3E3 2012 Cribs

Question 1

a) For both alternatives, we have an *M/M/*1 queueing system. Using one hour as the unit of time, we also have $\lambda = 30$ in both cases.

With candidate X: $\mu = 50$, so

$$W = \frac{1}{\mu - \lambda} = \frac{1}{20} = 3 \text{ minutes},$$

$$\text{Monthly revenue} = \frac{\$6,000}{3} = £2000.$$

With candidate Y: $\mu = 40$, so

$$W = \frac{1}{\mu - \lambda} = \frac{1}{10} = 6 \text{ minutes},$$

$$\text{Monthly revenue} = \frac{\$6,000}{6} = £1000.$$

Since the difference in monthly revenues is £1000, the upper bound on the difference in their monthly compensations that would justify hiring candidate X rather than candidate Y is £1000.

b)

Using one hour as the unit of time, we are given an M/M/1 queue with $L = 8$ and $W = 2$. Then, from Little's formula,

$$\lambda = \frac{L}{W} = 4.$$

$$W = \frac{1}{\mu - \lambda} = 2 \quad \Rightarrow \quad \mu = \frac{1}{2} + \lambda = 4\frac{1}{2}.$$

$$P\{T_s > \frac{1}{3}\} = e^{-\frac{9}{2}*\frac{1}{3}} = e^{-\frac{3}{2}}.$$

c) The states are E={cured, rem,sick,dead}={1,2,3,4}
with

$$P=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0.33 & 0.33 & 0 & 0.33 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

i) h1=1 and h4=0. Also h2=0.5+ 0.5h3 and h3=0.33 +0.33h2.
so h2=0.8 and h3=0.6
ii) k1=k4=0 . Also, k2=1+0.5k3 and k3=1+0.33k2, so k2=9/5 and k3=8/5.

d)

- Arises if an omitted variable both (i) is a determinant of Y and (ii) is correlated with at least one included regressor.
- Potential solutions to omitted variable bias
  - If the variable can be measured, include it as a regressor in multiple regression;
  - Possibly, use panel data in which each entity (individual) is observed more than once;
  - If the variable cannot be measured, use instrumental variables regression (not covered in this course);
  - Run a randomized controlled experiment.

Question 2

(i) The decision alternatives are

a_1: Guess that the coin is fair,
a_2: Guess that the coin is two-headed.

The states of nature are

θ_1: The coin is a fair coin,
θ_2: The coin is a two-headed coin.

The payoff table is given below.

Decision Alternative	State of Nature θ_1 Fair Coin	θ_2 2-headed coin
a_1 (guess fair)	5	-5
a_2 (guess 2-headed)	-5	5
Prior probability	0.5	0.5

(ii) The two decision alternatives tie for being optimal, since both give an expected payoff of £0.

(iii) The expected payoff with perfect information is

$$0.5(5) + 0.5(5) = 5.$$

Therefore, since the expected payoff without experimentation is 0,

$$\text{EVPI} = 5 - 0 = 5.$$

(iv) $$\text{P(fair | flip tail)} = \frac{\text{P(flip tail | fair)P(fair)}}{\text{P(flip tail | fair)P(fair)} + \text{P(flip tail | 2-headed) P(2-headed)}}$$

$$= \frac{0.5(0.5)}{0.5(0.5)+0(0.5)} = 1.$$

So P(2-headed | flip tail) = 0.

$$\text{P(fair | flip head)} = \frac{\text{P(flip head | fair) P(fair)}}{\text{P(flip head | fair)P(fair)} + \text{P(flip head | 2-headed) P(2-headed)}}$$

$$= \frac{0.5(0.5)}{0.5(0.5)+1(0.5)} = \frac{1}{3}.$$

So P(2-headed | flip head) = 2/3.

