# Engineering Tripos Part IIA
# 3D1 Geotechnical Engineering I
# 2012 Solutions

M D Bolton

1 (a) Assume zero friction on the sides of the core.

Vertical stress at the base is $10 \times 16 = 160$ kPa.

Ultimate pore pressure at base is $10 \times 10 = 100$ kPa

$\therefore \Delta\sigma_v'$ at base $= 60$ kPa

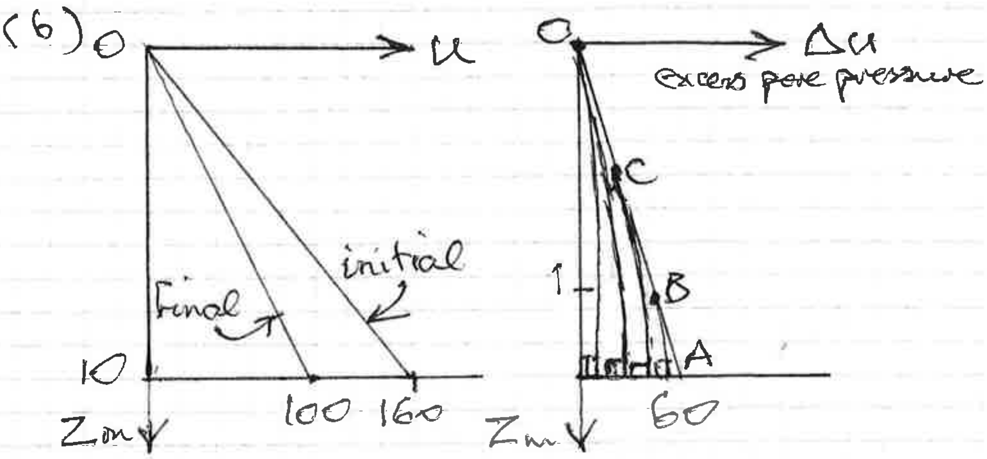
So <u>average</u> $\sigma_v'$ in the core increases $0 \rightarrow 30$ kPa

But average $E_0' = 800$ kPa

$\therefore \; \varepsilon_v = 30/800 = 3.75\%$

Ultimate settlement $= 3.75\% \times 10\text{m} = \underline{0.375\text{m}}$

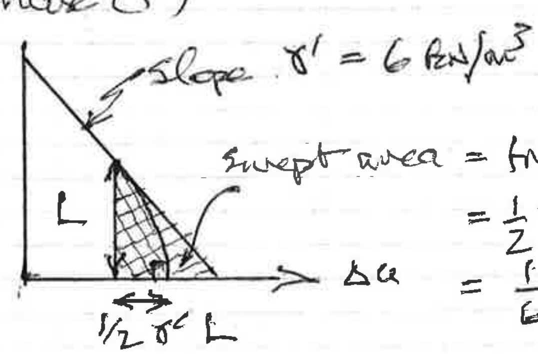
(b)




phase(i) : parabolas with apexes at base, tangential to initial triangle at points $A \rightarrow B \rightarrow C \rightarrow O$

phase(ii) : parabolas through O, diminishing at the base, reducing to zero.

phase (i)




swept area = triangle − parabola

$$= \frac{1}{2}\gamma' L^2 - \frac{2}{3}\frac{1}{2}\gamma' L^2$$

$$= \frac{1}{6}\gamma' L^2$$

1(b) cont.

