}$$

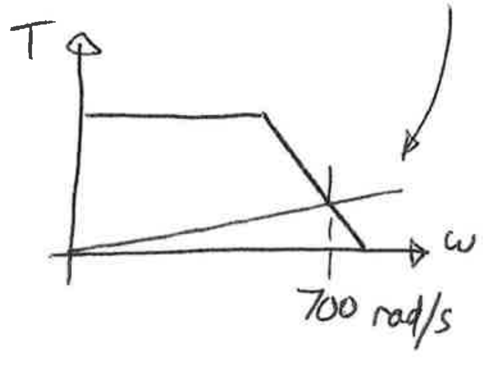
from question $\quad F = CV$

$$\frac{GT}{R} = C \omega \frac{R}{G}$$

$$T = \omega C \left(\frac{R}{G}\right)^2$$

$$T = \omega . \frac{200}{7} \left(\frac{0.3}{6}\right)^2$$

$$T = \frac{\omega}{14}$$



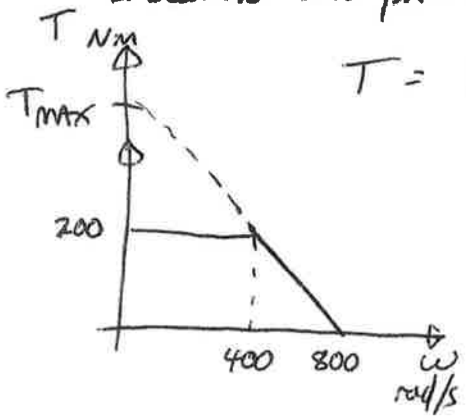
Hence max ω is 700 rad/s

$$\therefore \quad V = \frac{\omega R}{G} = 700 . \frac{0.3}{6} = \underline{\underline{35 m/s}}$$

b) Max. vehicle speed arises when operating point is at maximum power.

Power increases with ω along the flat part of the T-ω curve. Hence max power must be somewhere on the sloping part of the curve.

Idealise this part of the curve as:



$$T = 2T_{MAX} - m\omega \quad \text{where } m = \frac{2T_{MAX}}{800}$$

$$T = 2T_0 - \frac{2T_0}{800}.\omega$$

$$T = 2T_0\left(1 - \frac{\omega}{800}\right)$$

power $P = T\omega = 2T_0\left(\omega - \frac{\omega^2}{800}\right)$

$$\frac{dP}{d\omega} = 2T_0\left(1 - \frac{2\omega}{800}\right)$$

max P when $\frac{dP}{d\omega} = 0$. $\therefore \frac{2\omega}{800} = 1$

$$\omega = 400 \text{ rad/s}$$

$\therefore P_{MAX} = 400 \text{ rad/s} \times 200 \text{ Nm} = 80 \text{kW}$

but $F = CV \quad \therefore P = F.V = CV^2 = 80.10^3$

$$V^2 = \frac{80.10^3}{200/7}$$

$$V = \underline{\underline{52.9 \text{ m/s}}}$$

for speed ratio

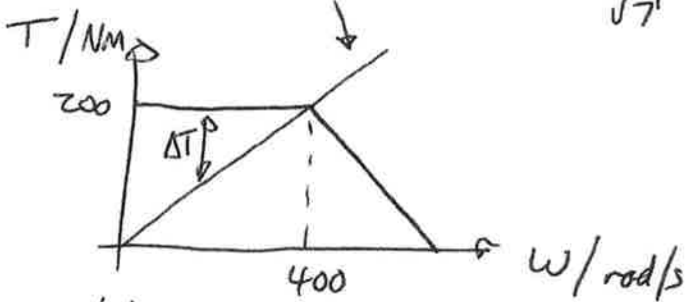
$$V = \frac{\omega R}{G} = \sqrt{\frac{80.10^3}{200/7}} \quad \text{(from preceding lines)}$$