If flipped tail:

For a_1: $E[p(a_1, \theta \mid \text{flip tail})] = 1(5) + 0(-5) = 5.$

For a_2: $E[p(a_2, \theta \mid \text{flip tail})] = 1(-5) + 0(5) = -5.$

If flipped head:

For a_1: $E[p(a_1, \theta \mid \text{flip head})] = (1/3)(5) + (2/3)(-5) = -5/3.$

For a_2: $E[p(a_2, \theta \mid \text{flip head})] = (1/3)(-5) + (2/3)(5) = 5/3.$

So, if tail occurs, guess fair coin; if head occurs, guess 2-headed coin.

P(flip tail) = P(flip tail | fair) P(fair) + P(flip tail | 2-headed) P(2-headed)
= 0.5(0.5) + 0(0.5) = 0.25,

so

$$P(\text{flip head}) = 1 - P(\text{flip tail}) = 0.75.$$

Therefore, using the optimal policy from part (e), the expected payoff with experimentation is

$$\begin{aligned}
&P(\text{flip tail})\, E[p(a_1, \theta) \mid \text{flip tail})] \\
&+ P(\text{flip head})\, E[p(a_2, \theta) \mid \text{flip head})] \\
&= 0.25(5) + 0.75\left(\frac{5}{3}\right) = 2.5.
\end{aligned}$$

Since the expected payoff without experimentation is 0, the expected value of experimentation is

$$EVE = 2.5 - 0 = 2.5.$$

Consequently, the most you should be willing to pay to see the demonstration flip is £2.50.

b) There are two fundamental reasons why prospect theory (which calculates value) is inconsistent with expected utility theory. Firstly, whilst utility is necessarily linear in the probabilities, value is not. Secondly, whereas utility is dependent on final wealth, value is defined in terms of gains and losses (deviations from current wealth). In particular, we observe risk-aversion for gains, risk-seeking for losses, and also s steeper for losses than for gains (loss aversion).

Question 3

a)

i) Predicted M&S return = 0.015 + 0.80Market return
ii) Specific risk is slightly higher for CT (6.6% versus 6.2% for M&S). When market falls by 1%, M&S return falls by 0.8% (versus 1.32% for CT). Therefore, a risk averse investor would prefer M&S to CT in his portfolio.
iii) Usually small companies tend to have lower beta values. So, one explanation could be that in the previous decade, CT was considered a startup but as it grew bigger it became more dependent on the FTSE.

b)
i)

- For every unit demand falls below 100,000 we lose £450 of revenue.
- For every unit demand rises above 100,000, our profit is the *same* as for demand 100,000 because our production capacity is fixed at 100,000.
- Thus there is no upside to balance off the downside.
- Thus on *average* the profit is less than the B-o-E number.
- This is an example of the Flaw of Averages (average inputs in the B-o-E calculation don't give the average value of the uncertain project profit).

