ENGINEERING TRIPOS PART IIA

Monday 30 April 2012 2:30 to 4:00

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: Data sheet (1 page).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) A solid has density ρ, specific heat capacity c, and thermal conductivity λ, which are independent of temperature T. For a cartesian coordinate system, the energy balance within a differential volume $\Delta V = \Delta x\, \Delta y\, \Delta z$ in the solid is given by the expression

$$\rho c \frac{\partial T}{\partial t} \Delta V = \Delta Q$$

where ΔQ is the net rate of heat transfer into the element. By considering the heat transfer by conduction in one direction only, then proceeding by analogy, show that the unsteady heat diffusion equation within the solid can be written as [15%]

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \nabla^2 T$$

where

$$\alpha = \frac{\lambda}{\rho c}$$

(b) Consider now a one-dimensional situation for a semi-infinite solid extending from $x = 0$ into the x-coordinate.The solid is initially in thermal equilibrium with its surroundings at a temperature T_∞. At a time $t = 0$, the surface of the solid is exposed to a sudden change in boundary condition. For each of the three cases below, sketch the temperature variation with depth into the surface, indicating how it changes with time after its boundary at $x = 0$ is subjected to:

(i) a constant temperature, $T_0 > T_\infty$;

(ii) a constant positive heat flux, q;

(iii) a convective heat flux with constant heat transfer coefficient, h, and far field fluid temperature, $T_0 > T_\infty$.

Indicate clearly the boundary conditions used at the interface of solid and surroundings. [30%]

(c) When the surface is exposed to the sudden constant temperature boundary condition of part (b) (i), the temperature is given by

$$\frac{T_0 - T(\eta)}{T_0 - T_\infty} = \mathrm{erf}(\eta)$$

where $\eta = \dfrac{x}{2\sqrt{\alpha t}}$ and erf is the error function.

(i) For a case where T_∞ = 300 K and T_0 = 350 K, the temperature at a depth 0.005 m below the surface reaches 310 K at 60 seconds. The material has density ρ = 1320 kg m^{-3} and specific heat capacity c = 2160 J kg $^{-1}$ K $^{-1}$. Using the result above, calculate the thermal conductivity of the material. Values for the error function are given in Table 1. [30%]

(ii) Assuming $T_0 > T_\infty$, if the surface were exposed to a convective boundary condition instead, would the temperature at the same location be higher or lower at 60 seconds? Comment on the practical realisability of the boundary conditions suggested in cases (b) (i) and (iii). [10%]

(d) By considering an approximate numerical solution scheme, derive an expression for the nodal equation at a point within the semi-infinite solid using a first order discretisation in both time and x-direction. [15%]

η	erf(η)	η	erf(η)	η	erf(η)
0.8000	0.7421	0.8500	0.7707	0.9000	0.7969
0.8050	0.7451	0.8550	0.7734	0.9050	0.7994
0.8100	0.7480	0.8600	0.7761	0.9100	0.8019
0.8150	0.7509	0.8650	0.7788	0.9150	0.8043
0.8200	0.7538	0.8700	0.7814	0.9200	0.8068
0.8250	0.7567	0.8750	0.7841	0.9250	0.8092
0.8300	0.7595	0.8800	0.7867	0.9300	0.8116
0.8350	0.7623	0.8850	0.7893	0.9350	0.8139
0.8400	0.7651	0.8900	0.7918	0.9400	0.8163
0.8450	0.7679	0.8950	0.7944	0.9450	0.8186

Table 1

2 A cylindrical shaft with rod of radius R rotates in a flooded, perfectly symmetric lubricated bearing with a spacing δ between the shaft and supporting wall. The fluid between the surfaces entirely fills the space, setting up a steady-state flow inside the cavity, which is independent of the local angular position. The fluid is Newtonian with constant density ρ, dynamic viscosity μ, thermal conductivity κ and specific heat c_p. Assume that $R >> \delta$, so that the flow in the lubricating channel can be considered planar, as shown in Fig. 1. The outer wall is stationary, and the shaft rotates with angular velocity ω. The pressure is uniform throughout the fluid.

(a) Starting from the mass and momentum conservation equations (see attached data sheet), show that the flow velocity profile is given by $u = \omega R y/\delta$. [20%]

(b) Assume that the outer wall of the bearing is kept at temperature T_0, and that the inner wall (the shaft) is adiabatic.