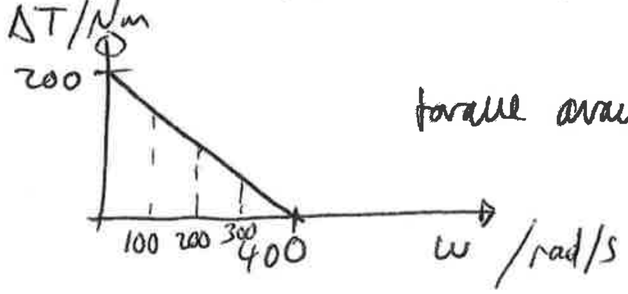
$\omega = 400$ rad/s $\quad \therefore \; 400 . \frac{R}{G} = 20\sqrt{7}$

$R = 0.3 \qquad G = \frac{400 . 0.3}{20\sqrt{7}}$

$$\underline{\underline{G = \frac{6}{\sqrt{7}}}}$$

c) load characteristic when $G = \frac{6}{\sqrt{7}}$:





torque available for acceleration

convert to vehicle acceleration and vehicle speed

$$\text{accn} = \frac{F}{\text{mass}} = \frac{G}{R}\frac{\Delta T}{m} = \frac{6}{\sqrt{7}.0.3.1000} . \Delta T$$

$$= 7.56 . 10^{-3} \, \Delta T$$

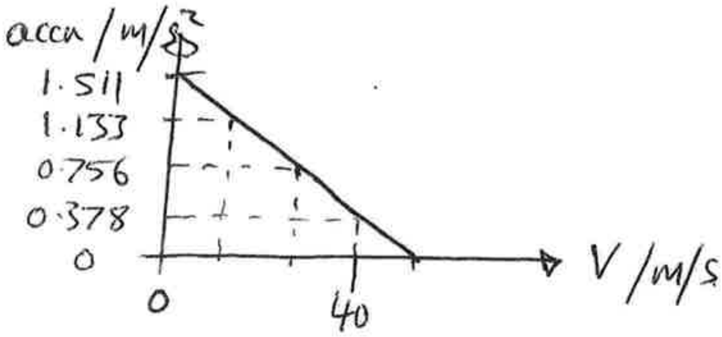
from (b) $\omega = 400$ rad/s is 52.92 m/s

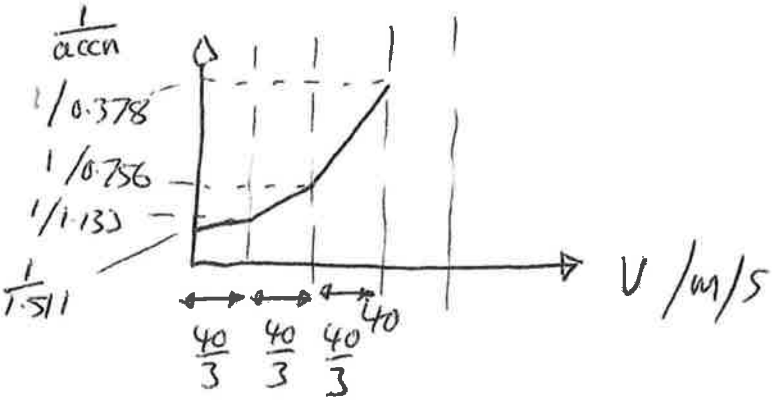
$\therefore$ 40 m/s is $\frac{40}{52.9} \times 400$ rad/s

$= 302$ rad/s

say 300.

hence:





$$\text{accn} = \frac{dV}{dt}$$

$$dt = \frac{1}{a}\,dV$$

$$t = \int_0^{V_{final}} \frac{1}{a}\,dV$$

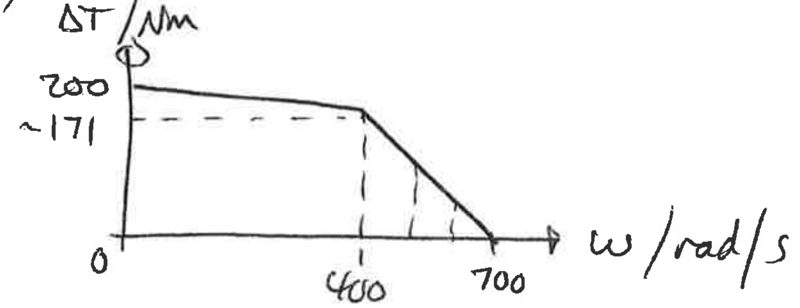
trapezoidal integration

$$t \simeq \Delta V \left(\frac{1}{2a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{2a_4} \right)$$

