ENGINEERING TRIPOS PART IIA

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Thursday 3 May 2012 9 to 10.30

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Module 3C8

MACHINE DESIGN

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments:*
*Module 3C8 data sheet (9 pages)*
*Copy of Fig. 1*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

FINAL version

1 Figure 1 shows the maximum torque as a function of speed for the drive motor of a vehicle. The radius of the vehicle's wheels is 0.3 m and the motor drives the wheels through a gearbox with speed ratio $G$ (motor speed divided by wheel speed). The vehicle has resistance to constant speed motion on horizontal ground given by $F = CV$, where $F$ is the resistance force in N, $V$ is the vehicle speed in m s$^{-1}$ and $C = (200/7)$ N s m$^{-1}$. The vehicle mass is 1000 kg.

(a) For $G = 6$, plot the load characteristic of the vehicle on the attached copy of Fig. 1 and hence determine the maximum speed of the vehicle for this speed ratio. [25%]

(b) Determine the speed ratio $G$ that allows the vehicle to travel at its maximum possible speed, and state this maximum speed. [20%]

(c) For $G = 6/\sqrt{7}$ plot a graph of maximum vehicle acceleration as a function of vehicle speed. Hence, using an approximate numerical method, or otherwise, estimate the minimum time taken for the vehicle to accelerate from rest to 40 m s$^{-1}$. [35%]

(d) The vehicle accelerates from rest, initially using speed ratio $G = 6$, then switching to $G = 6/\sqrt{7}$. Estimate the vehicle speed at which the ratio should be changed in order to minimise the time to reach 40 m s$^{-1}$. Explain why this speed is less than the maximum speed for $G = 6$. [20%]

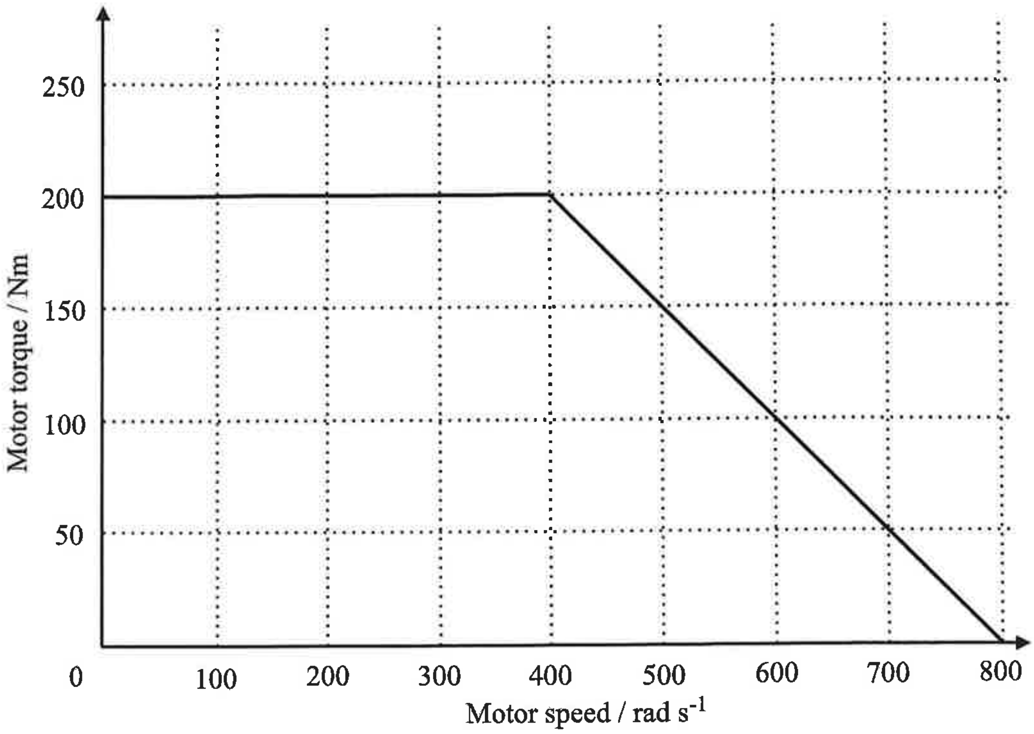



Fig. 1

A copy of Fig. 1 is attached after the data sheet.

2 (a) Summarize briefly the idealizations that are made in the Hertz theory of elastic contact. [10%]

(b) A roller bearing has an inner race of radius $R$ and $N$ cylindrical rollers each of radius $a$ and axial length $L$. A radial force $P$ acts on the bearing as illustrated in Fig. 2. $N$ is as large as possible so that the bearing is 'full'. The angular separation of rollers is then $\theta$.

(i) Confirm that if $N = 8$ then $a \approx 0.62R$. [10%]

(ii) By assuming a suitable variation of contact force with angular deviation from the direction of $P$, make an estimate of the contact force $F_1$ on the most heavily loaded roller. Confirm that your answer is consistent with the more general design rule that for even values of $N$,

$$P = 0.25 F_1 N .$$ [25%]

(iii) For the bearing with $N = 8$ show that on the roller in (ii) the maximum Hertz pressure $p_0$ is given by

$$p_0 = 0.645\sqrt{\frac{PE^*}{LR}}$$

where $E^*$ is the contact modulus. [25%]

(iv) A bearing with a significantly larger number of rollers has the same applied load $P$, inner race radius $R$ and roller length $L$. At any value of $N$, which can be assumed even, the radius of the rollers is such that the bearing is again full. When $N$ is large, so that $a \ll R$, estimate the maximum reduction in the peak Hertz pressure that might be expected compared to the value when $N = 8$. [30%]

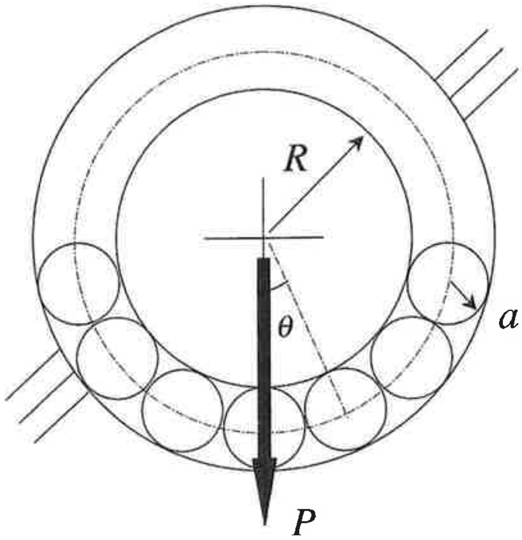



Fig. 2

3 (a) List four important criteria by which rolling-element bearings might be selected for a particular application. Using these four criteria construct a table comparing the performance of: (i) deep-groove ball bearings; (ii) double-row angular contact ball bearings; (iii) cylindrical roller bearings; and (iv) taper roller bearings. [15%]