So settlement $\rho = \frac{1}{6}\frac{\gamma' L^2}{E_0'}$ ①

But rate of settlement equals rate of upward seepage. So!

$$\frac{d\rho}{dt} = \frac{k\gamma'}{\gamma_w}$$ ②

Differentiating ① and equating to ②:

$$\frac{1}{6}\gamma' \frac{2L}{E_0'}\frac{dL}{dt} = \frac{k\gamma'}{\gamma_w}$$

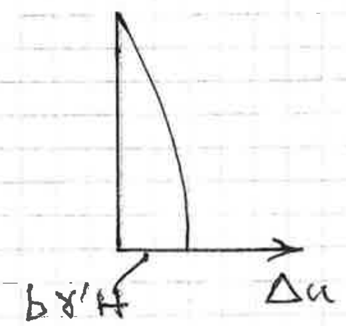
Integrating again:

$$\left.\begin{aligned} L^2 &= 6 C_v t \\ \text{where } C_v &= \frac{E_0' k}{\gamma_w} \end{aligned}\right\}$$ ③

At the end of phase (i) $L = H = 10\text{m}$

$$\text{so } t_{(i)} = \frac{H^2}{6C_v}$$

phase (ii)




Let base excess pwp be $b\gamma' H$

Then exit gradient will be:

$$\frac{b\gamma' H}{H/2} = 2b\gamma'$$

$$\rho = \frac{1}{E_0}\left[\frac{1}{2}\gamma' H^2 - \frac{2}{3}\gamma' b H^2\right]$$ ④

$$\frac{d\rho}{dt} = \frac{k}{\gamma_w}.2b\gamma'$$

$$\therefore \frac{k}{\gamma_w} 2b\gamma' = -\frac{2}{3}\frac{\gamma' H^2}{E_0'}\frac{db}{dt}$$

1(b) cont.

$$\int_{0.5}^{b} \frac{db}{b} = -\frac{3C_v}{H^2}\left[t - t_{(i)}\right]$$

$$\therefore \ln 2b = \frac{1}{2} - 3T_v$$ (5)

(c) (i) $C_v = \frac{E_0' k}{\gamma_w} = \frac{800 \times 10^{-8}}{10} = 8 \times 10^{-7}\, m^2/s$

At 6 months, $t = \frac{1}{2} \times 365 \times 24 \times 3600 = 1.6 \times 10^7\, s$

So $T_v \simeq 0.126 < 0.167$

So this is in phase (i) and $L = \sqrt{6 \times 8 \times 1.6}$

So $L = 8.8\, m$

And $\rho = \frac{1}{6} \times \frac{6 \times 8.8^2}{800} = 0.097\, m$

(ii) If $\frac{2}{3} b \gamma' H^2 = 0.1 \frac{1}{2} \gamma' H^2$

Then $b = 0.075$

So $T_v = \frac{1}{3}\left[0.5 - \ln 0.15\right] = 0.80$

and $t_{90\%} = \frac{0.80 \times 10^2}{8 \times 10^{-7}} = 10^8\, s \doteq 3.1\, y$

(d) As the core consolidates it will "hang up" on the sides of the trench, reducing stresses at depth, thereby tending to reduce settlements and consolidation times. In detail, $\gamma'$, $E_0'$ and $k$ will vary with $\sigma_v'$ changes during the process. Drainage might occur horizontally, also.

1 Feedback

(a) Marks are lost for using the stresses at the base instead of the average stresses at mid-depth. Final pore pressures are hydrostatic below the crest: it is a serious mistake not to take these into account when deriving the change in vertical effective stress.