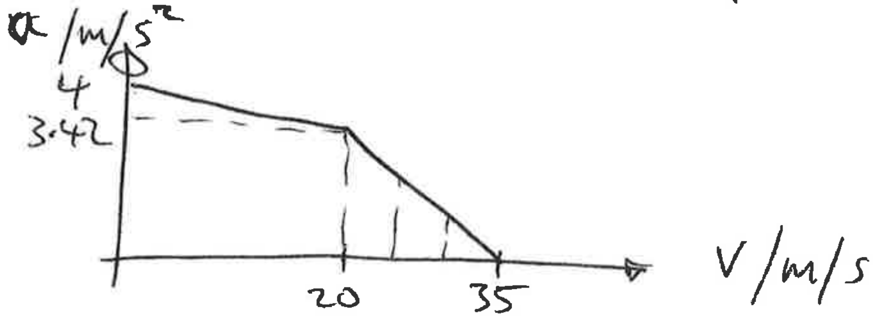
$$t = \frac{40}{3}\left(\frac{1}{2(1.511)} + \frac{1}{1.133} + \frac{1}{0.756} + \frac{1}{2(0.378)}\right)$$

$$\underline{\underline{t = 51.5\,\text{s}}} \qquad \text{(exact integration gives } 49.4\,\text{s)}$$

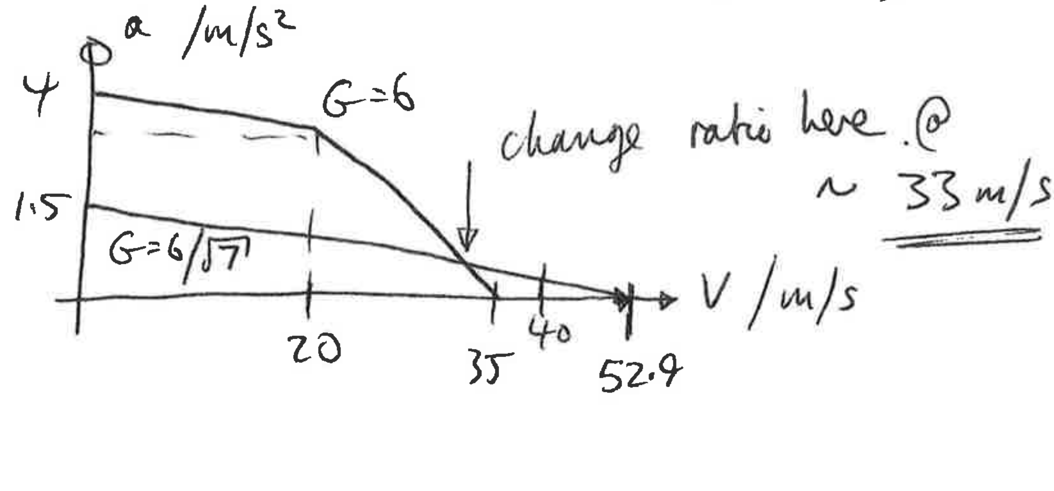
d) For $G = 6$, use load characteristic from part (a)



convert to vehicle accn and speed.



superimpose on results from part (c)

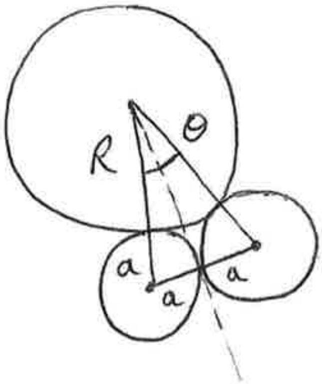


Ratio is changed at 33 m/s and
not 35 m/s (max speed for G=6) because
acceleration is greater for G = 6/√7 above
about 33 m/s.