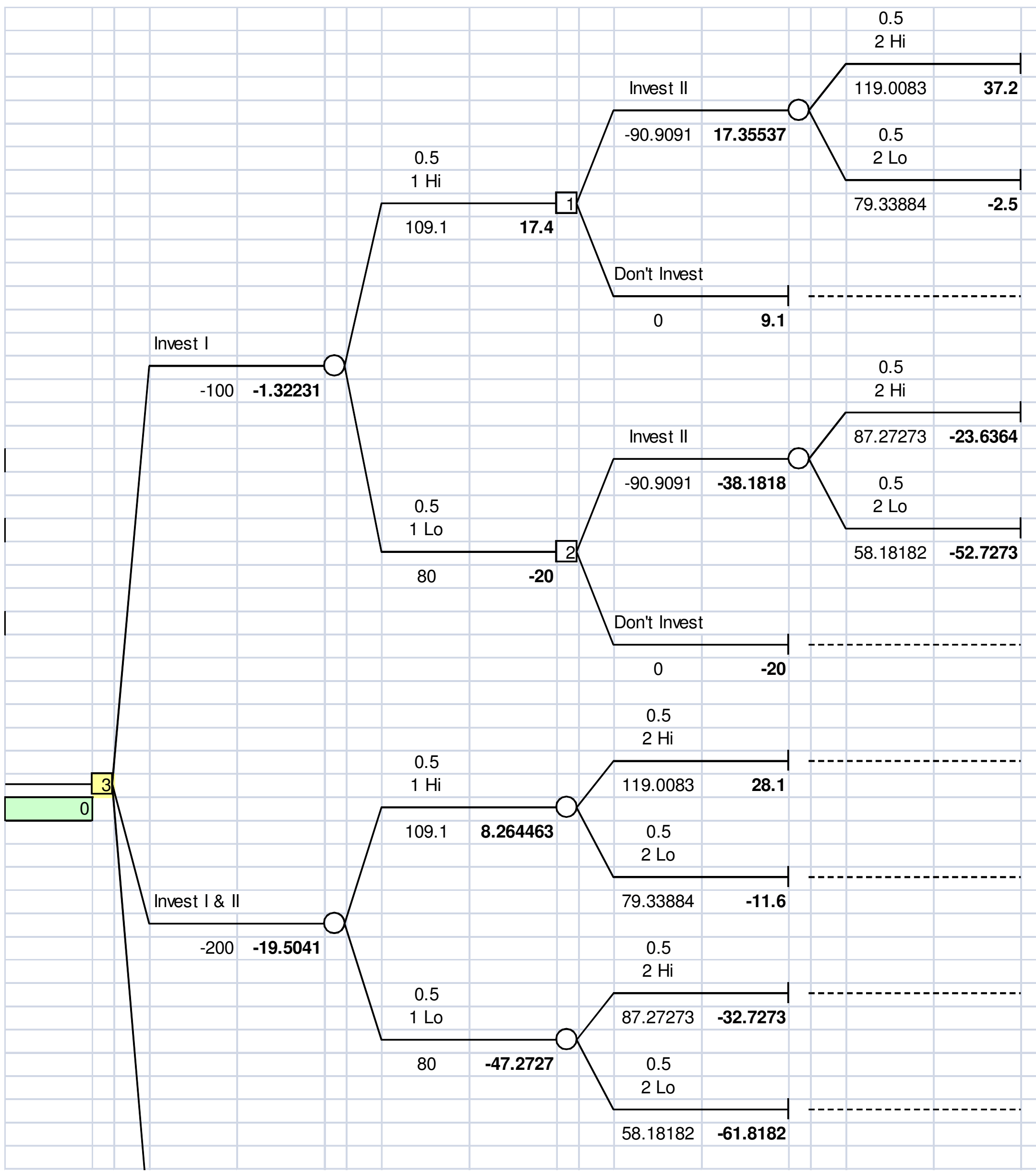
ii)

- For every pound the price falls below £450 we lose £100,000 of revenue.
- But for every pound that the price rises above £450 we gain £100,000 of revenue.
- Thus if the distribution of price is symmetric about $p = £450$, the upside and downside balance – there is no Flaw of Averages.
- The B-o-E value and the value you would obtain with simulation are the same. There is no Flaw of Averages here.
- In fact, even if the price distribution is not symmetric, if the average price is £450 then there will still be no Flaw because expected value of revenue will just be expected value of price p times demand quantity 100,000.