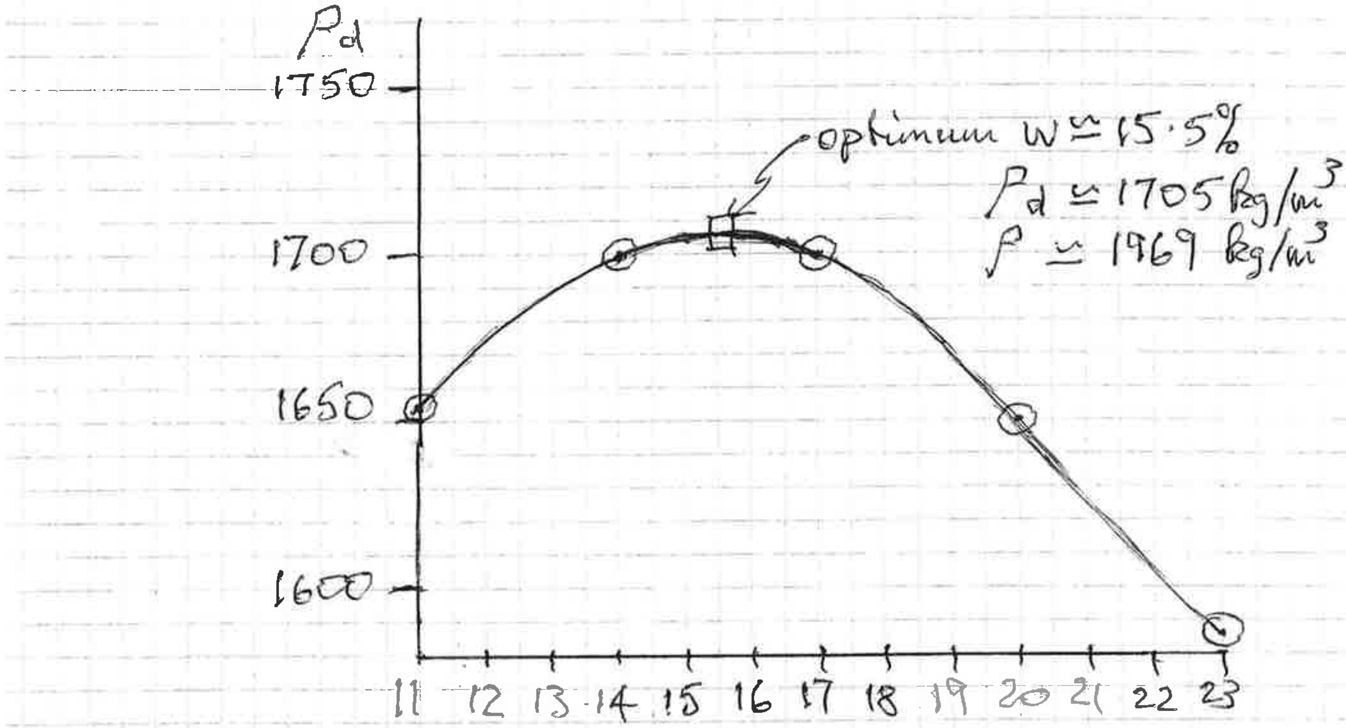
(b) A maximum of 5/8 could be given for thorough bookwork derivations relating to an initially rectangular isochrone. Few candidates correctly pursued the triangle through both phases of transient flow.

(c) Correct use of Databook solutions for an initially rectangular isochrone could attract 4/4 marks in this numerical application.

(d) Few candidates mentioned side friction. Many correctly referred to the need for a layered analysis with soil properties varying with depth. Some mentioned that lateral drainage would occur.

2(a)

| | | | | | |
|---|---|---|---|---|---|
| w % | 11 | 14 | 17 | 20 | 23 |
| $\rho$ kg/m$^3$ | 1835 | 1938 | 1989 | 1980 | 1950 |
| $\rho_d = \rho/(1+w)$ | 1653 | 1700 | 1700 | 1650 | 1585 |




At optimum: $1 + e = 2680/1705$

$\therefore e = 0.57$

And saturation $S_r = \dfrac{wG_s}{e} = 73\%$

(b) The fill will be most stable when wetted if it is a little wetter than optimum, with a higher $S_r$. e.g at $w = 18\%$, $\rho_d = 1690$ kg/m$^3$, $e = 0.59$, $S_r = 82\%$. So air voids are reduced — better for flood embankments.