2 a) Hertz idealizations:

smooth surfaces
small strains
linear elasticity
negligible surface shear stresses

b) i)



$N=8 \quad \therefore \theta = \frac{2\pi}{8} = \frac{\pi}{4}$

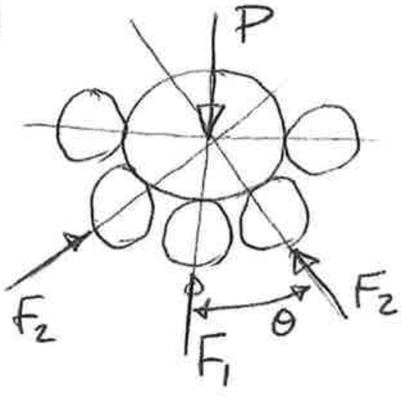
$$\sin\frac{\pi}{8} = \frac{a}{R+a} = 0.38$$

$$a = 0.38(R+a)$$

$$(1-0.38)\,a = 0.38R$$

$$\underline{\underline{a = 0.62R}}$$

ii)



$F_2 = F_1 \cos\theta$

vertical equilibrium

$$P = F_1 + 2(F_1\cos\theta)\cos\theta$$

$$= F_1(1+2\cos^2\theta)$$

$$P = F_1\left(1+2\cos^2\frac{\pi}{4}\right)$$

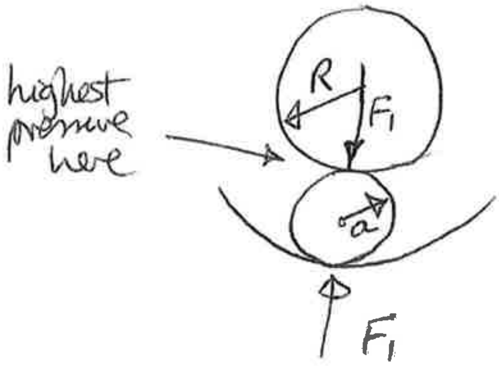
$$\underline{\underline{P = 2F_1}}$$

If $N = 8$, $P = 0.25 F_1 N$

$= 0.25 F_1 . 8$

$$\underline{\underline{P = 2F_1}}$$

iii) Max. Hertz pressure



$a = 0.62R$

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{0.62R}$$

$$= \frac{1}{R}\left(1 + \frac{1}{0.62}\right)$$

$$R_{eq} = R . \frac{0.62}{1.62} = R . 0.383$$

line contact

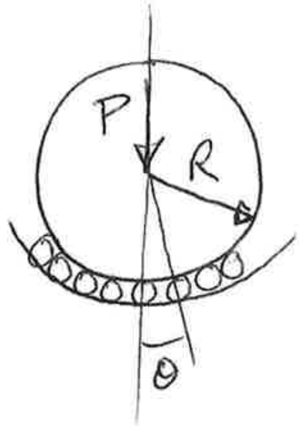
$$p_0 = \left\{\frac{P' E^*}{\pi R_{eq}}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{F_1/L \quad E^*}{\pi R . 0.383}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{\frac{P}{2L} \quad E^*}{\pi R \, 0.383}\right\}^{\frac{1}{2}} = \sqrt{\frac{1}{2\pi 0.383}} \sqrt{\frac{PE^*}{LR}}$$

$$\underline{\underline{p_0 = 0.645 \sqrt{\frac{PE^*}{LR}}}}$$

iv)



N large
$a \ll R$

consider radius of roller

$$\sin\frac{\theta}{2} = \frac{a}{R+a} \quad \text{(from part (b)(i))}$$

small angle: $\frac{\theta}{2} = \frac{a}{R+a}$

$a \ll R$: $\frac{\theta}{2} = \frac{a}{R}$

$\therefore \quad \underline{a = \frac{R\theta}{2}}$

consider equivalent contact radius

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{a}$$

$a \ll R \therefore \quad \frac{1}{R_{eq}} \simeq \frac{1}{a}$

$$\underline{R_{eq} \simeq a \simeq \frac{R\theta}{2}.}$$

consider Hertz pressure

$$p_0 = \left\{ \frac{\frac{F_1}{L} \cdot E^*}{\pi R_{eq}} \right\}^{\frac{1}{2}}$$

where $F_1 = \frac{P}{N} . 4$ (from (b)(ii))

so $$p_0 = \left(\frac{\frac{4P}{NL} \quad E^*}{\pi \frac{R\theta}{2}} \right)^{\frac{1}{2}}$$