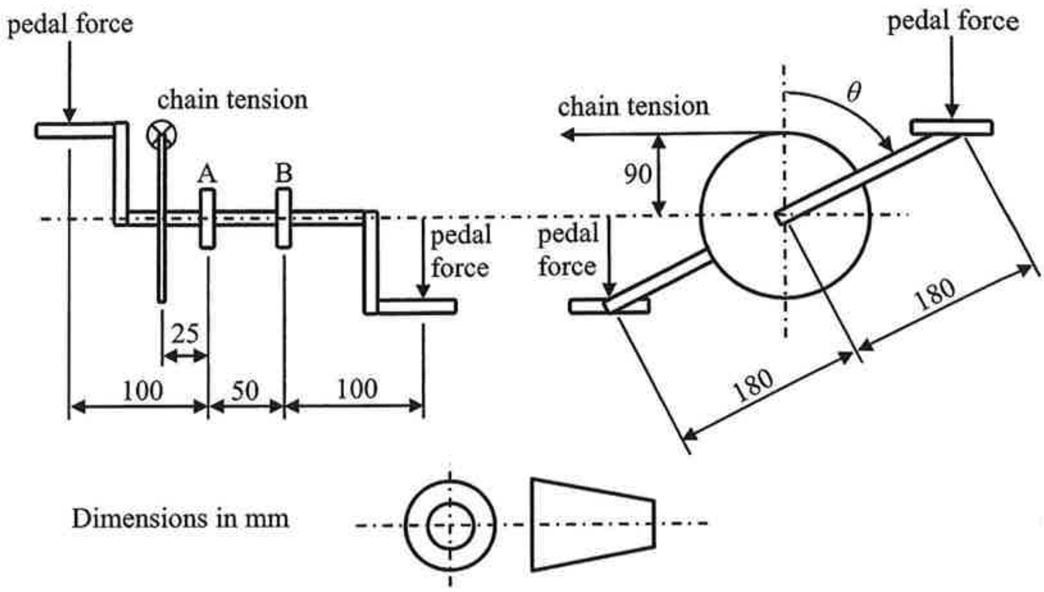
(b) Figure 3 shows the side and front elevations of the crank, pedal and sprocket assembly of a bicycle. The crank is supported by two bearings whose radial forces act at A and B. The crank is shown at angle $\theta$ to the vertical and rotates at constant speed in the direction of positive $\theta$. A *constant* vertical force $F$ is applied alternately to the left and right pedals by the rider as the crank rotates. The force $F$ is applied to the downward-moving pedal; the upward-moving pedal has zero applied force. The force is reacted by the radial bearing forces and by tensile force $T$ in the top section of chain; there is zero force in the bottom section of chain. Inertia forces are negligible. Dimensions are given in the figure.

(i) Explain why the radial force on each bearing is a maximum when the crank is horizontal ($\theta = \pi/2$ or $3\pi/2$). [10%]

(ii) Show that the maximum radial force on bearing A is $3\sqrt{2}F$ and on bearing B is $\sqrt{10}F$. For each bearing state the corresponding crank angle. [35%]

(iii) The bearings at A and B are identical deep groove ball bearings. Force $F$ is 100 N, the wheel diameter is 0.7 m, the crank rotates once for every three rotations of the wheel, lubrication is ideal, and the bicycle travels 20,000 km. Determine the required dynamic load rating of the bearing if a 1% probability of failure is allowed. [25%]

(iv) Describe, with the aid of a sketch, where damage in bearing A is likely to occur first. [15%]




Note that the force applied to the upward-moving pedal is zero.

Fig. 3

4 Consider an epicyclic gear with sun, annulus, and four planet wheels. The planet wheels each have pitch radius $r$ and 15 teeth. The sun wheel has the same radius as the planet wheels. The gear teeth have a standard involute geometry with facewidth $w$, pressure angle $\phi = 20°$ and addendum equal to the module. The contact modulus is $E^*$.

(a) The sun wheel is driven by a torque $T$ while the planet carrier is held fixed. Losses in the gear can be neglected.

(i) Find expressions for the line load $P'$ along each of the eight pressure lines, in terms of $r$, $w$ and $T$. [15%]

(ii) Due to inaccuracies in the gear profiles, load is carried by only one contact per pressure line. Find an expression for the maximum Hertz pressure in terms of $r$, $w$, $T$ and $E^*$. [30%]

(b) Find expressions for the torques acting on the annulus and planet carrier in the following cases.

(i) The sun wheel is driven by a torque $T$ while the planet carrier is held fixed. Assume no losses in the gear. [15%]

(ii) The sun wheel is driven by a torque $T$ while the planet carrier is held fixed. The gear has an overall efficiency of 95%. [10%]

(iii) The sun wheel is driven by a torque $T$ and the planet carrier is driven in the opposite direction to the sun wheel at 1/10th of the speed of the sun wheel. The gear has an overall efficiency of 95%. [30%]

**END OF PAPER**

# ENGINEERING TRIPOS Part IIA

## Module 3C8 Data Sheet

### ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

Effective curvature $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where $R_1, R_2$ are the radii of curvature of the two bodies (convex positive).

Contact modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

where $E_1, E_2$ and $\nu_1, \nu_2$ are Young's moduli and Poisson's ratios

| | <u>Line contact</u><br>width $2b$ ; load $P'$ per unit length | <u>Circular contact</u><br>diameter $2a$ ; load $P$ |
|---|---|---|
| Semi contact width or contact radius | $b = 2\left\{\frac{P'R}{\pi E^*}\right\}^{1/2}$ | $a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$ |
| Maximum contact pressure ('Hertz stress') | $p_0 = \left\{\frac{P'E^*}{\pi R}\right\}^{1/2}$ | $p_0 = \frac{1}{\pi}\left\{\frac{6PE^{*2}}{R^2}\right\}^{1/3}$ |
| Approach of centres | $\delta = \frac{2P'}{\pi}\left[\frac{1-\nu_1^2}{E_1}\left\{\ln\left(\frac{4R_1}{b}\right) - \frac{1}{2}\right\} + \frac{1-\nu_2^2}{E_2}\left\{\ln\left(\frac{4R_2}{b}\right) - \frac{1}{2}\right\}\right]$ | $\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{P^2}{E^{*2}R}\right\}^{1/3}$ |
| Mean contact pressure | $\bar{p} = \frac{P'}{2b} = \frac{\pi}{4}p_0$ | $\bar{p} = \frac{P}{\pi a^2} = \frac{2}{3}p_0$ |
| | $\tau_{max} = 0.300\,p_0$<br>at $x = 0, z = 0.79b$ | $\tau_{max} = 0.310\,p_0$<br>at $r = 0, z = 0.48a$ for $\nu = 0.3$ |
| Maximum tensile stress | zero | $\frac{1}{3}(1-2\nu)p_0$ at $r = a, z = 0.79b$ |

