ENGINEERING TRIPOS   PART IIA

Friday 27 April 2012   9 to 12

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: Compressible Flow Data Book (38 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Ultimate version

1 Figure 1 shows a streamwise section through the tailplane aerofoil of a subsonic aircraft along with the associated cartesian (x, y) coordinate system. The aerofoil chord is c and τ is a measure of its thickness. The shape function describing the aerofoil surface is $g(x/c)$.

(a) Carefully describe the conditions under which the two-dimensional compressible flow in cartesian coordinates around the aerofoil shown in Fig. 1 may be given by

$$(1-M_\infty^2)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$$

where M_∞ is the flight Mach number and φ is the flow potential. [10%]

(b) By considering the boundary condition for the flow on the aerofoil perimeter, derive a suitable boundary condition for φ. [30%]

(c) By making appropriate substitutions and considering the linearised x-component of momentum, show that the pressure coefficient, C_p, of the flow around the aerofoil may be written as

$$C_p=\frac{k\tau}{c\sqrt{1-M_\infty^2}}$$

where k is an expression to be determined. [40%]

(d) The real tailplane can tolerate a maximum loading of $10\ \mathrm{kN\,m^{-2}}$. Calculate the maximum permitted flight speed at sea level, assuming that the tailplane operates at its optimum lift coefficient of 0.5 and treating the flow as incompressible. [10%]

(e) Using your solution to part (c), recalculate your solution to part (d) allowing for the effects of compressibility. Briefly comment on the disparity between the two calculated values of the maximum permitted flight speed. [10%]

(Cont.

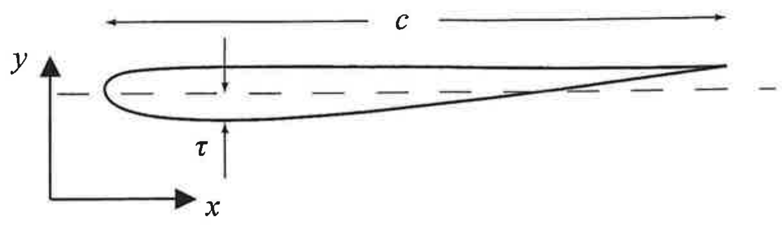

Fig. 1

2 A supersonic aircraft flies at a Mach number of 2. Figure 2 shows a section through the centre-line of a symmetric two-dimensional bump of height 1.5 mm, effective half-width 3 mm and ramp angle 20° on the outer skin of the aircraft.

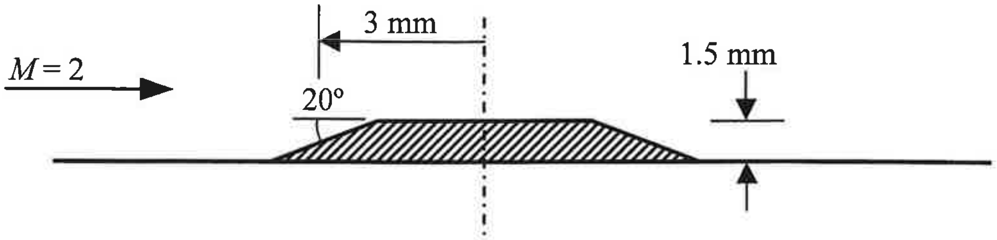

Fig. 2 (not to scale)

(a) Assuming that the flow remains attached, carefully sketch and label the principal supersonic flow features around the bump. [25%]

(b) Calculate the form drag per unit length caused by the bump in terms of the local static pressure p_∞. You may ignore all boundary layer effects. [50%]

(c) The aircraft has 1000 rivet heads protruding through the outer skin in aerodynamically significant locations. The total aerodynamic drag due to these rivets can be estimated by assuming that each rivet head has an effective radius of 3 mm and they are arranged in a single unbroken line transverse to the on coming flow. By using the result in part (b), calculate the additional power required (over a smooth-skinned aircraft) for steady, level flight at an altitude of 15000 m. [25%]

3 (a) A convergent-divergent nozzle is supplied with air from a reservoir which is held at a fixed stagnation pressure. The flow is adiabatic and quasi-one-dimensional. Describe the flow regimes which occur inside the nozzle as the back pressure is reduced relative to the upstream stagnation pressure. Sketch the variation of Mach number and the static pressure along the length of the nozzle. [20%]

(b) In a certain type of rocket engine, a convergent-divergent nozzle is connected to a reservoir of gas at a stagnation pressure of 1.8 bar. The nozzle is designed to provide isentropic acceleration of the gas to supersonic speed at exit conditions corresponding to atmospheric pressure at an altitude of 12 km above sea level. The diameter of the throat is 0.2 m. Assuming that the gas has the same properties as air with $\gamma = 1.4$ calculate:

(i) the area of the nozzle exit; [20%]

(ii) the thrust produced by the nozzle. [20%]

(c) The rocket engine is also required to operate at sea level. Assuming that the reservoir conditions are unchanged, determine:

(i) that a shock will occur in the nozzle; [10%]

(ii) the Mach number ahead of the shock; [15%]

(iii) the thrust produced by the nozzle at sea level. [15%]

4 (a) The inlet of a convergent-divergent nozzle is connected to a reservoir of air in which the stagnation pressure and stagnation temperature are constant. The outlet of the nozzle is connected to a straight pipe of constant cross-sectional area. The pipe exit is at a fixed pressure. The nozzle is frictionless, but the pipe has a uniform non-zero friction coefficient. Describe, with the aid of a *T-s* diagram, the flow regimes that will occur in the nozzle and pipe as the friction coefficient is varied. [30%]

(b) The pipe is 7.5 m long and has an inside diameter of 0.1 m. The reservoir contains air with $\gamma = 1.4$ at a stagnation pressure of 3.14 bar. At the pipe exit the static pressure is 1 bar and the Mach number is 1. There are no shocks in either the pipe or the nozzle.

(i) Determine the two possible values of the Mach number at the pipe inlet and the corresponding friction coefficients. [20%]

(ii) Comment on the relationship between the friction coefficient and the flow regime for the two solutions in (b)(i). [10%]

(c) When manufactured the inside surface of the pipe has the lower of the two friction coefficients obtained in (b). Later, however, corrosion on the inside surface of the pipe is found to have changed the friction coefficient to a value of $c_f = 0.0015$. During operation with the same reservoir conditions as before, a shock wave is formed within the pipe. The pressure and the Mach number at the pipe exit are also the same as before.

(i) Show that the shock wave is between 1 m and 2 m from the inlet of the pipe. [20%]

(ii) Using the two solutions considered in (c)(i), estimate the location of the shock wave and the Mach number ahead of it. [10%]

(iii) Explain carefully the relationship between the mass flow rates and the pressure drops for the "as manufactured" and "corroded" conditions of the pipe. [10%]

5 (a) Consider an infinitesimal wave on the surface of shallow water. The velocity of the water is V, taken positive to the right, while the speed of wave propagation is c. Show that the quantity $V - 2c$ is invariant across a right-running wave. [30%]

(b) A long water channel of constant width is filled to a depth h_0. At the left-hand end of the channel there is a closed sluice gate to prevent the water escaping. Initially the water is stationary. At time t_0 the gate is opened instantaneously, causing the water level at the gate to drop *slightly* to a new depth h_1. Draw a space-time diagram to illustrate the subsequent wave and particle trajectories. [20%]

(c) What is the value of the ratio h_1/h_0 for the maximum flow rate of water escaping through the open sluice gate? Comment on the validity of the analysis. [20%]

(d) With the sluice gate closed, the initial depth of water in the channel is measured as 0.3 m. When the gate is opened suddenly, the water level at the gate falls by 0.05 m. After a delay of 10 s the gate is suddenly closed. Calculate the distance from the gate at which the waves begin to interact. [30%]

6 The development of a steady boundary layer on a flat plate is to be simulated numerically by solving the continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

together with the axial momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}$$

where x is the coordinate parallel to the plate, y is the coordinate normal to the plate, ν is the kinematic viscosity, p is the pressure and ρ is the (uniform) fluid density.

(a) Discretise the continuity equation using second order finite differences on a uniform mesh with spacing Δx and Δy. Give an example of an alternative second order expression explaining why, in this problem, it is less useful. [30%]

(b) Discretise the axial momentum equation on the same mesh using first order finite differences where necessary and second order finite differences where possible. [30%]

(c) Describe a solution algorithm that can be used to solve these two equations for the downstream development of the boundary layer in a specified pressure field. [40%]

7 *Note that part (a) of this question is not related to part (b).*

(a) The unsteady heat conduction equation in one-dimension is

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2}$$

where θ is the normalised temperature difference and α is the thermal diffusivity. A discrete form of this equation is proposed as

$$\theta_i^{n+1} = \theta_i^n + \beta\left(\theta_{i+1}^n - 2\theta_i^n + \theta_{i-1}^n\right)$$

where $t = n\Delta t,\ x = i\Delta x$ and $\beta = \alpha\Delta t(\Delta x)^{-2}$. Determine:

(i) the order of accuracy of this discrete form, in space and in time; [30%]

(ii) the stability limit on the parameter β. [20%]

(b) A pump for an incompressible fluid has impeller diameter D, rotational speed Ω, and produces a stagnation pressure rise of Δp_0. When operating at its design flow coefficient, the irreversibility within the pump varies according to

$$\frac{\Delta p_0^{isen} - \Delta p_0}{\rho \Omega^2 D^2} \propto Re^{-0.2}$$

where Re is the Reynolds number and ρ is the fluid density. It may be assumed that $\Delta p_0^{isen} / \rho\Omega^2 D^2$ depends only on the flow coefficient.