2(c) Under the crest of the embankment, considering 1D conditions:

$$\Delta\sigma = 5 \times 1969 \times 9.8 = 96.5 \text{ kPa}$$

At 2.5 m in the clay:

$$\sigma'_{v,0} \simeq 2.5 \times 10.2 = 25 \text{ kPa}$$

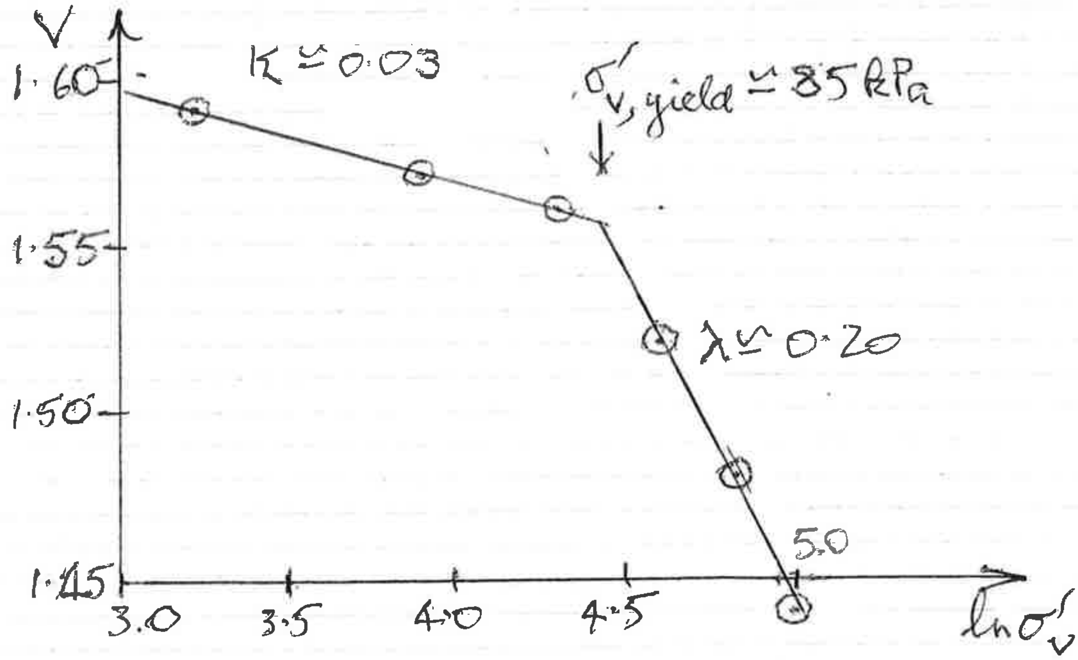
And if $\gamma_0 \simeq 20 \text{ kN/m}^3$, $\gamma_w \simeq 9.8 \text{ kN/m}^3$:

$$V_0 = 1 + e_0 = \frac{(G_s - 1)\gamma_w}{(\gamma_0 - \gamma_w)} = 1.585$$

At later stages of the oedometer test

$$v = V_0 \frac{h}{h_0}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\sigma'_v$ kPa | 25 | 50 | 75 | 100 | 125 | 150 |
| $h$ mm | 20.0 | 19.8 | 19.7 | 19.2 | 18.6 | 18.2 |
| $v$ | 1.59 | 1.57 | 1.56 | 1.52 | 1.48 | 1.44 |
| $\ln \sigma'_v$ | 3.22 | 3.91 | 4.32 | 4.61 | 4.83 | 5.01 |



2(d) – If we use the sample as representative, and take $\sigma_v'$ from 25 kPa to 122 kPa, we get to $\ln \sigma_v' = 4.80$, $v = 1.49$. So treating 10m of clay like an oedometer

$$\frac{H}{H_0} = \frac{1.49}{1.59}$$

$$\therefore H = 9.37\,m$$

So settlement $\Delta H \simeq 0.63$ m

(e) If $\sigma_v'$ falls by 10 kPa, and neglecting any change in saturation of the embankment:

$$\Delta v = \kappa \ln \frac{122}{112} \simeq 2.6 \times 10^{-3}$$

So $\Delta \rho = \dfrac{9.37 \times 2.6 \times 10^{-3}}{1.49} \simeq 16$ mm

The embankment might heave 16mm

2. Feedback

(a) A few candidates forgot the whole point of optimum water content, and associated it with maximum bulk density instead of dry density.