now $N\theta = 2\pi \rightarrow \theta = \frac{2\pi}{N}$

$$\therefore \quad p_0 = \left\{ \frac{\frac{4P}{\cancel{N}L} \quad E^*}{\pi R \frac{2\pi}{2\cancel{N}}} \right\}^{\frac{1}{2}}$$

$$= \sqrt{\frac{4}{\pi^2}} \sqrt{\frac{PE^*}{LR}}$$

$$\underline{\underline{p_0 = 0.6366 \sqrt{\frac{PE^*}{LR}}}}$$

which is approximately 1.3% less than when $N = 8$.

3.

a)

	Radial Load	Axial Load	Stiffness	Axial Displacement
Deep groove ball	+	+	+	– –
Dbl row ang contact ball	++	++	+	– –
Cylindrical roller	+++	– –	++	+++
Taper roller	+++	+++ (one dirn only)	++	– –

b) i) For moment equilibrium about rotation axis of crank:

$$T.\ 90 = F.\ 180\,|\sin\theta|$$

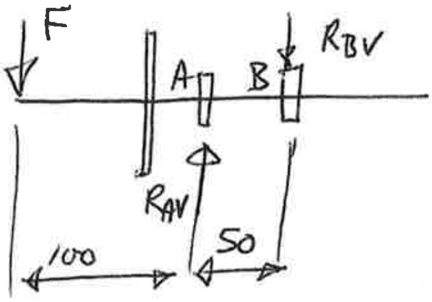
F is const, hence T is max when $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

Bearing radial forces are the resultant of forces arising from F and T, hence radial force is max when T is max.

$$T_{MAX} = F.\ \frac{180}{90} = 2F$$

ii) Consider F.B.D of crank in front elevation

$\theta = \frac{\pi}{2}$



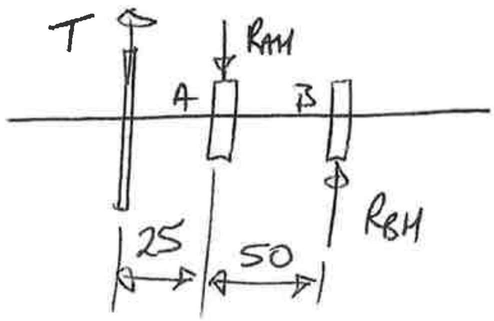
Mts about B $\quad 50 . R_{AV} = F . 150$

$\underline{R_{AV} = 3F \quad \therefore \quad R_{BV} = 2F.}$

When $\theta = \frac{3\pi}{2}$, F applied to other pedal, so by inspection

$\underline{R_{AV} = -2F \quad , R_{BV} = -3F}$

F.B.D in plan view.
(note, independent of whether $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$)



Mts about B. $\quad T . 75 = R_{AH} . 50$

note $T = T_{MAX} = 2F$ $\quad \underline{R_{AH}} = 2F . \frac{75}{50} = \underline{3F}$
(see part (i))

$\underline{R_{BH} = F.}$

Resultant Forces. – side elevation shown.

Bearing A, $\theta = \frac{\pi}{2}$

$R_{AH} = 3F$

$R_{AV} = 3F$

resultant $R_A = 3\sqrt{2}\,F$

Bearing A, $\theta = \frac{3\pi}{2}$

$R_{AH} = 3F$

$R_{AV} = -2F.$

$R_A = \sqrt{13}\,F$

Maxima

$$R_A = 3\sqrt{2}\,F, \quad \theta = \frac{\pi}{2}$$

Bearing B, $\theta = \frac{\pi}{2}$

$R_{BV} = 2F$

$R_{BH} = F$

$R_B = \sqrt{5}\,F.$

Bearing B, $\theta = \frac{3\pi}{2}$

$R_{BH} = F.$

$R_{BV} = -3F$

$R_B = \sqrt{10}\,F$

$$R_B = \sqrt{10}\,F, \quad \theta = \frac{3\pi}{2}$$

iii) $F = 100N$, so largest bearing force is $3\sqrt{2}!.100 = 424.3N$

wheel $\phi = 0.7m$, travel = 20,000km

number of wheel revs = $\dfrac{20.10^6}{\pi.0.7} = 9.1.10^6$

number of crank revs = $\dfrac{9.1.10^6}{3} \sim 3.10^6$

$\therefore L = 3$

life formula (data sheet) $L = a_1 a_{23} \left(\frac{C}{P}\right)^p$

$P = 424.3N$

$a_1 = 0.21$ (1% reliability)