<u>Mildly elliptical contacts</u>

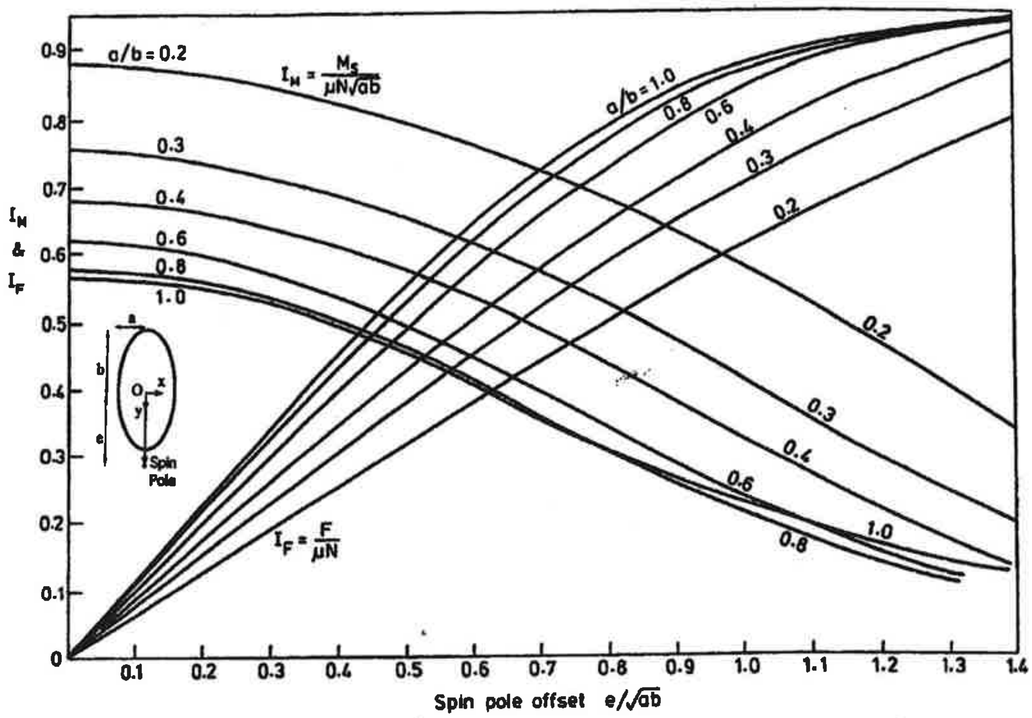
If the gap at zero load is $h = \frac{1}{2}\mathrm{A}x^2 + \frac{1}{2}\mathrm{B}y^2$ and $0.2 < \mathrm{A/B} < 5$ then

ratio of semi-axes $b/a \cong (\mathrm{A/B})^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with $R = (\mathrm{AB})^{-1/2}$ or better $R_e = [\mathrm{AB}(\mathrm{A}+\mathrm{B})/2]^{-1/3}$ .

For **approach** use circular contact equation with $R = (\mathrm{AB})^{-1/2}$ (**not** $R_e$)

Hertzian contact frictional losses




## INVOLUTE GEARING

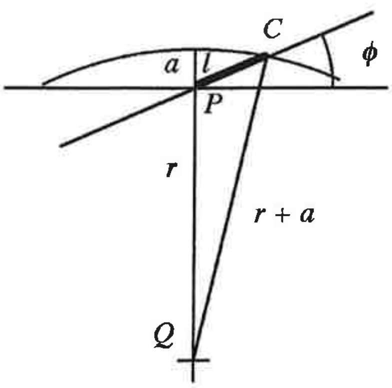
Spur gears

pitch cylinder radii $r$
base cylinder radii $r_b$
addendum cylinder radii $r_a$
number of teeth $N$
(with suffix 1 or 2)

addendum $a = r_a - r$
pressure angle $\phi$

circumferential pitch $p = 2\pi r/N$
base pitch $p_b = p\cos\phi$
module $m = p/\pi = 2r/N$
ratio of contact $r_c$
radius of curvature at pitch point $\rho = r\sin\phi$

Path of contact




$$l = \left\{r^2\sin^2\phi + a(2r+a)\right\}^{1/2} - r\sin\phi$$

For a standard 20° spur wheel with $N$ teeth of module $m$ this becomes

$$\frac{l}{m} = \left(0.02924N^2 + N + 1\right)^{1/2} - 0.1710N$$

Standard tooth forms

Addendum $a = m$, Dedendum $= \frac{7}{6}m$, pressure angle = 20°.

Modules:

1.0 – 4.0 mm in 0.25 mm steps

7.0 – 16.0 mm in 1.0 mm steps

24.0 – 45.0 mm in 3.0 mm steps

0.3 – 1.0 mm in 0.1 mm steps

4.0 – 7.0 mm in 0.5 mm steps

16.0 – 24.0 mm in 2.0 mm steps

45.0 – 75.0 mm in 5.0 mm steps

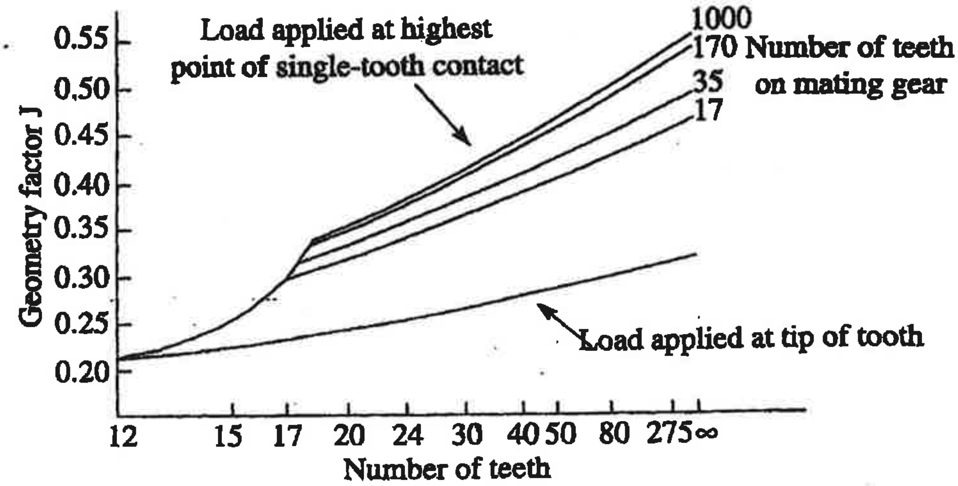
Friction in spur gears

$$\frac{\text{average friction loss}}{\text{power transmitted}} \approx \mu\pi\left\{\frac{1}{N_1} + \frac{1}{N_2}\right\}$$

Tooth failure

Allowable bending stress $\sigma_b$ according to AGMA guidelines given by $\sigma_b = \frac{P'_T}{Jm}$

where $P'_T$ is force per unit face-width acting tangentially to pitch circle and $J$ given in the figure below for 20° spur gears. Typical values of $\sigma_b$ shown in table.