(b) Well - remembered.

(c) This is an unusual case, where taking $\gamma_w = 10$ kN/m$^3$ instead of 9.8 kN/m$^3$ gives significantly different answers (but without losing marks).

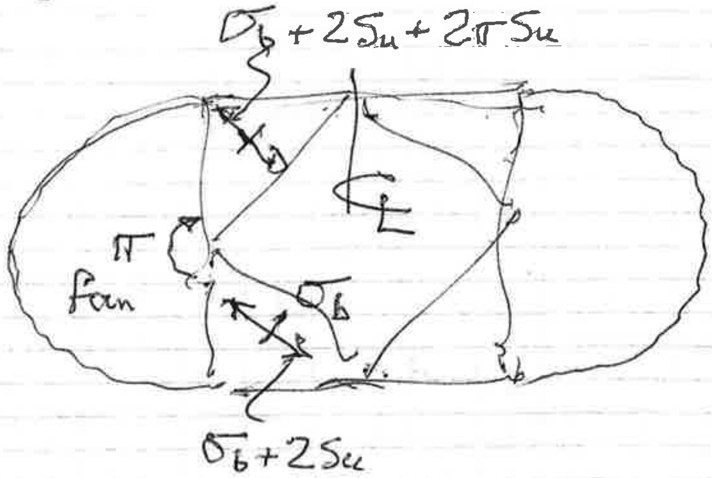
(d) Some candidates went to equations instead of reading V of their graph, and made mistakes.

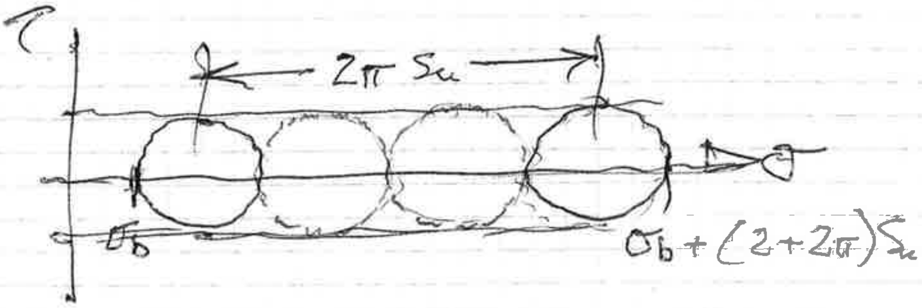
Candidates were not expected to take other representative depths.

(e) Many candidates failed to make any calculation, but got the idea right.

3 (a)

(i)



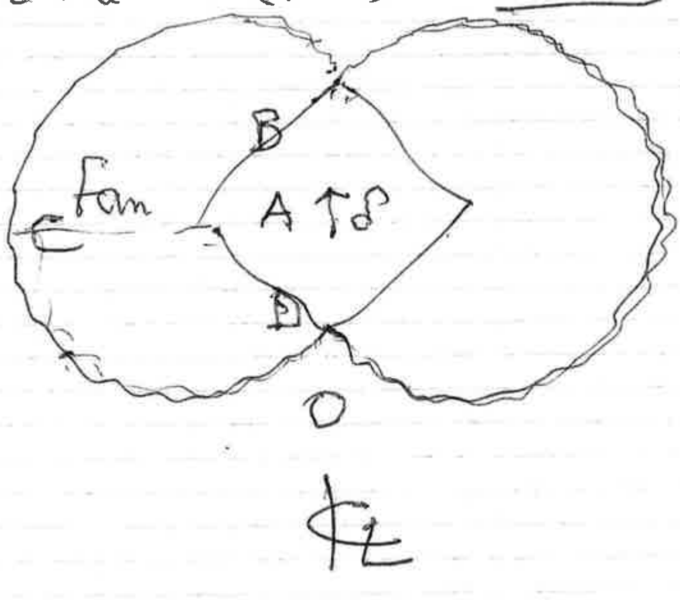

Resolving forces vertically:

