2012 IIA 3D5 - Water Engineering Dr D. Liang

1.

(a)

There is no excess rainfall at the start of the rainfall.
Runoff occurs when the instantaneous infiltration rate is smaller than the rainfall rate.
At the critical point: $f = f_c + (f_0 - f_c)e^{-K_f t} = 2 + (20-2)e^{-0.8t} = 10$

$t = 1.0$ hour

Given plenty of rainfall available, the infiltration rate will drop to 10 mm/hr in 1 hour.
Then, the total amount of water infiltrated is:

$$\int_0^1 f \cdot dt = f_c(1-0) - \frac{1}{K_f}(f_0 - f_c)(e^{-K_f \times 1} - 1) = 2 - \frac{18}{0.8}(e^{-0.8} - 1) = 14.4 \text{ mm}$$

We assume that the infiltration rate depends only on the amount of water infiltrated.
As long as the water infiltrated is 14.4 mm, the infiltration rate will drop to 10 mm/hr.
For constant rain of 10 mm/hr, it requires 1.44 hr to get this amount of water.
So, runoff occurs only after 1.44 hr after the rainfall.

Runoff is generated 1.44 hr after the rainfall, when the infiltration rate is 10 mm/hr.
The infiltration rate between 1.44 hr and 4 hr can be expressed as:

$$f = f_c + (f_0 - f_c)e^{-K_f t} = 2 + (10-2)e^{-0.8(t-1.44)}$$

The total amount of water infiltration between 1.44hr and 4 hr is:

$$\int_{1.44}^4 f \cdot dt = 2(4-1.44) - \frac{1}{0.8}(10-2)\left(e^{-0.8\times(4-1.44)} - e^{-0.8\times 0}\right) = 13.83 \text{ mm}$$

The runoff is $10 \times (4-1.44) - 13.83 = 11.77$ mm

The total volume is $20 \times 10^6 \times 11.77 \times 10^{-3} = 2.35 \times 10^5$ m^3

(b)

The base flow is 2 m^3/s, so the hydrograph of the 3 hour excess rainfall is:

Duration (hr)	0-3	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Discharge (m^3 s^{-1})	8	48	28	8	1	0	0	0
Discharge (%)	8.6	51.6	30.1	8.6	1.1	0	0	0

First, need to find the runoff proportions due to the 2 hour uniform rainfall.

Construct the S curve:

Time (hour)	0	1.5	4.5	7.5	10.5	13.5	16.5
Runoff (%)	0	8.6	60.2	90.3	98.9	100	100

Read from S curve:

Time (hour)	0	1	3	5	7	9	11	13	15	...
S-curve value	0	4	34.5	66.5	86.5	96	99.5	100	100	...
Shift S-curve by 2 hour		0	4	34.5	66.5	86.5	96	99.5	100	...
2-hour hydrograph	0	4	30.5	32	20	9.5	3.5	0.5	0	...

So, the peak flow rate occurs 5 hours (between 4 and 6 hours) after the rainfall starts. And flow in these two hours is 32% of the total runoff.

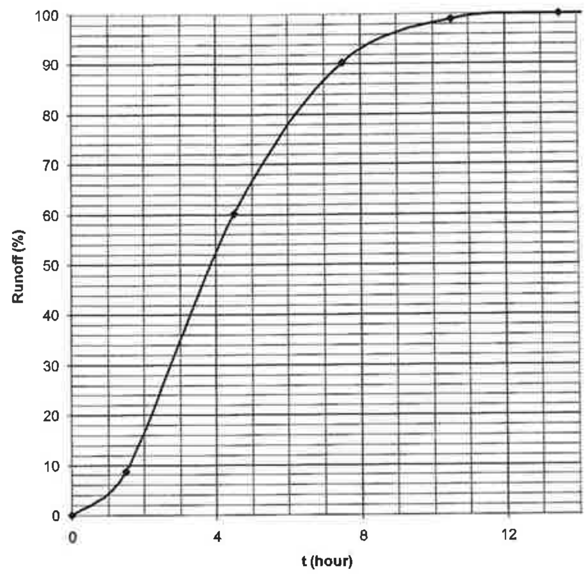
For 3-hour excess rain, the total volume is $(8+48+28+8+1)\text{m}^3/\text{s} \times 3\text{hr}$