Typical allowable tooth stresses (AGMA)

| Material | Condition | Bending fatigue strength $\sigma_b$ (MPa) | Surface fatigue strength $\sigma_s$ (MPa) |
|---|---|---|---|
| Steel | Through hardened and tempered | 170-390 | 590-1200 |
| | Carburised and case hardened | 380-480 | 1250-1550 |
| Cast iron | As cast | 69-90 | 450-590 |
| Nodular iron | Quenched, annealed and tempered | 150-300 | 500-800 |
| Malleable iron | Pearlitic | 70-145 | 500-650 |

## EPICYCLIC SPEED RULE

$$\omega_s = (1+R)\omega_c - R\omega_a \quad \text{where} \quad R = \frac{A}{S}$$

## ROLLING ELEMENT BEARINGS

Fatigue life

$L = a_1 a_{23}(C/P)^p$ p = 3 for ball and 10/3 for roller bearings

| | | | | | | |
|---|---|---|---|---|---|---|
| Fatigue probability % | 10 | 5 | 4 | 3 | 2 | 1 |
| Life adjust factor $a_1$ | 1 | 0.62 | 0.53 | 0.44 | 0.33 | 0.21 |

Bearing choice

The information on the following pages concerning loads, viscosities and standard bearing sizes and ratings is extracted from the SKF General Bearing Catalogue and is copied with permission. 

MPFS, DJC, JAW
November 07

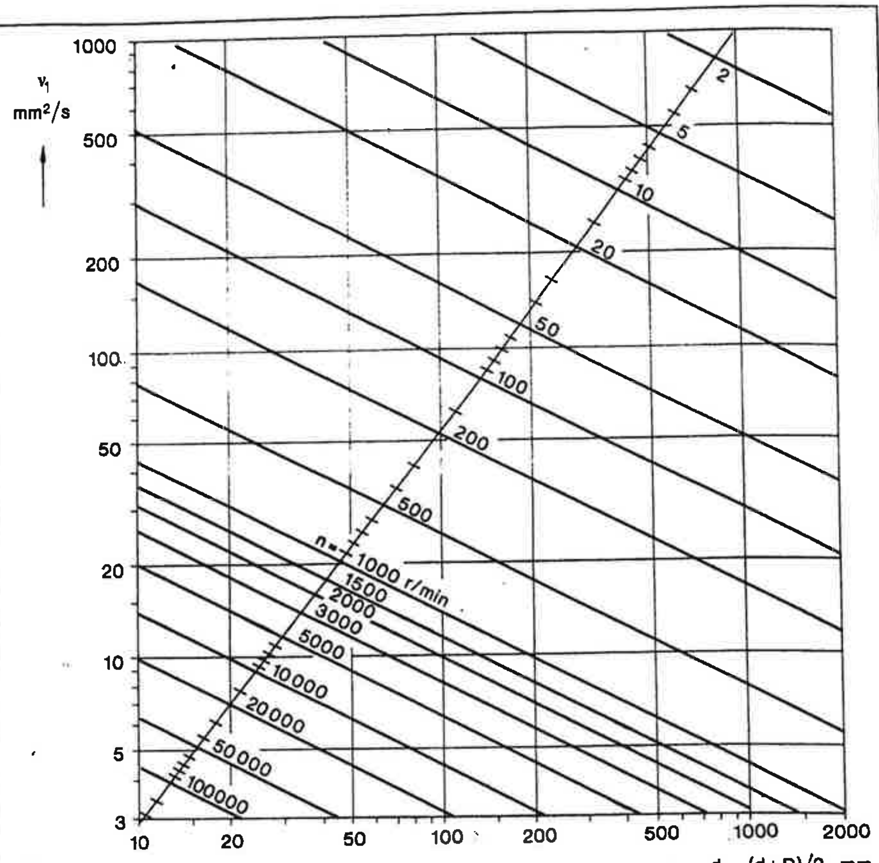



Diagram 1

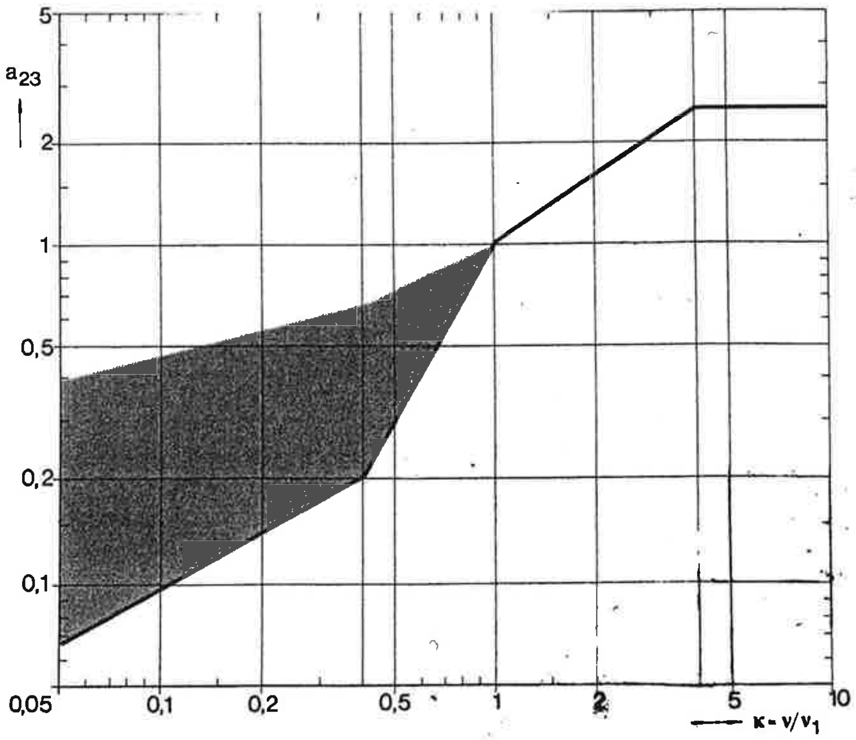
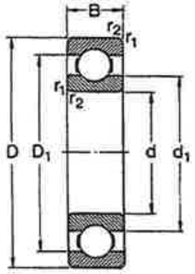