$$T_f = A\left[\sigma_b + S_u(2+2\pi) - \sigma_b\right]$$

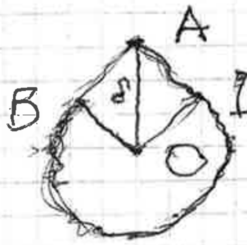
or $T_f = A\, S_u N_a$

where $N_a = 2(1+\pi) = \underline{8.28}$

(ii)



hodograph for half-anchor




Equating work and dissipation:

$$\frac{1}{2} T_f . \delta = \frac{s_u A}{\sqrt{2}} \frac{\delta}{\sqrt{2}} . 2 + 2 . s_u \frac{\delta}{\sqrt{2}} . \frac{3\pi}{2} \frac{\delta}{\sqrt{2}}$$

$\therefore T_f = A\, s_u N_a$

where $N_a = 2 + 3\pi = \underline{11 \cdot 42}$

(b) A circular anchor body mobilizes additional strength at its ends. This is analagous to a circular footing compared with a strip footing, where $N_c$ increases by factor 6.05 / 5.14 for a rough footing. A shape factor of 1.18 is therefore involved.

Applied to (a ii) this gives $N_a = 1.18 \times 11 \cdot 42$
So an upper bound should be $\underline{N_a = 13 \cdot 4}$

It is less clear by what factor the lower bound in (a i) should be increased. This would require an axisymmetric stress field.

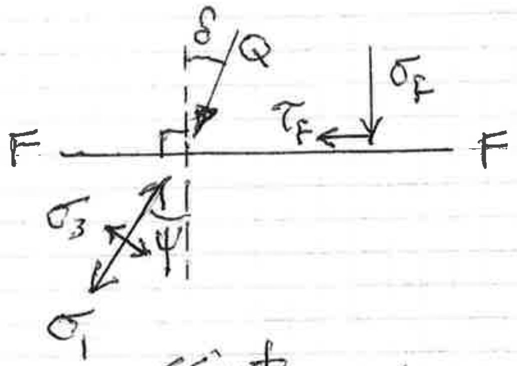
3 Feedback

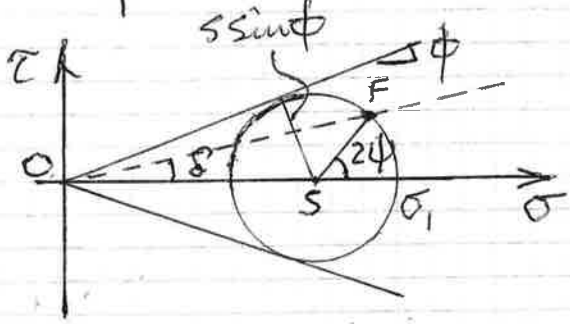
(ai) Most candidates failed to clarify that the principal stresses in the 45° triangles are parallel/perpendicular to the faces of the anchor. But they at least got credit for recognising a plastic fan, rotating π radians.

(aii) Many candidates made a poor attempt of a hodograph, but wrote down some reasonable work equation.

(b) Many recognised the shape factor of 1.18 from Lockings.

4 (a)