$a_{23} = 1$ (ideal lubrication)

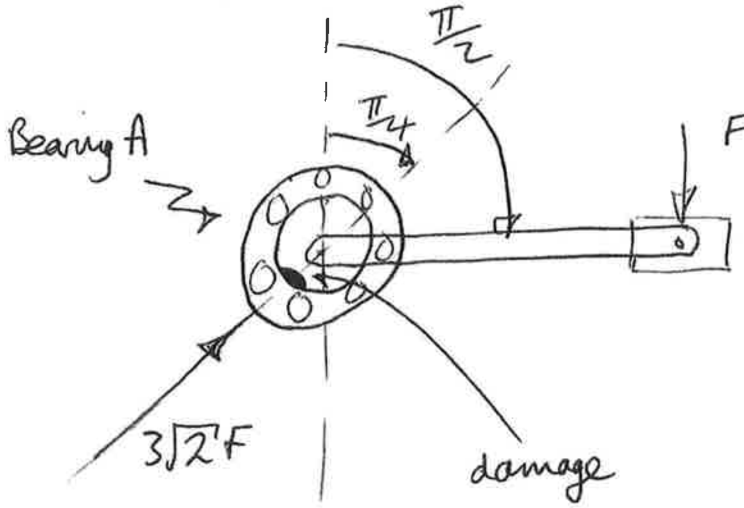
$p = 3$ (ball bearing)

$$3 = 0.21\left(\frac{C}{424.3}\right)^3$$

$$C = \left(\frac{3}{0.21}\right)^{\frac{1}{3}} 424.3$$

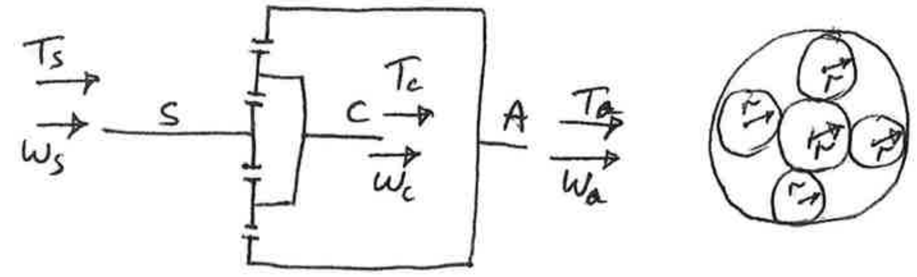
$$\underline{\underline{C = 1029\ N}}$$

iv) Max bearing force is on bearing A when $\Theta = \frac{\pi}{2}$
(see part ii)

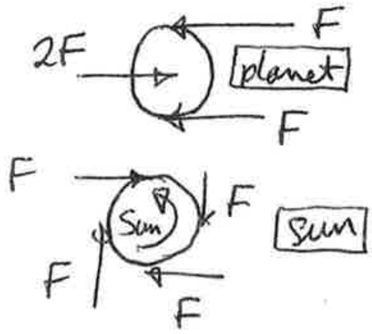


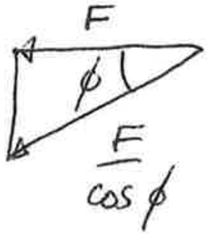
Damage likely to occur first on inner track of bearing A at location shown. The contact stress is highest here. Although the inner track is rotating, damage is not uniformly distributed around the inner track because the bearing force is also varying in magnitude and direction.