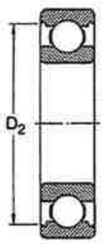

Diagram 3

6

## Deep groove ball bearings
**single row**
d 35–55 mm



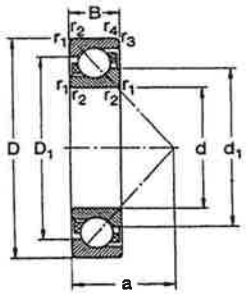

With full outer ring shoulders




With recessed outer ring shoulders

| Principal dimensions d | D | B | Basic load ratings dynamic C | static $C_0$ | Fatigue load limit $P_u$ | Speed ratings Lubrication grease | oil | Mass | Designation |
|---|---|---|---|---|---|---|---|---|---|
| mm | | | N | | N | r/min | | kg | – |
| 35 | 47 | 7 | 4 750 | 3 200 | 166 | 13 000 | 16 000 | 0,030 | 61807 |
| | 55 | 10 | 9 560 | 6 200 | 290 | 11 000 | 14 000 | 0,080 | 61907 |
| | 62 | 9 | 12 400 | 8 150 | 375 | 10 000 | 13 000 | 0,11 | 16007 |
| | 62 | 14 | 15 900 | 10 200 | 440 | 10 000 | 13 000 | 0,16 | 6007 |
| | 72 | 17 | 25 500 | 15 300 | 655 | 9 000 | 11 000 | 0,29 | 6207 |
| | 80 | 21 | 33 200 | 19 000 | 815 | 8 500 | 10 000 | 0,46 | 6307 |
| | 100 | 25 | 55 300 | 31 000 | 1 290 | 7 000 | 8 500 | 0,95 | 6407 |
| 40 | 52 | 7 | 4 940 | 3 450 | 186 | 11 000 | 14 000 | 0,034 | 61808 |
| | 62 | 12 | 13 800 | 9 300 | 425 | 10 000 | 13 000 | 0,12 | 61908 |
| | 68 | 9 | 13 300 | 9 150 | 440 | 9 500 | 12 000 | 0,13 | 16008 |
| | 68 | 15 | 16 800 | 11 600 | 490 | 9 500 | 12 000 | 0,19 | 6008 |
| | 80 | 18 | 30 700 | 19 000 | 800 | 8 500 | 10 000 | 0,37 | 6208 |
| | 90 | 23 | 41 000 | 24 000 | 1 020 | 7 500 | 9 000 | 0,63 | 6308 |
| | 110 | 27 | 63 700 | 36 500 | 1 530 | 6 700 | 8 000 | 1,25 | 6408 |
| 45 | 58 | 7 | 6 050 | 4 300 | 228 | 9 500 | 12 000 | 0,040 | 61809 |
| | 68 | 12 | 14 000 | 9 800 | 465 | 9 000 | 11 000 | 0,14 | 61909 |
| | 75 | 10 | 15 600 | 10 800 | 520 | 9 000 | 11 000 | 0,17 | 16009 |
| | 75 | 16 | 20 800 | 14 600 | 640 | 9 000 | 11 000 | 0,25 | 6009 |
| | 85 | 19 | 33 200 | 21 600 | 915 | 7 500 | 9 000 | 0,41 | 6209 |
| | 100 | 25 | 52 700 | 31 500 | 1 340 | 6 700 | 8 000 | 0,83 | 6309 |
| | 120 | 29 | 76 100 | 45 000 | 1 900 | 6 000 | 7 000 | 1,55 | 6409 |
| 50 | 65 | 7 | 6 240 | 4 750 | 250 | 9 000 | 11 000 | 0,052 | 61810 |
| | 72 | 12 | 14 600 | 10 400 | 500 | 8 500 | 10 000 | 0,14 | 61910 |
| | 80 | 10 | 16 300 | 11 400 | 560 | 8 500 | 10 000 | 0,18 | 16010 |
| | 80 | 16 | 21 600 | 16 000 | 710 | 8 500 | 10 000 | 0,26 | 6010 |
| | 90 | 20 | 35 100 | 23 200 | 980 | 7 000 | 8 500 | 0,46 | 6210 |
| | 110 | 27 | 61 800 | 38 000 | 1 600 | 6 300 | 7 500 | 1,05 | 6310 |
| | 130 | 31 | 87 100 | 52 000 | 2 200 | 5 300 | 6 300 | 1,90 | 6410 |
| 55 | 72 | 9 | 8 840 | 6 800 | 360 | 8 500 | 10 000 | 0,083 | 61811 |
| | 80 | 13 | 15 900 | 11 400 | 560 | 8 000 | 9 500 | 0,19 | 61911 |
| | 90 | 11 | 19 500 | 14 000 | 695 | 7 500 | 9 000 | 0,26 | 16011 |
| | 90 | 18 | 28 100 | 21 200 | 900 | 7 500 | 9 000 | 0,39 | 6011 |
| | 100 | 21 | 43 600 | 29 000 | 1 250 | 6 300 | 7 500 | 0,61 | 6211 |
| | 120 | 29 | 71 500 | 45 000 | 1 900 | 5 600 | 6 700 | 1,35 | 6311 |
| | 140 | 33 | 99 500 | 62 000 | 2 600 | 5 000 | 6 000 | 2,30 | 6411 |

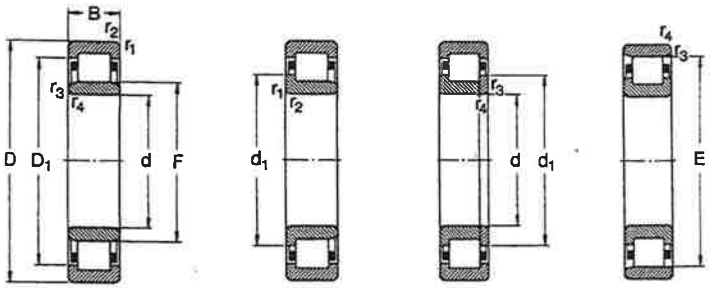
## Angular contact ball bearings
**single row**
d 10–65 mm




