ENGINEERING TRIPOS PART IIA

Tuesday 8 May 2012 9.00 to 10.30

Module 3C6

VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Data Sheet: 3C5 Dynamics and 3C6 Vibration (6 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final Version

1 (a) A string of circular cross-section is made from material of density ρ. It has radius a and is stretched with tension P between fixed points $x = 0$ and $x = L$. It can undergo small transverse vibrations with displacement $w(x)$. Assuming that the lowest mode of vibration takes the form

$$w(x) = \sin\frac{\pi x}{L},$$

use information from the Data Sheet to calculate the potential and kinetic energies for this mode, and hence use the Rayleigh quotient to find the natural frequency. [25%]

(b) Express this frequency in terms of the tensile stress in the string, and hence show that a string of given length, made of material with a breaking stress σ_f, always breaks at the same frequency regardless of the diameter. [20%]

(c) The fixed point at $x = 0$ is now replaced by a dashpot of rate R, representing energy transfer to the body of a musical instrument. Assuming that R is sufficiently high that the mode shape is unchanged, write down the force acting on the dashpot and hence find the rate of energy dissipation in the lowest mode. Making use of the potential energy from part (a) show that the proportion of energy absorbed by the dashpot during one cycle of vibration of the string is given approximately by

$$\frac{K\sqrt{P\rho}}{R},$$

where K is a constant to be determined. [30%]

(d) A piano uses strings of different lengths for different notes, whereas a guitar uses strings of the same length tuned to different notes. Discuss briefly the implications of the results of parts (b) and (c) for the design of these instruments and their strings. Issues of practical convenience, the desired frequency range of the instrument and the relative loudness of notes should be considered. [25%]

2 (a) A flexible beam with mass per unit length m and bending stiffness EI can execute small-amplitude bending motion with displacement $w(x)$. It has freely pinned boundaries at $x = 0$ and $x = L$. Find expressions for the mode shapes and natural frequencies of the beam. [30%]

(b) The beam from part (a) is now placed under tension P. Explain briefly why the equation governing small transverse vibration becomes

$$m\frac{\partial^2 w}{\partial t^2} - P\frac{\partial^2 w}{\partial x^2} + EI\frac{\partial^4 w}{\partial x^4} = 0 \quad .$$

Show that the mode shapes found in (a) still satisfy the governing equation and boundary conditions, and obtain an expression for the new natural frequencies. [30%]

(c) Explain what happens when the tension P becomes negative, and hence obtain an expression for the buckling threshold of the beam. Sketch the shape in which the beam will buckle. [20%]

(d) Explain briefly how a similar approach could be used to analyse the buckling of a free-standing column under its self-weight. (Detailed calculations are not required.) [10%]

(e) An underground car park has a roof supported by a large number of identical concrete columns. It is suspected that the load is not being carried equally by these columns. Suggest a non-destructive way to evaluate inequalities of compressive load in the columns, and to assess whether any are close to their buckling threshold. [10%]

3 Figure 1a shows a model for examining torsional vibration of a 4-cylinder diesel engine with a flywheel driving a generator. The engine is represented by four discs, each with polar moment of inertia J connected by light elastic shaft sections, each with torsional stiffness k. The flywheel and generator have polar moments of inertia I_1 and I_2 respectively and are connected by a light shaft with torsional stiffness S. The system is supported by frictionless bearings and gravity may be neglected. The shape of the first mode with non-zero natural frequency and the system parameters have been measured. These are provided in Fig. 1b.

(a) Derive mass $[M]$ and stiffness $[K]$ matrices for the system in terms of the generalized coordinates $\mathbf{y} = [\theta_1 \quad \theta_2 \quad .... \quad \theta_6]^{\mathrm{T}}$ where θ_{j} is the absolute angular displacement of disc j. [20%]

(b) By considering a free vibration solution of the form $\mathbf{y} = \mathbf{u}e^{i\omega t}$, show that the natural frequencies and natural mode shapes can be determined from the equation

$$\left([K] - \omega^2[M]\right)\mathbf{u} = 0$$

Hence verify that the rigid body rotation mode with eigenvector $\omega = 0$ satisfies this equation. [20%]

(c) Using the measured mode shape in Fig. 1b, calculate the first torsional natural frequency. [20%]

(d) A simple approximation to the first torsional mode shape is apparent from the data in Fig. 1b. Use this approximation to calculate the first torsional natural frequency. Compare this frequency with the result of part (c). Comment on the orthogonality of this mode with respect to other torsional vibration modes of the system. [25%]