For 2-hour rain of the same intensity, the total volume is 2/3 of this value, which is:

$$(8+48+28+8+1)\text{m}^3/\text{s} \times 2\text{hr}$$

The peak flow rate above base flow is: $(8+48+28+8+1) \times 32\% = 29.76\ \text{m}^3/\text{s}$

The total discharge (including the base flow) is: $31.76\ \text{m}^3/\text{s}$



(c)

Theissen polygons are constructed by drawing the perpendicular bisectors of all the lines connecting adjacent rain gauges. Given a set of sites, they divide the domain into areas. All the points in one area are closer to the given site inside the area than to any other given sites.

They are used to associate the precipitation across an area with rain gauge measurements at scattered points. Rainfall is assumed to be spatially uniform over each polygon.

(d.i)

Froude number is the ratio of the flow velocity over the speed that small disturbances travel.

(d.ii)

In supercritical flows, both characteristic lines point downstream. According to the method of characteristics, h and U at one location are only determined by the upstream sections.

In subcritical flows, two characteristic lines point to opposite directions, so h and U at one location are determined by both upstream and downstream sections.

2 (a.i)

Manning formula: $U = \frac{1}{n} \cdot R_h^{2/3} \cdot S_b^{1/2} = \frac{1}{n} \cdot \left(\frac{B \cdot h_0}{B + 2h_0} \right)^{2/3} \cdot S_b^{1/2}$

$$Q = U \cdot B \cdot h_0 = \frac{1}{n} \cdot \frac{(B \cdot h_0)^{5/3}}{(B + 2h_0)^{2/3}} \cdot S_b^{1/2}$$

$$41.6 = \frac{1}{0.013} \cdot \frac{(3 \cdot h_0)^{5/3}}{(3 + 2h_0)^{2/3}} \cdot 0.01^{1/2}$$

$h_0 = 2$ m is the solution.

(a.ii)

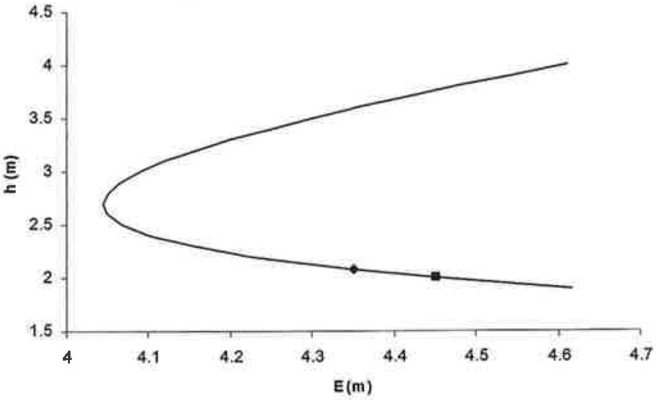
The flow varies rapidly close to the hump, so neglect the energy loss.

Specific discharge is: $\frac{41.6}{3} = 13.87$ m^2/s

Specific energy in front of the hump: $h + \frac{q^2}{h^2 2g} = 2 + \frac{13.87^2}{2^2 \cdot 2 \cdot 9.81} = 4.45$ m

Specific energy above the hump: $4.45 - 0.1 = 4.35$ m

Plot specific energy vs. water depth:



Flow is supercritical, water depth above the hump is 2.076 m.