| Principal dimensions d | D | B | Basic load ratings dynamic C | static $C_0$ | Fatigue load limit $P_u$ | Speed ratings Lubrication grease | oil | Mass | Designation |
|---|---|---|---|---|---|---|---|---|---|
| mm | | | N | | N | r/min | | kg | – |
| 10 | 30 | 9 | 7 020 | 3 350 | 140 | 19 000 | 28 000 | 0,030 | 7200 BE |
| 12 | 32 | 10 | 7 610 | 3 800 | 160 | 18 000 | 26 000 | 0,036 | 7201 BE |
| | 37 | 12 | 10 600 | 5 000 | 208 | 17 000 | 24 000 | 0,060 | 7301 BE |
| 15 | 35 | 11 | 8 840 | 4 800 | 204 | 17 000 | 24 000 | 0,045 | 7202 BE |
| | 42 | 13 | 13 000 | 6 700 | 280 | 15 000 | 20 000 | 0,080 | 7302 BE |
| 17 | 40 | 12 | 11 100 | 6 100 | 260 | 15 000 | 20 000 | 0,065 | 7203 BE |
| | 47 | 14 | 15 900 | 8 300 | 355 | 13 000 | 18 000 | 0,11 | 7303 BE |
| 20 | 47 | 14 | 14 000 | 8 300 | 355 | 12 000 | 17 000 | 0,11 | 7204 BE |
| | 52 | 15 | 19 000 | 10 400 | 440 | 11 000 | 16 000 | 0,14 | 7304 BE |
| 25 | 52 | 15 | 15 600 | 10 200 | 430 | 10 000 | 15 000 | 0,13 | 7205 BE |
| | 62 | 17 | 26 000 | 15 600 | 655 | 9 000 | 13 000 | 0,23 | 7305 BE |
| 30 | 62 | 16 | 23 800 | 15 600 | 655 | 8 500 | 12 000 | 0,20 | 7206 BE |
| | 72 | 19 | 34 500 | 21 200 | 900 | 8 000 | 11 000 | 0,34 | 7306 BE |
| 35 | 72 | 17 | 30 700 | 20 800 | 880 | 8 000 | 11 000 | 0,28 | 7207 BE |
| | 80 | 21 | 39 000 | 24 500 | 1 040 | 7 500 | 10 000 | 0,45 | 7307 BE |
| 40 | 80 | 18 | 36 400 | 26 000 | 1 100 | 7 000 | 9 500 | 0,37 | 7208 BE |
| | 90 | 23 | 49 400 | 33 500 | 1 400 | 6 700 | 9 000 | 0,63 | 7308 BE |
| 45 | 85 | 19 | 37 700 | 28 000 | 1 200 | 6 700 | 9 000 | 0,42 | 7209 BE |
| | 100 | 25 | 60 500 | 41 500 | 1 730 | 6 000 | 8 000 | 0,85 | 7309 BE |
| 50 | 90 | 20 | 39 000 | 30 500 | 1 290 | 6 000 | 8 000 | 0,47 | 7210 BE |
| | 110 | 27 | 74 100 | 51 000 | 2 200 | 5 300 | 7 000 | 1,10 | 7310 BE |
| 55 | 100 | 21 | 48 800 | 38 000 | 1 630 | 5 600 | 7 500 | 0,62 | 7211 BE |
| | 120 | 29 | 85 200 | 60 000 | 2 550 | 4 800 | 6 300 | 1,40 | 7311 BE |
| 60 | 110 | 22 | 57 200 | 45 500 | 1 930 | 5 000 | 6 700 | 0,80 | 7212 BE |
| | 130 | 31 | 95 600 | 69 500 | 3 000 | 4 500 | 6 000 | 1,75 | 7312 BE |
| 65 | 120 | 23 | 66 300 | 54 000 | 2 280 | 4 500 | 6 000 | 1,00 | 7213 BE |
| | 140 | 33 | 108 000 | 80 000 | 3 350 | 4 300 | 5 600 | 2,15 | 7313 BE |

7

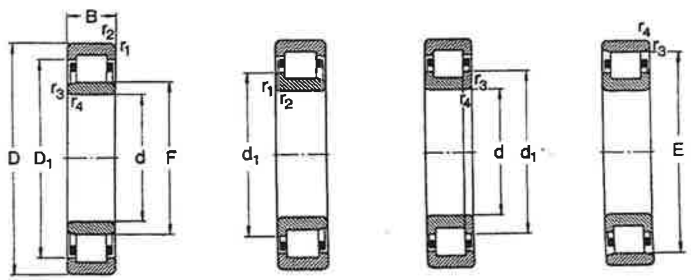
## Cylindrical roller bearings
**single row**
d **40–45** mm




Type NU | Type NJ | Type NUP | Type N

| Principal dimensions d | D | B | Basic load ratings dynamic C | static $C_0$ | Fatigue load limit $P_u$ | Speed ratings Lubrication grease | oil | Mass | Designation |
|---|---|---|---|---|---|---|---|---|---|
| mm | | | N | | N | r/min | | kg | – |
| 40 (cont.) | 90 | 23 | 80 900 | 78 000 | 10 200 | 6 700 | 8 000 | 0,65 | NU 308 EC |
| | 90 | 23 | 80 900 | 78 000 | 10 200 | 6 700 | 8 000 | 0,67 | NJ 308 EC |
| | 90 | 23 | 80 900 | 78 000 | 10 200 | 6 700 | 8 000 | 0,68 | NUP 308 EC |
| | 90 | 23 | 80 900 | 78 000 | 10 200 | 6 700 | 8 000 | 0,64 | N 308 EC |
| | 90 | 33 | 112 000 | 120 000 | 15 300 | 6 300 | 7 500 | 0,94 | NU 2308 EC |
| | 90 | 33 | 112 000 | 120 000 | 15 300 | 6 300 | 7 500 | 0,96 | NJ 2308 EC |
| | 90 | 33 | 112 000 | 120 000 | 15 300 | 6 300 | 7 500 | 0,98 | NUP 2308 EC |
| | 110 | 27 | 96 800 | 90 000 | 11 600 | 6 000 | 7 000 | 1,30 | NU 408 |
| | 110 | 27 | 96 800 | 90 000 | 11 600 | 6 000 | 7 000 | 1,30 | NJ 408 |
| | 110 | 27 | 96 800 | 90 000 | 11 600 | 6 000 | 7 000 | 1,35 | NUP 408 |
| 45 | 75 | 16 | 44 600 | 52 000 | 6 300 | 9 000 | 11 000 | 0,26 | NU 1009 EC |
| | 85 | 19 | 60 500 | 64 000 | 8 150 | 6 700 | 8 000 | 0,43 | NU 209 EC |
| | 85 | 19 | 60 500 | 64 000 | 8 150 | 6 700 | 8 000 | 0,44 | NJ 209 EC |
| | 85 | 19 | 60 500 | 64 000 | 8 150 | 6 700 | 8 000 | 0,45 | NUP 209 EC |
| | 85 | 19 | 60 500 | 64 000 | 8 150 | 6 700 | 8 000 | 0,43 | N 209 EC |
| | 85 | 23 | 73 700 | 81 500 | 10 600 | 6 700 | 8 000 | 0,52 | NU 2209 EC |
| | 85 | 23 | 73 700 | 81 500 | 10 600 | 6 700 | 8 000 | 0,54 | NJ 2209 EC |
| | 85 | 23 | 73 700 | 81 500 | 10 600 | 6 700 | 8 000 | 0,55 | NUP 2209 EC |
| | 85 | 23 | 73 700 | 81 500 | 10 600 | 6 700 | 8 000 | 0,52 | N 2209 EC |
| | 100 | 25 | 99 000 | 100 000 | 12 900 | 6 300 | 7 500 | 0,90 | NU 309 EC |
| | 100 | 25 | 99 000 | 100 000 | 12 900 | 6 300 | 7 500 | 0,92 | NJ 309 EC |
| | 100 | 25 | 99 000 | 100 000 | 12 900 | 6 300 | 7 500 | 0,95 | NUP 309 EC |
| | 100 | 25 | 99 000 | 100 000 | 12 900 | 6 300 | 7 500 | 0,88 | N 309 EC |
| | 100 | 36 | 138 000 | 153 000 | 20 000 | 5 600 | 6 700 | 1,30 | NU 2309 EC |
| | 100 | 36 | 138 000 | 153 000 | 20 000 | 5 600 | 6 700 | 1,30 | NJ 2309 EC |
| | 100 | 36 | 138 000 | 153 000 | 20 000 | 5 600 | 6 700 | 1,35 | NUP 2309 EC |
| | 120 | 29 | 106 000 | 102 000 | 13 400 | 5 600 | 6 700 | 1,65 | NU 409 |
| | 120 | 29 | 106 000 | 102 000 | 13 400 | 5 600 | 6 700 | 1,65 | NJ 409 |
| | 120 | 29 | 106 000 | 102 000 | 13 400 | 5 600 | 6 700 | 1,70 | NUP 409 |

