ENGINEERING TRIPOS PART IIA

Friday 4 May 2012 9 to 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) What is the physical significance of the *wavefunction* of a particle? [20%]

(b) The wavefunction, ψ, of a particle moving along an arbitrary x-direction through free space may be expressed as

$$\psi = A\exp[\mathrm{j}(kx - \omega t)]$$

where k is the wavenumber, ω is the angular frequency, t is time and A is a constant.

(i) Using this wavefunction, show that the momentum of the particle, p, may be found from

$$p\psi = -\mathrm{j}\hbar\frac{\partial\psi}{\partial x}$$ [20%]

(ii) Using the same wavefunction, show that the total energy of the particle, E, may be found from

$$E\psi = \mathrm{j}\hbar\frac{\partial\psi}{\partial t}$$ [20%]

(c) With reference to your answers to parts (a) and (b), explain why the wavefunction must be a single-valued function of space coordinates and time, and why both the wavefunction and its first derivative with respect to space coordinates must be continuous. [20%]

(d) Hence, explain why a particle in an infinitely deep potential well of finite width can only exist with particular values of total energy. [20%]

2 (a) Why must *Fermi-Dirac statistics* be used to determine the probability that an electron occupies a particular quantum mechanical state rather than Boltzmann statistics? Explain the meaning of the *Fermi energy*. [25%]

(b) Using the *Free Electron Theory*, show that the electron Fermi energy, E_F, in a metal at a temperature of absolute zero is given by

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

where m is the electron mass and n is the number of free electrons per unit volume. [35%]

(c) Calculate the Fermi energy of magnesium, which is in Group II of the Periodic Table and has an atomic density of 3.92×10^{28} m^{-3}. [20%]

(d) Calculate the wavelength and velocity of an electron in magnesium whose energy is the Fermi energy which you have calculated in part (c). Comment on your answers. [20%]

NOTE: The density of states according to the Free Electron Theory is given by

$$g(E)dE = \frac{V}{2\pi^2\hbar^3}(2m)^{3/2}E^{1/2}dE$$

where V is the volume of material.

3 (a) Sketch the band diagram for an unbiased and forward biased p^+n junction diode. Indicate the position of the Fermi levels and the amount of band bending in both cases. [10%]

(b) The Continuity equation for excess holes in a semiconductor may be written as

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_h} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}$$

Explain briefly what the various terms represent and under what conditions the equation is valid in its stated form. [15%]

(c) The injected hole concentration at the edge of the depletion region on the n-type side of a p^+n junction is given by

$$p(x=0) = p_{n0} \exp\left(\frac{eV}{kT}\right)$$

where p_{n0} is the equilibrium concentration of holes in the bulk of the n-type semiconductor and V is the applied forward bias. Starting from the Continuity equation as stated in part (b), derive an expression for the variation in excess hole concentration $\Delta p = p - p_{n0}$ with distance x away from the edge of the depletion region into the n-type region. State all assumptions made and sketch the solution. [40%]

(d) The current across the p^+n junction can be expressed as

$$I = I_S\left[\exp\left(\frac{eV}{\eta kT}\right) - 1\right]$$

where I_s is the reverse saturation current and η is the ideality factor. Explain why η is typically measured to be larger than 1 and discuss why real diodes further deviate from this ideal equation. [15%]

(cont.

(e) For device fabrication, an Ohmic contact is required to the p-type semiconductor. The semiconductor is silicon with a band gap of 1.1 eV, an electron affinity of 4.05 eV, an effective density of states in the valence band of 1.04×10^{25} m^{-3} and an acceptor density of 10^{23} m^{-3}. Calculate the minimum work function the metal should ideally have to form an Ohmic contact. State all assumptions made. Draw a band diagram for such a contact. [20%]

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4 An ideal metal oxide semiconductor (MOS) capacitor is formed on an n-type Si substrate with a doping density of $N_D = 10^{23}$ m^{-3}. The SiO$_2$ thickness is 10 nm and its relative permittivity is 3.9. The intrinsic carrier concentration of silicon is 1.5×10^{16} m^{-3}. Assume room temperature (298 K) operation.

(a) (i) Explain what is meant by the terms *inversion* and *strong inversion* and state if they occur for a negative or positive gate bias for the given MOS capacitor.

(ii) Draw a graph of the distribution of charge as a function of position across the MOS capacitor at the point of strong inversion.

(iii) Draw a band diagram for the MOS capacitor at the point of strong inversion. In the band diagram, indicate the externally applied bias, the voltage drop across the n-type Si and the position of the Fermi level with respect to an intrinsic level that lies midgap. [20%]

(b) Calculate the channel conductivity near the Si-SiO$_2$ interface at strong inversion. Assume a carrier field effect mobility of 0.02 m^{2}V^{-1}s^{-1} and state any further assumptions. [10%]

(c) Calculate the threshold voltage which has to be applied to the gate to induce strong inversion. Assume that the width of the depletion region in the n-type Si is 100 nm at strong inversion and that the work functions of the metal and the semiconductor are equal. State any other assumptions made. [35%]

(d) Sketch the capacitance-voltage (C-V) characteristics of the MOS capacitor. Discuss how and why the C-V characteristics will change for measurements at high frequencies. [25%]

(e) Consider an MOS capacitor for which the work function of the gate metal is 1 eV smaller than that of the n-type Si. Calculate the shift in threshold voltage compared to the ideal MOS capacitor above. [10%]

END OF PAPER