4. a) i)



Free body diagram of sun and planet:





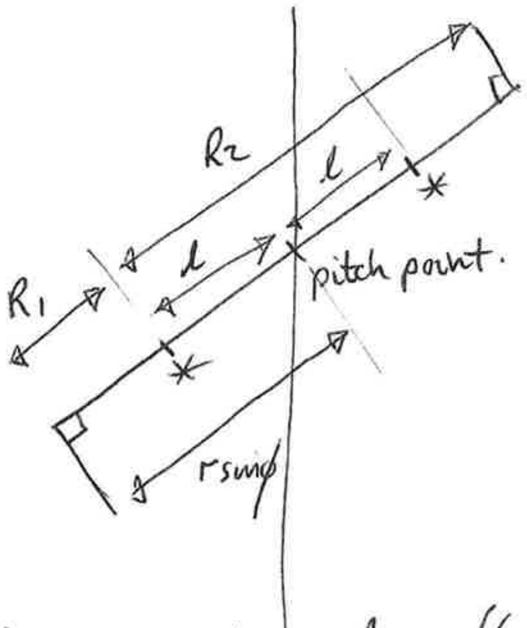
$$T = 4Fr$$

$$\therefore F = \frac{T}{4r}$$

$$p' = \frac{F}{w\cos\phi} = \underline{\underline{\frac{T}{4rw\cos\phi}}}$$ at all eight pressure lines

ii) Max Hertz pressure occurs on the sun / planet pressure line because there is non-conformal contact here. The contact between planet and annulus is conformal.

Max Hertz pressure will be at the addendum circle ✱:



$N = 15$

pitch circle radius

$r = \frac{mN}{2} = 7.5\text{m}$

$r\sin\phi = 7.5\text{m} \sin 20°$

$r\sin\phi = 2.565\text{m}$

from data sheet $l = m\left((0.0292 + N^2 + N + 1)^{\frac{1}{2}} - 0.171N\right)$

$l = 2.187\text{m}$

$\therefore R_1 = r\sin\phi - l = 0.378\text{m}$

$R_2 = r\sin\phi + l = 4.752\text{m}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.378m} + \frac{1}{4.752m}$$

$$\therefore R_{eq} = 0.350m = 0.350\frac{r}{7.5}$$

$$p_0 = \sqrt{\frac{P' E^*}{\pi R_{eq}}}$$

$$= \sqrt{\frac{T}{4rw\cos 20} \frac{E^*}{\pi 0.350 \frac{r}{7.5}}}$$

$$\underline{\underline{p_0 = 1.35\sqrt{\frac{TE^*}{wr^2}}}}$$

b) i) $\omega_s = (1+R)\omega_c - R\omega_a$ (data sheet)

$$R = \frac{A}{S} = \frac{45}{15} = 3$$

$$\therefore \omega_s = 4\omega_c - 3\omega_a$$

carrier is fixed, $\omega_c = 0 \therefore \omega_s = -3\omega_a$

power $T_s\omega_s + T_a\omega_a = 0$ (assume +ve power is input)

$$T_s 3\omega_a = T_a\omega_a$$

$T_s = T \quad \therefore \underline{\underline{T_a = 3T}}$

$T_s + T_C + T_A = 0 \quad \therefore \underline{\underline{T_c = -4T}}$

ii) power: $T_s\omega_s + T_a\omega_a + T_c\omega_c = 0$

$\omega_c = 0 \qquad \underset{(\text{input})}{T_s\omega_s} = \underset{(\text{output})}{-T_a\omega_a}$

consider efficiency

$$T_s\omega_s \,.\, 0.95 = -T_a\omega_a$$

$T_s = T$ and $\omega_s = -3\omega_a$

so $\quad -T \,.\, 3 \,.\, 0.95 = -T_a$

$$T_a = 2.85\,T$$

$T_s + T_c + T_A = 0 \qquad \therefore \; T_c = -3.85\,T$

iii) consider 100% efficiency initially to establish direction of power at each shaft.