## Cylindrical roller bearings
**single row**
d **50–55** mm

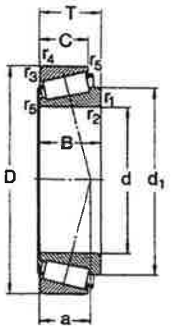



Type NU | Type NJ | Type NUP | Type N

| Principal dimensions d | D | B | Basic load ratings dynamic C | static $C_0$ | Fatigue load limit $P_u$ | Speed ratings Lubrication grease | oil | Mass | Designation |
|---|---|---|---|---|---|---|---|---|---|
| mm | | | N | | N | r/min | | kg | – |
| 50 | 80 | 16 | 30 600 | 34 500 | 4 000 | 8 500 | 10 000 | 0,31 | NU 1010 |
| | 90 | 20 | 64 400 | 69 500 | 8 800 | 6 300 | 7 500 | 0,48 | NU 210 EC |
| | 90 | 20 | 64 400 | 69 500 | 8 800 | 6 300 | 7 500 | 0,49 | NJ 210 EC |
| | 90 | 20 | 64 400 | 69 500 | 8 800 | 6 300 | 7 500 | 0,51 | NUP 210 EC |
| | 90 | 20 | 64 400 | 69 500 | 8 800 | 6 300 | 7 500 | 0,48 | N 210 EC |
| | 90 | 23 | 78 100 | 88 000 | 11 400 | 6 300 | 7 500 | 0,56 | NU 2210 EC |
| | 90 | 23 | 78 100 | 88 000 | 11 400 | 6 300 | 7 500 | 0,58 | NJ 2210 EC |
| | 80 | 23 | 78 100 | 88 000 | 11 400 | 6 300 | 7 500 | 0,59 | NUP 2210 EC |
| | 110 | 27 | 110 000 | 112 000 | 15 000 | 5 000 | 6 000 | 1,15 | NU 310 EC |
| | 110 | 27 | 110 000 | 112 000 | 15 000 | 5 000 | 6 000 | 1,15 | NJ 310 EC |
| | 110 | 27 | 110 000 | 112 000 | 15 000 | 5 000 | 6 000 | 1,20 | NUP 310 EC |
| | 110 | 27 | 110 000 | 112 000 | 15 000 | 5 000 | 6 000 | 1,15 | N 310 EC |
| | 110 | 40 | 161 000 | 186 000 | 24 500 | 5 000 | 6 000 | 1,70 | NU 2310 EC |
| | 110 | 40 | 161 000 | 186 000 | 24 500 | 5 000 | 6 000 | 1,75 | NJ 2310 EC |
| | 110 | 40 | 161 000 | 186 000 | 24 500 | 5 000 | 6 000 | 1,80 | NUP 2310 EC |
| | 130 | 31 | 130 000 | 127 000 | 16 600 | 5 000 | 6 000 | 2,00 | NU 410 |
| | 130 | 31 | 130 000 | 127 000 | 16 600 | 5 000 | 6 000 | 2,05 | NJ 410 |
| 55 | 90 | 18 | 57 200 | 69 500 | 8 300 | 7 000 | 8 500 | 0,40 | NU 1011 EC |
| | 100 | 21 | 84 200 | 95 000 | 12 200 | 6 000 | 7 000 | 0,66 | NU 211 EC |
| | 100 | 21 | 84 200 | 95 000 | 12 200 | 6 000 | 7 000 | 0,67 | NJ 211 EC |
| | 100 | 21 | 84 200 | 95 000 | 12 200 | 6 000 | 7 000 | 0,69 | NUP 211 EC |
| | 100 | 21 | 84 200 | 95 000 | 12 200 | 6 000 | 7 000 | 0,66 | N 211 EC |
| | 100 | 25 | 99 000 | 118 000 | 15 300 | 6 000 | 7 000 | 0,79 | NU 2211 EC |
| | 100 | 25 | 99 000 | 118 000 | 15 300 | 6 000 | 7 000 | 0,81 | NJ 2211 EC |
| | 100 | 25 | 99 000 | 118 000 | 15 300 | 6 000 | 7 000 | 0,82 | NUP 2211 EC |
| | 100 | 25 | 99 000 | 118 000 | 15 300 | 6 000 | 7 000 | 0,79 | N 2211 EC |
| | 120 | 29 | 138 000 | 143 000 | 18 600 | 4 800 | 5 600 | 1,45 | NU 311 EC |
| | 120 | 29 | 138 000 | 143 000 | 18 600 | 4 800 | 5 600 | 1,50 | NJ 311 EC |
| | 120 | 29 | 138 000 | 143 000 | 18 600 | 4 800 | 5 600 | 1,55 | NUP 311 EC |
| | 120 | 29 | 138 000 | 143 000 | 18 600 | 4 800 | 5 600 | 1,45 | N 311 EC |