(a.iii)

Gradually varied flow: $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$

When h =2.1 m: $U = \frac{41.6}{3 \times 2.1} \approx 6.60$ m/s

$$R_h = \frac{3 \times 2.1}{3 + 2 \times 2.1} \approx 0.875 \text{ m}$$

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.013^2 \cdot 6.6^2}{0.875^{4/3}} \approx 0.00880$$

$$Fr = \frac{U}{\sqrt{gh}} = \frac{6.60}{\sqrt{9.81 \times 2.1}} \approx 1.454$$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.01 - 0.0088}{1 - 1.454^2} \approx -0.0011$$

When h = 2.2 m: $$U = \frac{41.6}{3 \times 2.2} \approx 6.303 \text{ m/s}$$

$$R_h = \frac{3 \times 2.2}{3 + 2 \times 2.2} \approx 0.892 \text{ m}$$

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.013^2 \cdot 6.303^2}{0.892^{4/3}} \approx 0.00782$$

$$Fr = \frac{U}{\sqrt{gh}} = \frac{6.303}{\sqrt{9.81 \times 2.2}} \approx 1.357$$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.01 - 0.00782}{1 - 1.357^2} \approx -0.0026$$

So, the depth decreases with distance.
2.1 m section is downstream of 2.2 m section.

$$\frac{2.1 - 2.2}{\Delta x} = \frac{-0.0011 - 0.0026}{2} \approx -0.00185$$

$\Delta x = 54$ m

Or $$\frac{2.1 - 2.2}{\Delta x} = \frac{0.01 - \frac{0.0088 + 0.00782}{2}}{1 - \left(\frac{1.454 + 1.357}{2}\right)^2} = -0.00173$$

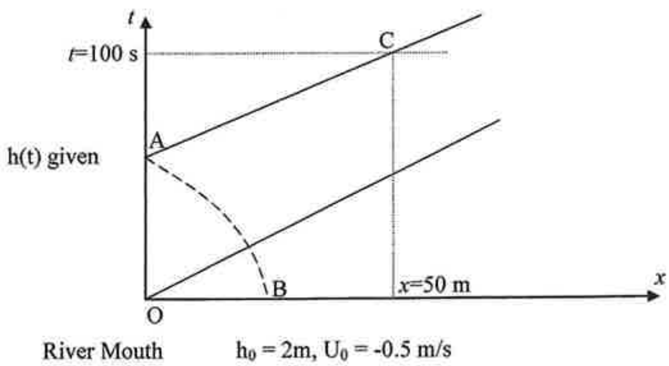
$\Delta x = 57.8$ m

Or using $$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

$$\frac{\left(2.1 + \frac{6.6^2}{2 \times 9.81}\right) - \left(2.2 + \frac{6.303^2}{2 \times 9.81}\right)}{\Delta x} = 0.01 - \frac{0.0088 + 0.00782}{2}$$

$\Delta x = 56.4$ m

(b.i)



Along the negative line AB: $\left(U - 2\sqrt{gh}\right) = const$

$$U_A - 2\sqrt{9.81 h_A} = -0.5 - 2\sqrt{9.81 \times 2} = -9.359$$

Also, $h_A = \left(\sqrt{2} - 0.002 \cdot t\right)^2$

$$U_A = -9.359 + 2\sqrt{9.81} \times \left(\sqrt{2} - 0.002t\right) = -0.5 - 0.0125t$$

x axis in the graph is towards upstream. So, the velocity towards downstream is:

$$U(t) = 0.5 + 0.0125 \cdot t$$

(b.ii) According to the straight positive line AC: $\dfrac{dx}{dt} = U_A + \sqrt{gh_A}$

$$\frac{50}{100 - t_A} = -0.5 - 0.0125 t_A + \sqrt{9.81} \times \left(\sqrt{2} - 0.002 \cdot t_A\right)$$

$$\frac{50}{100 - t_A} = -0.5 - 0.0125 t_A + 4.429 - 0.00626 t_A$$

$$\frac{50}{100 - t_A} = 3.929 - 0.001875 t_A$$

$$0.001875 t_A{}^2 - 4.1165 t_A + 342.9 = 0$$

$t_A = 79.5$ s or 230 s (neglect),

At $t_A = 79.5$ s, $h_A = 1.95$ m.

Water depth at C is also 1.576 m.