(e) Comment on the applicability of the idealisation in Fig. 1 for modelling the torsional vibration of a real diesel engine - flywheel - generator system. [15%]

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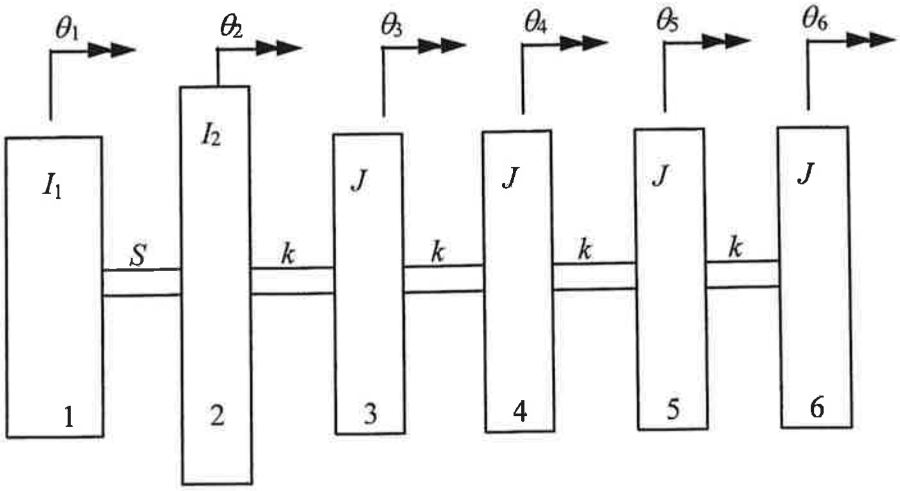

Fig. 1a

First natural mode shape:

$\theta_1 = 1.0$	$I_1 = 2715 \text{ kgm}^2$
$\theta_2 = -0.281$	$I_2 = 8484 \text{ kgm}^2$
$\theta_3 = -0.284$	$J = 289.6 \text{ kgm}^2$
$\theta_4 = -0.286$	
$\theta_5 = -0.288$	$S = 1.53 \times 10^6$ Nm/rad
$\theta_6 = -0.289$	$k = 7.64 \times 10^7$ Nm/rad

Fig. 1b

4 Three masses are connected together and to fixed points by springs, as shown in Fig. 2. Each mass can move in the horizontal direction only, without rotation. The small displacements from equilibrium are denoted by the coordinate vector $[x_1 \quad x_2 \quad y]^T$.

(a) Show that the stiffness matrix for small vibration of this system is

$$\begin{bmatrix} 2k & 0 & -k \\ 0 & 2k & -k \\ -k & -k & 6k \end{bmatrix}$$ [20%]

(b) Find the natural frequencies and corresponding mode shapes. Illustrate these mode shapes with sketches. [40%]

(c) Sketch log-amplitude plots of the transfer functions describing:

(i) the displacement response of mass $4m$ due to harmonic forcing of mass $4m$ [20%]

(ii) the velocity response of one mass m due to harmonic forcing of the other mass m. [20%]

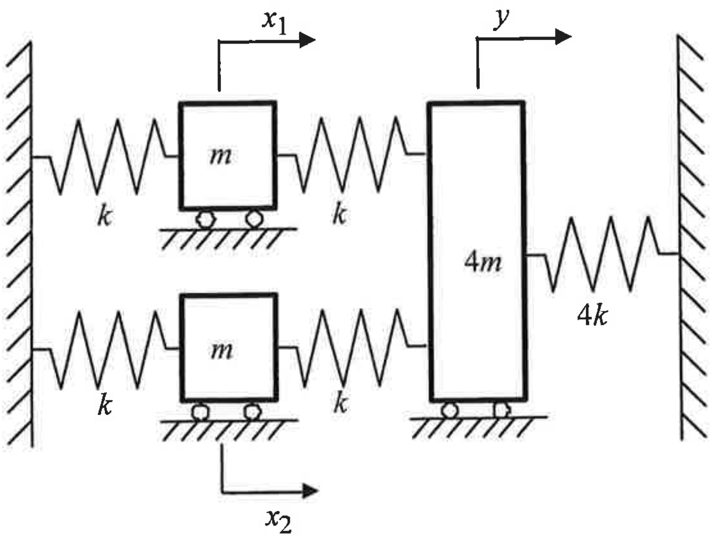

Fig. 2

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