speed equation $\qquad \omega_s = 4\omega_c - 3\omega_a$

but $\omega_c = -\dfrac{\omega_s}{10} \qquad \omega_s = -\dfrac{4\omega_s}{10} - 3\omega_a$

$$\omega_a = -\frac{1.4}{3}\,\omega_s$$

torque equilibrium

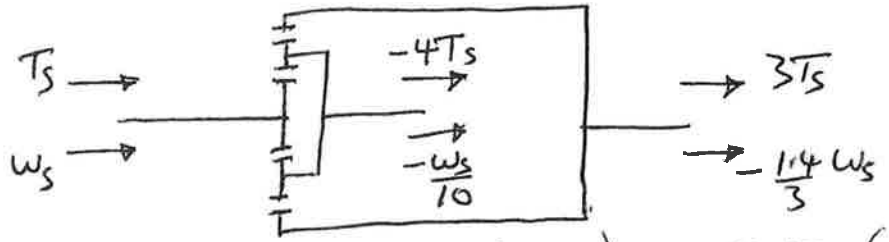
$$T_s + T_c + T_a = 0$$

but $T_a = 3T_s \qquad$ (part (b)(i))

$$T_s + T_c + 3T_s = 0$$

$$T_c = -4T_s$$

Summary, to establish power directions:



power in ($T.\omega$ is +ve) power in ($T.\omega$ is +ve) power out ($T.\omega$ is -ve)

now consider 95% efficiency

$$\underbrace{(T_s\omega_s + T_c\omega_c)}_{\text{power in}}0.95 + \underbrace{T_a\omega_a}_{\text{power out}} = 0$$

speeds are as before, find T_a and T_c

$$\left(T.\omega_s - T_c\frac{\omega_s}{10}\right)0.95 - \frac{1.4\omega_s}{3}T_a = 0$$

$$0.95T = T_c\frac{0.95}{10} + T_a\frac{1.4}{3}$$

equilibrium of torques:

$$T + T_c + T_a = 0$$

$$\therefore\ T_c = -T_a - T$$

hence $\quad 0.95T = (-T_a - T)\dfrac{0.95}{10} + T_a\dfrac{1.4}{3}$

$$T_a = 2.81T$$

hence $\quad \underline{\underline{T_c = -3.81T}}$

3C8 2012 Examiner's comments

Q1 Power matching
A popular question, but often poorly answered. A number of candidates didn't know how to relate wheel speed in rad/s to vehicle speed in m/s. In part (b), maximum vehicle speed was often incorrectly assumed to occur at the maximum motor speed of 800 rad/s (at which speed the motor power is zero). In part (c) many candidates replaced $F=CV$ with $F=ma$, instead of $F=CV+ma$.

Q2 Contact mechanics
Another popular question. In part (b)(i) the distance $2a$ between centres of adjacent rollers was often incorrectly assumed to lie on an arc of the pitch circle instead of a chord. In part (b)(iii) some candidates analysed the conformal contact between the roller and the outer track, instead of the contact between the roller and inner track. There were very few correct answers to the last part of the question.

Q3 Rolling element bearings
The first parts of the question involved static equilibrium of forces and were usually answered satisfactorily. In part (b)(iii) common errors were to omit the speed ratio between the crank and the wheel, and to forget that the bearing life equation gives life in units of 10^6 revolutions. Descriptions and sketches of the damage location usually lacked detail.

Q4 Epicyclic gear
In part (a)(i) a common error was not to divide by 4 (the number of planet wheels). In part (a)(ii) candidates often failed to account for the absence of double contacts when determining the location of maximum pressure. Calculation of torques in part (b) was generally okay, although in part (iii) few candidates checked the direction of power at the carrier.

3C8 2012 Answers

1	(a)	35 m/s		
	(b)	$G = 6/\sqrt{7}$ maximum speed 52.9 m/s		
	(c)	49.4 s		
	(d)	approximately 33 m/s		
2	(b)	(ii)	$P = 2F_1$	
		(iv)	1.3% reduction	
3	(b)	(ii)	$\theta = \pi/2$ for bearing A, $\theta = 3\pi/2$ for bearing B.	
		(iii)	$C = 1029$ N	
4	(a)	(i)	$P' = \dfrac{T}{4rw\cos\phi}$ at all eight pressure lines	

(ii) $p_0 = 1.35\sqrt{\dfrac{TE^*}{wr^2}}$ at a single contact on the sun/planet pressure line at the addendum circle.

(b)	(i)	$T_a = 3T$	$T_c = -4T$
	(ii)	$T_a = 2.85T$	$T_c = -3.85T$
	(iii)	$T_a = 2.81T$	$T_c = -3.81T$