(i) Starting from the thermal energy equation, and assuming that temperature varies only with y, show that the temperature within the fluid is given by [30%]

$$T(y) = T_0 + \frac{\mu}{\kappa}(\omega R)^2 \left(\frac{y}{\delta}\right)\left(1 - \frac{y}{2\delta}\right)$$

(ii) Determine the heat flux at the outer walls, $y = 0$, and show that it is equal to the total dissipative work rate integrated across the fluid layer. [20%]

(c) During another period of operation, the outer wall is maintained at T_0, while heat flows equally to the inner and outer wall. Determine the temperature distribution $T(y)$ as a function of the variables given, and the heat flux to each wall. Sketch the temperature distribution, indicating any features or boundary conditions used. [20%]

(d) Considering the situation where both inner cylinder and outer wall start from temperature T_0, but only the outer temperature is kept fixed as the shaft rotates, explain and sketch how the temperature of the fluid and the shaft will evolve as a function of time. Add to your description any equations or boundary conditions to be applied. [10%]

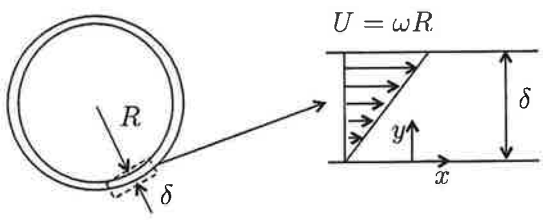

Fig. 1

3 A biological test kit involves the continuous injection of reactant into a uniform, one-dimensional carrier flow well-mixed with dye, as shown in Fig. 2. The dye D reacts with the reactant R forming a product P, according to $R+D \rightarrow P$. The rate of reactant disappearance per unit volume is $w_R = k\rho^2 Y_R Y_D$, where ρ is the constant density of the mixture, Y_i is the mass fraction of component i, and k is the reaction rate constant. The reactant and product have the same diffusivity $\mathscr{D}$. Reactants and products flow uniformly and steadily with velocity U along the x-direction. The concentration of reactant at $x=0$ is $Y_{R,0}$ and the net mass flux of reactant at the origin is $\dot{m}_{R,0}$.

(a) Consider a differential element dx along the flow path. Show that the governing equation for the reactant mass fraction Y_R is given by

$$\rho U \frac{dY_R}{dx} = \rho \mathscr{D} \frac{d^2 Y_R}{dx^2} - w_R .$$

State all assumptions made along your derivation. [30%]

(b) In the following, assume that the dye mass fraction remains constant and equal to its initial concentration $Y_{D,0}$ along the whole reactor.

(i) Determine the variation of mass fraction of reactant with x, when diffusion is negligible compared to the other terms. [30%]

(ii) For the case in which *both* reactive and diffusive fluxes are not negligible, obtain a functional expression for the concentration as a function of x and the parameters given, and explain clearly how the constants in the equation are obtained from the boundary conditions. Do not solve for the final constants. [30%]

(c) Explain how you would obtain a solution to the reactant concentration as a function of distance if the dye concentration were allowed to vary along the reactor. [10%]

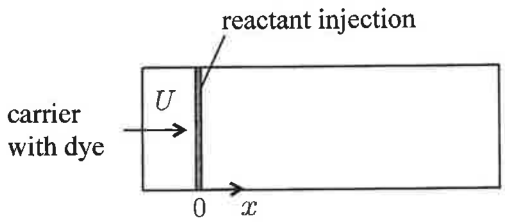

Fig. 2

4 A stream of fluid of mass flow rate $\dot{m}_h$ at an initial temperature $T_{h,i}$ is to be cooled by a stream of cooling flow rate $\dot{m}_c$ at an initial temperature $T_{c,i}$ in a heat exchanger with a concentric tube counterflow arrangement. The outlet temperatures for the hot and cold streams are $T_{h,o}$ and $T_{c,o}$, respectively. The hot and cold streams have constant specific heat capacities c_h and c_c, respectively. Assume that $C_h = \dot{m}_h c_h < \dot{m}_c c_c = C_c$.

(a) Determine the maximum possible overall rate of heat transfer between the two streams. [10%]

(b) Write an expression for the effectiveness ε of the heat exchanger, defined as the ratio of the actual to the maximum rate of heat transfer, as a function of the stream inlet and outlet temperatures. [10%]

(c) Show that for the counterflow heat exchanger,

$$\ln\left(\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}}\right) = -n(1 - C_r)$$

where $n = UA/C_h$ and $C_r = C_h/C_c$, U is the overall heat transfer coefficient and A is the total heat exchange area. [20%]

(d) Starting from the equation in (c), and expressing the ratio of temperature differences as a function of ε and C_r, show that [20%]

$$\varepsilon = \frac{1 - \exp[-n(1 - C_r)]}{1 - C_r \exp[-n(1 - C_r)]}$$

(e) Explain how the inner heat transfer coefficient h_i can be estimated if only the friction factor C_f and the fluid properties are known. [20%]

(f) Explain what techniques can be used to increase the heat transfer rate per unit volume of the heat exchanger. [20%]

END OF PAPER

Final version

Data Sheet: Conservation equations for steady, 2D constant density flows

Mass balance

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

x-Momentum balance

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial x^2} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + g_x$$

Thermal energy balance

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \alpha\frac{\partial^2 T}{\partial x^2} + \frac{\mu}{\rho c_p}\Phi$$

$$\mu\Phi = \tau_{xx}\frac{\partial u}{\partial x} + \tau_{yy}\frac{\partial v}{\partial y} + \tau_{xy}\frac{\partial u}{\partial y} + \tau_{yx}\frac{\partial v}{\partial y}$$

$$\tau_{xx} = \tau_{yy} = -\frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad \text{and} \quad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$