3. (a)

The Darcy-Weisbach Equation: $H_f = \lambda \frac{L}{D}\frac{U^2}{2g}$

can be rearranged for λ: $\lambda = \frac{H_f}{L} \cdot D \cdot \frac{2g}{U^2} = S_f \cdot D \cdot \frac{2g}{U^2}$

Hence: $\sqrt{\lambda} = \sqrt{2g} \cdot \frac{\sqrt{S_f D}}{U}$

Substitute this expression of $\sqrt{\lambda}$ and $\mathrm{Re} = \frac{UD}{\nu}$ into the Colebrook-White equation:

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

Then one gets:

$$\frac{1}{\sqrt{2g} \cdot \frac{\sqrt{S_f D}}{U}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\frac{UD}{\nu} \cdot \sqrt{2g} \cdot \frac{\sqrt{S_f D}}{U}}\right)$$

$$U = -2 \cdot \sqrt{2g} \cdot \sqrt{S_f D} \cdot \log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51 \cdot \nu}{\sqrt{2g} \cdot D \cdot \sqrt{S_f D}}\right)$$

Taking g = 9.81 m/s^2: $U = -8.86 \cdot \sqrt{S_f D} \cdot \log_{10}\left(\frac{k_s}{3.7D} + \frac{0.57 \cdot \nu}{D \cdot \sqrt{S_f D}}\right)$

Together with equation: $Q = U \cdot \frac{\pi D^2}{4}$

There are 2 equations for 6 variables: k_s, ν, U, S_f, D and Q.
In order for these variables to be described on a chart, simplifications are made:

i. Water temperature is known, then ν can be held constant

ii. Separate chart is made for different roughness, so k_s is constant for each chart.

This leaves 4 unknowns. Given any 2 of them, the other 2 can be found from 2 eqs.
This is the basis for producing the pipeline design charts.

(b.i)

At 27 litre s^{-1}, the velocity is: $U = \frac{0.027}{3.14 \times 0.25^2/4} = 0.55$ m^2/s

From $\frac{k_s}{D} = \frac{0.06}{250} = 0.00024$ and $\mathrm{Re} = \frac{UD}{\nu} = \frac{0.55 \times 0.25}{10^{-6}} = 1.375 \times 10^5$,
the friction factor is: $\lambda = 0.0185$

The head losses of the system are:

$$H_f = \lambda \frac{L}{D}\frac{U^2}{2g} = 0.0185 \times \frac{2000}{0.25} \times \frac{0.55^2}{2 \times 9.81} = 2.282 \text{ m}$$

$$H_L = \sum \zeta \frac{U^2}{2g} = 10 \times \frac{0.55^2}{2 \times 9.81} = 0.154 \text{ m}$$

Head demand by the system is: $H = 15 + H_f + H_L = 17.44$ m

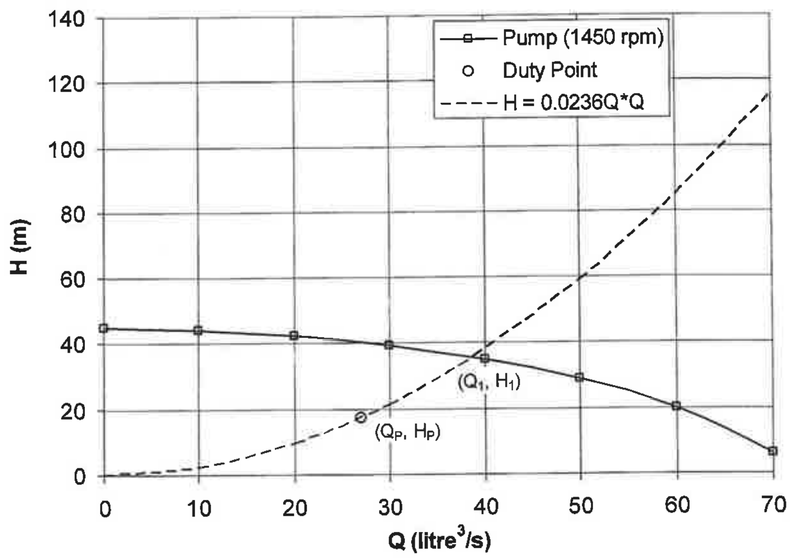
So, the duty point of the pump is: Q_P= 27 litre s^{-1}, H_P = 17.44 m (Pump speed N_P=?)