(i) Show that when operating at its design flow coefficient, the variation of the total-to-total isentropic efficiency, η, may be estimated using

$$(1-\eta) \propto Re^{-0.2}$$ [15%]

(ii) A water pump with impeller diameter of $3.8\ \text{m}$ is to produce a stagnation pressure rise corresponding to a height of $500\ \text{m}$ of water at a volumetric flow rate of $50\ \text{m}^3\text{s}^{-1}$ with a total-to-total efficiency of 90% at a rotational speed of $500\ \text{rpm}$. A geometrically similar pump with an impeller diameter of $0.4\ \text{m}$ operating at $3000\ \text{rpm}$ is to be tested. For the test configuration calculate the power required, the efficiency and hence the head produced. [35%]

8 A single-stage axial flow turbine has a mean radius of 0.3 m. At the design operating point the turbine rotates at 15400 rpm, the reaction is 50% and the axial velocity of the flow at the mean radius is constant. The inlet stagnation temperature is 1200 K and the inlet stagnation pressure is 8.4 bar.

(a) The Mach number of the flow at the exit of the turbine stator is 0.9 and the flow angle is 65°. Calculate the flow coefficient. [15%]

(b) Calculate the stage loading coefficient of the turbine. [20%]

(c) If the stator has a stagnation pressure loss coefficient of 0.06, calculate the stagnation pressure at the exit of the stator. [15%]

(d) If the design mass flow rate is $40\ \mathrm{kg\,s^{-1}}$, calculate the radial height of the flow path at the exit of the stator. [15%]

(e) Assuming that the stagnation conditions at entry to the turbine stator and its stagnation pressure loss coefficient do not change, estimate the maximum mass flow rate through the stator. [15%]

(f) The turbine drives a compressor and, at the design point, the fuel mass flow rate entering the combustion chamber is $0.5\ \mathrm{kg\,s^{-1}}$. The flow entering the compressor has stagnation temperature of 288 K and inlet stagnation pressure of 1 bar. Calculate the isentropic total-to-total efficiency of the compression process. You may ignore all pressure losses across the combustor. [20%]

You may assume that the working fluid has $R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$ and:
within the turbine $\gamma = 1.333$ with $c_p = 1149\ \mathrm{J\,kg^{-1}\,K^{-1}}$;
within the compressor $\gamma = 1.4$ with $c_p = 1005\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

END OF PAPER

2012 IIA Paper 3A3 (Fluid Mechanics II) Answers

Q1: a) steady, isentropic, irrotational, linearised.

b) $\left.\dfrac{\partial \phi}{\partial y}\right|_{y=0} = \dfrac{\tau}{c} g' u_\infty$

c) $k = -2\dfrac{\partial \tilde{\phi}}{\partial \tilde{x}}$ where $\tilde{x} = x/c$ and $\tilde{\phi} = \dfrac{\sqrt{1-M_\infty^2}}{\tau u_\infty}\phi$

d) $u_\infty = 180.7$ m/s $\quad (M_\infty = 0.531)$

e) $u_\infty = 168.3$ m/s $\quad (M_\infty = 0.495)$

Q2: b) $drag/length = 3.82 \times 10^{-3}\, p_\infty$ N/m

c) 163.8 kW $\quad (drag = 277.5$ N $\;$ for $\;6$ m long bump)

Q3: b) i) 0.0582 m^2 $\;(M_{exit} = 2.11)$

ii) 7026 N

c) i) just choked with isentropic subsonic flow in divergent section $\;p_{exit} = 1.667$ bar

ii) $M_{ahead} = 2.075$ $\;(p_{0exit}/p_{0inlet} = 0.6856)$

iii) 2397 N

Q4: b) i) $M_{inlet} = 0.3800,\; c_f = 0.0090$ or $M_{inlet} = 1.9792,\; c_f = 0.0010$

c) i) $1\,\text{m}:\; M_{ahead-shock} = 1.7909,\; M_{after-shock} = 0.6186,\; \left.\dfrac{4c_f L}{D}\right|_{exit} = 0.0322$

$2\,\text{m}:\; M_{ahead-shock} = 1.6180,\; M_{after-shock} = 0.6631,\; \left.\dfrac{4c_f L}{D}\right|_{exit} = -0.0398$

ii) $1.45\,\text{m},\; M_{ahead-shock} = 1.71$

Q5: c) $h_1/h_0 = 0.444$ (Not a small height change.)

d) $d = 179$ m $\;(\Delta t = 104.4$ s$)$

Q6: c) Get u_j^{i+1} from momentum equation, then v_{j+1}^{i+1} from continuity

Q7: a) i) Second order in space, first order in time.

ii) $0 < \beta < 0.5$

b) ii) $Power|_{test} = 0.761$ MW, $\eta|_{test} = 82.8\%$, $Head|_{test} = 184$ m $\;(\dot{Q}|_{test} = 0.350$ m^{3}s$^{-1})$

Q8: a) $\phi = 0.500$

b) $\psi = 1.145$

c) 8.20 bar

d) 53.8 mm

e) 40.4 kg/s

f) 89.2%