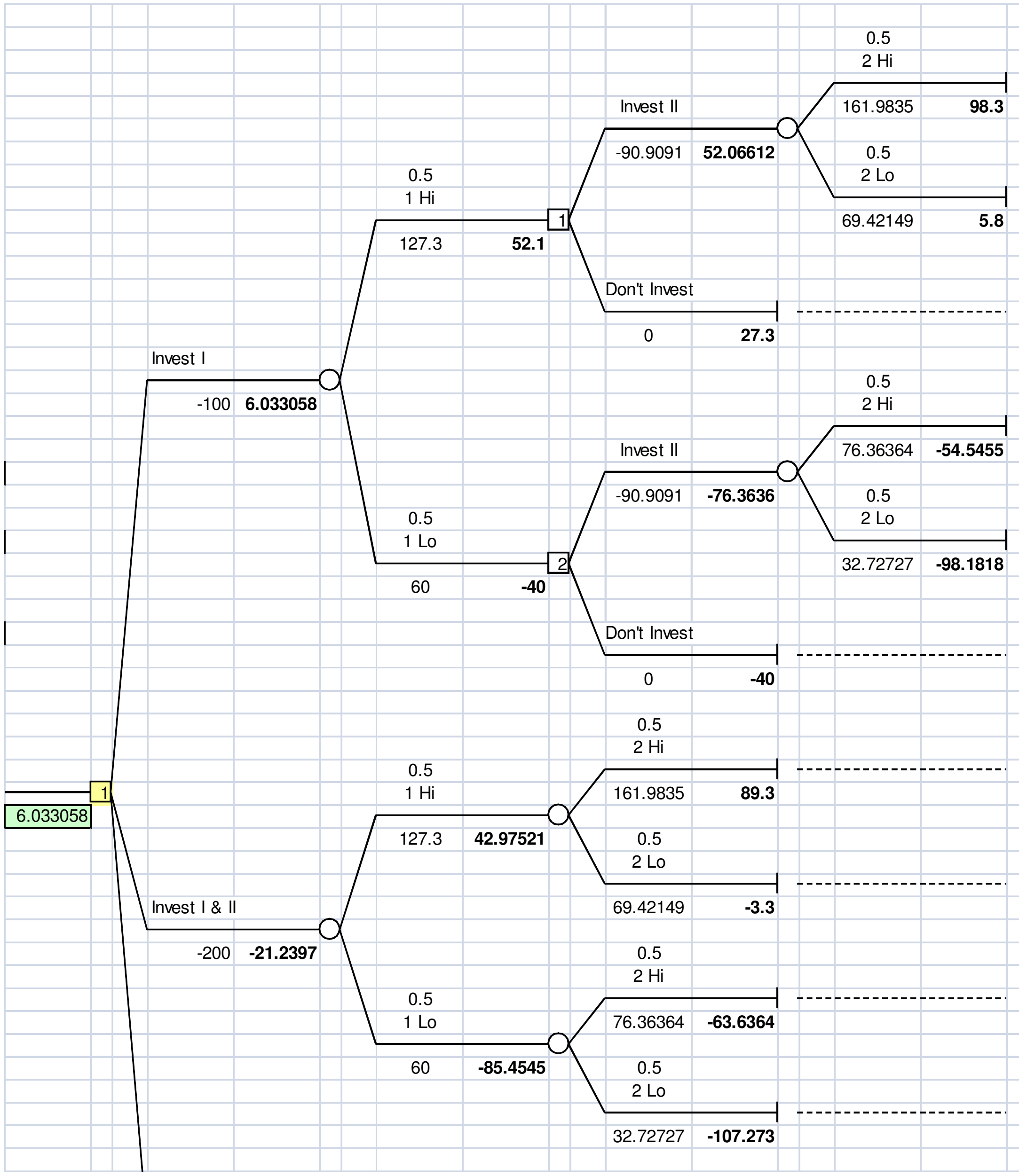
c) One way is to find the SML/CAPM line $y= 5.5 + 7.5*(\text{beta})$ and notice that for beta=1.5 , the CAPM would predict a return of $16.5\% < 17\%$ so stock Y is currently undervalued. On the other hand, for stock Z the CAPM would predict a return of $11.5>10.5\%$ so stock Z is overvalued.
The reward-to-risk ratio for the two stocks has to be the same:
$(17- Rf)/ 1.5 = (10.5-Rf)0.8$ from which we get $Rf= 3.07\%$.

Question 4

a) i) The decision tree is shown below. Optimal Strategy not to invest.



ii) Optimal strategy to invest in Phase I.



b) The tangency portfolio is the portfolio of risky assets on the efficient frontier at the point where the capital market line is tangent to the efficiency frontier. Combinations of the tangency portfolio and the risk-free asset compose the capital market line where all the rational investors should be placed on.