On the Q-H graph, plot the parabola through the duty point:

$$H = \left(\frac{H_P}{Q_P^2}\right)Q^2 = \left(\frac{17.44}{27^2}\right)Q^2 = 0.0239Q^2$$

The intersection with the pump curve gives the dynamically similar point at 1450 rpm:

Q_1 = 38.9 litre s^{-1} , H_1 = 35.7 m (Pump speed N_1 = 1450 rpm)



According to $\frac{Q_P}{N_P} = \frac{Q_1}{N_1}: \frac{27}{N_P} = \frac{38.9}{1450}$. So, $N_P = 1006.4$ rpm

According to $\frac{H_P}{N_P^2} = \frac{H_1}{N_1^2}: \frac{17.19}{N_P^2} = \frac{35.7}{1450^2}$. So, $N_P = 1006.2$ rpm

These two values are almost the same. The pump speed should be around 1006 rpm.

(b.ii)

Bernoulli's equation between the lake surface and the suction side of the pump:

$$\frac{p_a}{\rho g} + \frac{0}{\rho g} + z_0 = \frac{p_P}{\rho g} + \frac{U_P^2}{\rho g} + z_P + H_l + H_f$$

At the critical condition, $\frac{p_P}{\rho g} + \frac{U_P^2}{\rho g} = \frac{p_a}{\rho g} - 3$

So, $$\frac{p_a}{\rho g} + z_0 = \frac{p_a}{\rho g} - 3 + z_P + H_l + H_f$$

$$z_P - z_0 = 3 - H_l - H_f$$

$$H_f = \lambda\frac{L}{D}\frac{U^2}{2g} = 0.0165 \times \frac{100}{0.25} \times \frac{0.55^2}{2 \times 9.81} = 0.102 \text{ m}$$

$$H_l = \sum\zeta\frac{U^2}{2g} = 10 \times \frac{0.55^2}{2 \times 9.81} = 0.154 \text{ m}$$

So, $z_P - z_0 = 3 - 0.102 - 0.154 = 2.744 \text{ m}$

The maximum level of the pump location is 2.744 m above the lake surface.

4 (a) Turbulent diffusion emerges when describing turbulent transport phenomena using the Reynolds-averaged approach. It represents the effect of turbulent fluctuations on the Reynolds-averaged distributions.

Longitudinal dispersion arises from the combined effect of the velocity non-uniformity in the longitudinal direction and the diffusion in the vertical (or lateral) direction.

(b.i)

This is a 3-D problem. Also need to consider the boundary – water surface.
The image is located at the same location as the real source.

$$\bar{c}(x,y,z,t) = \frac{2M}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}}\exp\left(-\frac{x^2}{4D_x t}-\frac{y^2}{4D_y t}-\frac{z^2}{4D_z t}\right)$$

$$\bar{c}(50,0,2,600) = \frac{2\times 50}{(4\pi\times 600)^{3/2}\sqrt{1}}\exp\left(-\frac{50^2}{4\times 600}-\frac{0^2}{4\times 600}-\frac{2^2}{4\times 600}\right) = 5.4\times 10^{-5}\ \text{kg/m}^3$$

(b.ii)

$$\bar{c}(50,0,2,t) = \frac{2\times 50}{(4\pi t)^{3/2}\sqrt{1}}\exp\left(-\frac{50^2}{4\times t}-\frac{0^2}{4\times t}-\frac{2^2}{4\times t}\right)$$

$$\bar{c}(t) = \frac{2\times 50}{(4\pi)^{3/2}\cdot t^{3/2}}\exp\left(-\frac{50^2+2^2}{4t}\right)$$

$$\bar{c}(t) = At^{-3/2}\exp\left(-626t^{-1}\right)$$

From $\dfrac{d\bar{c}}{dt} = A\left(-\dfrac{3}{2}\right)t^{-5/2}\exp\left(-626t^{-1}\right) + At^{-3/2}\exp\left(-626t^{-1}\right)\left(626t^{-2}\right) = 0$