$$\sigma_f = S + S\sin\phi\cos 2\psi$$

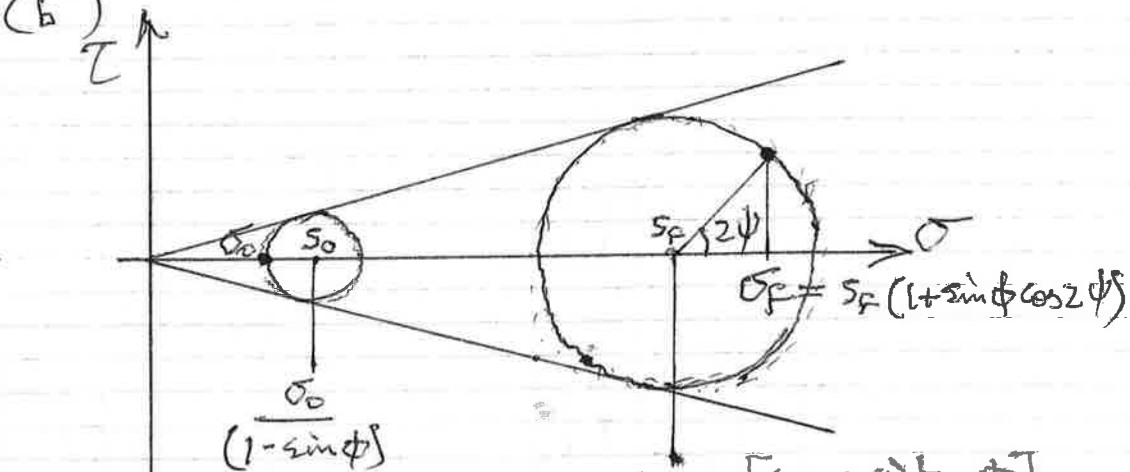
$$\tau_f = S\sin\phi\sin 2\psi$$

And for triangle OFS:

$$\frac{S\sin\phi}{\sin\delta} = \frac{S}{\sin(2\psi-\delta)}$$

$$\therefore \sin(2\psi-\delta) = \frac{\sin\delta}{\sin\phi}$$

(b)




$$S_f = S_0\exp[(\pi-2\psi)\tan\phi]$$

$$\text{So}\quad \frac{\sigma_f}{\sigma_0} = \frac{(1+\sin\phi\cos 2\psi)}{(1-\sin\phi)}\exp[(\pi-2\psi)\tan\phi]$$

4 (c) Using (a) with $\delta = 20°$, $\phi = 35°$
we get: $\psi = 28.3°$

Putting this in (b) we get:

$$\frac{\sigma_F}{\sigma_0} = \frac{1.315}{0.426} \times \exp(1.508) = 14.0$$

Now compare with $\delta = 0$, $\phi = 35°$

$$\frac{\sigma_F}{\sigma_0} = \frac{1.574}{0.426} \times \exp(2.200) = 33.3$$

So the reduction factor is 0.42

The Data Book quotes:

$$\frac{H}{V_{ult} \tan 35°} = \frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)$$

So if $H = V \tan 20°$

$$0.52 = 1 - \frac{V}{V_{ult}}$$

and the predicted reduction factor

$$\frac{V}{V_{ult}} = 0.48$$

Butterfield & Gottardi would have been aiming to produce an approximately correct V-H-M interaction, presumably with soil self-weight included.

4. Feeback.

(a) Some candidates failed to define $\phi$ on a sketch, or failed to work through the required algebra. Otherwise well handled.

(b) Only a minority got the right expression, but most got elements correct.

(c) Few candidates made the calculations.

Answers to 3D1, 2012

1. (a) 0.375 mm

   (c) (i) 0.097 m (ii) 3.1 years

2. (a) 15.5%, 73%

   (c) 0.03, 0.20, 85 kPa

   (d) 0.63 m settlement

   (e) 16 mm heave

3. (a) (i) $2 + 2\pi$ (ii) $2 + 3\pi$

   (b) multiply by 1.18

4. (a) $\sin(2\psi - \delta) = \sin\delta / \sin\phi$

   (b) $\frac{\sigma_f}{\sigma_o} = \frac{(1 + \sin\phi \ \cos 2\psi)}{(1 - \sin\phi)} \exp[(\pi - 2\psi) \ \tan\phi]$

   (c) 33.3 reduces to 14 as $\delta \to 20°$, a reduction factor of 0.42

   whereas the factor derived from the Databook formula is 0.48