So, $t = 626\times\dfrac{2}{3} = 417\ \text{s}$

(c.i)

$$d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.15\times 10^{-3}\times\left(\frac{9.81\times(2.65-1)}{10^{-12}}\right)^{1/3} = 3.794$$

$$\theta_c = \frac{0.30}{1+1.2d_*} + 0.055\left[1-\exp(-0.02d_*)\right] = \frac{0.30}{1+1.2\times 3.794} + 0.055\left[1-e^{(-0.02\times 3.794)}\right]$$
$$= 0.0540 + 0.004 = 0.058$$

Grain-related value: $C' = 7.8\ln\left(\dfrac{12h}{k_s'}\right) = 7.8\ln\left(\dfrac{12\times 10}{0.3\times 10^{-3}}\right) = 100.6\ \text{m}^{1/2}\ \text{s}^{-1}$

$$\tau_b' = \rho g\frac{U^2}{C'^2} = 1000\cdot 9.81\cdot\frac{0.65^2}{100.6^2} = 0.410\ \text{Pa}$$

$$\theta' = \frac{\tau_b'}{g(\rho_s-\rho)d} = \frac{0.410}{9.81\cdot(2650-1000)\cdot 0.00015} = 0.169$$

$$T = \frac{\tau_b'-\tau_{bc}}{\tau_{bc}} = \frac{\theta'-\theta_c}{\theta_c} = \frac{0.169-0.058}{0.058} = 1.91$$

According to d_* and T values, ripples occur.

(c.ii)

$$k_s'' = 100 \times 0.15 \times 10^{-3} = 0.015 \text{ m}$$

$$k_s = k_s' + k_s'' = 102 \times 0.15 \times 10^{-3} = 0.0153$$

$$C = 7.8 \ln\left(\frac{12h}{k_s}\right) = 7.8 \ln\left(\frac{12 \times 10}{0.0153}\right) = 69.9 \text{ m}^{1/2} \text{ s}^{-1}$$

$$\tau_b = \rho g \frac{U^2}{C^2} = 1000 \times 9.81 \times \frac{0.65^2}{69.9^2} = 0.85 \text{ Pa}$$

(c.iii)

$$\bar{c}(2d) = \frac{0.331 \cdot (\theta' - 0.045)^{1.75}}{1 + 0.72 \cdot (\theta' - 0.045)^{1.75}}$$

$$\bar{c}(0.0003) = \frac{0.331 \cdot (0.169 - 0.045)^{1.75}}{1 + 0.72 \cdot (0.169 - 0.045)^{1.75}} = \frac{0.008577}{1 + 0.01866} = 0.00842$$

This is volumetric concentration.

$$\bar{c}(0.0003) = 0.00842 \times 2650 = 20.3 \text{ kg/m}^3$$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot d_*^{\,3}} - 10.36\right]$$

$$w_s = \frac{10^{-6}}{0.15 \times 10^{-3}}\left[\sqrt{10.36^2 + 1.049 \times 3.794^3} - 10.36\right] = 0.0165 \text{ m/s}$$

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{0.85}{1000}} = 0.029 \text{ m/s}$$

Let $a = 0.0003$ m, $\bar{c}(a) = 20.3$ kg m^{-3}, $z = 1$:

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h - z}{z} \cdot \frac{a}{h - a}\right)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{\bar{c}(z=1)}{20.3} = \left(\frac{10 - 1}{1} \cdot \frac{0.0003}{10 - 0.0003}\right)^{\frac{0.0165}{0.4 \times 0.029}} = 0.000222$$

$$\bar{c}(z=1) = 0.0045 \text{ kg/m}^3$$

2012 IIA 3D5 - Water Engineering Dr D. Liang

Answers

1. (a.ii) 2.35×10^5 m^3
   (b) 31.76 m^3/s

2. (a.ii) 2.076 m
   (a.iii) $\Delta x \approx 55$ m; 2.1 m section is downstream of 2.2 m section.
   (b.ii) 1.576 m

3. (b.i) 1006 rpm
   (b.ii) 2.744 m

4. (b.i) 5.4×10^{-5} kg/m^3
   (c.ii) 0.85 Pa
   (c.iii) 0.0045 kg/m^3