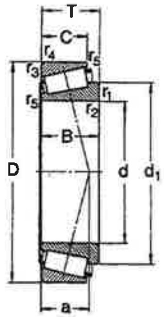
8

**Taper roller bearings**
**single row**
d 35–50 mm



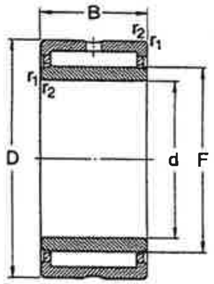

| Principal dimensions | | | Basic load ratings dynamic | static | Fatigue load limit | Speed ratings Lubrication grease | oil | Mass | Designation | Dimension Series to ISO 355 |
|---|---|---|---|---|---|---|---|---|---|---|
| d | D | T | C | $C_0$ | $P_u$ | | | | | |
| mm | | | N | | N | r/min | | kg | – | – |
| 35 (cont.) | 80 | 22,75 | 72 100 | 73 500 | 8 500 | 5 000 | 6 700 | 0,52 | **30307** | 2FB |
| | 80 | 22,75 | 61 600 | 67 000 | 7 800 | 4 500 | 6 000 | 0,52 | **31307** | 7FB |
| | 80 | 32,75 | 95 200 | 106 000 | 12 200 | 4 800 | 6 300 | 0,73 | **32307** | 2FE |
| | 80 | 32,75 | 93 500 | 114 000 | 13 200 | 4 500 | 6 000 | 0,80 | **32307 B** | 5FE |
| 40 | 68 | 19 | 52 800 | 71 000 | 7 800 | 5 300 | 7 000 | 0,27 | **32008 X** | 3CD |
| | 75 | 26 | 79 200 | 104 000 | 11 600 | 5 000 | 6 700 | 0,51 | **33108** | 2CE |
| | 80 | 19,75 | 61 600 | 68 000 | 7 650 | 4 800 | 6 300 | 0,42 | **30208** | 3DB |
| | 80 | 24,75 | 74 800 | 86 500 | 9 800 | 4 800 | 6 300 | 0,53 | **32208** | 3DC |
| | 80 | 32 | 105 000 | 132 000 | 15 300 | 4 300 | 5 600 | 0,77 | **33208** | 2DE |
| | 85 | 33 | 121 000 | 150 000 | 17 300 | 4 500 | 6 000 | 0,90 | **T2EE 040** | 2EE |
| | 90 | 25,25 | 85 800 | 95 000 | 11 000 | 4 500 | 6 000 | 0,72 | **30308** | 2FB |
| | 90 | 25,25 | 73 700 | 81 500 | 9 650 | 4 000 | 5 300 | 0,72 | **31308** | 7FB |
| | 90 | 35,25 | 117 000 | 140 000 | 16 300 | 4 000 | 5 300 | 1,00 | **32308** | 2FD |
| | 90 | 35,25 | 108 000 | 140 000 | 16 300 | 4 000 | 5 300 | 1,10 | **32308 B** | 5FD |
| 45 | 75 | 20 | 58 300 | 80 000 | 8 800 | 4 800 | 6 300 | 0,34 | **32009 X** | 3CC |
| | 80 | 26 | 84 200 | 114 000 | 12 900 | 4 500 | 6 000 | 0,56 | **33109** | 3CE |
| | 85 | 20,75 | 66 000 | 76 500 | 8 650 | 4 500 | 6 000 | 0,46 | **30209** | 3DB |
| | 85 | 24,75 | 80 900 | 98 000 | 11 200 | 4 500 | 6 000 | 0,58 | **32209** | 3DC |
| | 85 | 24,75 | 73 700 | 93 000 | 11 000 | 4 300 | 5 600 | 0,60 | **32209 B** | 5DC |
| | 85 | 32 | 108 000 | 143 000 | 16 300 | 4 000 | 5 300 | 0,82 | **33209** | 3DE |
| | 95 | 29 | 89 700 | 112 000 | 12 900 | 3 600 | 4 800 | 0,92 | **T7FC 045** | 7FC |
| | 95 | 36 | 147 000 | 186 000 | 21 200 | 4 000 | 5 300 | 1,20 | **T2ED 045** | 2ED |
| | 100 | 27,25 | 108 000 | 120 000 | 14 600 | 4 000 | 5 300 | 0,97 | **30309** | 2FB |
| | 100 | 27,25 | 91 300 | 102 000 | 12 500 | 3 400 | 4 500 | 0,95 | **31309** | 7FB |
| | 100 | 38,25 | 140 000 | 170 000 | 20 400 | 3 600 | 4 800 | 1,35 | **32309** | 2FD |
| | 100 | 38,25 | 134 000 | 176 000 | 20 000 | 3 600 | 4 800 | 1,45 | **32309 B** | 5FD |
| 50 | 80 | 20 | 60 500 | 88 000 | 9 650 | 4 500 | 6 000 | 0,37 | **32010 X** | 3CC |
| | 80 | 24 | 69 300 | 102 000 | 11 400 | 4 500 | 6 000 | 0,45 | **33010** | 2CE |
| | 82 | 21,5 | 72 100 | 100 000 | 11 000 | 4 500 | 6 000 | 0,43 | **K-JLM 104948/K-JLM 104910** | – |
| | 85 | 26 | 85 800 | 122 000 | 13 700 | 4 300 | 5 600 | 0,59 | **33110** | 3CE |
| | 90 | 21,75 | 76 500 | 81 500 | 10 400 | 4 300 | 5 600 | 0,54 | **30210** | 3DB |
| | 90 | 24,75 | 82 500 | 100 000 | 11 600 | 4 300 | 5 600 | 0,61 | **32210** | 3DC |
| | 90 | 24,75 | 82 500 | 104 000 | 12 500 | 4 000 | 5 300 | 0,65 | **32210 B** | 5DC |
| | 90 | 28 | 106 000 | 140 000 | 16 300 | 4 000 | 5 300 | 0,75 | **K-JM 205149/K-JM 205110** | – |
| | 90 | 28 | 106 000 | 140 000 | 16 300 | 4 000 | 5 300 | 0,75 | **K-JM 205149/K-JM 205110 A** | – |
| | 90 | 32 | 114 000 | 160 000 | 18 300 | 3 800 | 5 000 | 0,90 | **33210** | 3DE |
| | 100 | 36 | 154 000 | 200 000 | 22 800 | 3 800 | 5 000 | 1,30 | **T2ED 050** | 2ED |
| | 105 | 32 | 108 000 | 137 000 | 16 000 | 3 200 | 4 300 | 1,20 | **T7FC 050** | 7FC |

**Taper roller bearings**
**single row**
d 50–65 mm

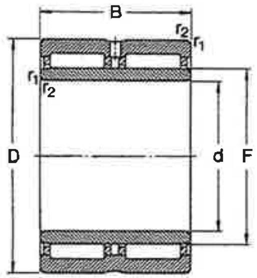



| Principal dimensions | | | Basic load ratings dynamic | static | Fatigue load limit | Speed ratings Lubrication grease | oil | Mass | Designation | Dimension Series to ISO 355 |
|---|---|---|---|---|---|---|---|---|---|---|
| d | D | T | C | $C_0$ | $P_u$ | | | | | |
| mm | | | N | | N | r/min | | kg | – | – |
| 50 (cont.) | 110 | 29,25 | 125 000 | 140 000 | 17 000 | 3 600 | 4 800 | 1,25 | **30310** | 2FB |
| | 110 | 29,25 | 106 000 | 120 000 | 14 300 | 3 200 | 4 300 | 1,20 | **31310** | 7FB |
| | 110 | 42,25 | 172 000 | 212 000 | 24 500 | 3 200 | 4 300 | 1,80 | **32310** | 2FD |
| | 110 | 42,25 | 161 000 | 216 000 | 25 000 | 3 200 | 4 300 | 1,85 | **32310 B** | 5FD |
| 55 | 90 | 23 | 78 100 | 112 000 | 12 500 | 4 000 | 5 300 | 0,56 | **K-JLM 506849/K-JLM 506810** | – |
| | 90 | 23 | 80 900 | 116 000 | 13 200 | 4 000 | 5 300 | 0,55 | **32011 X** | 3CC |
| | 90 | 27 | 89 700 | 137 000 | 15 300 | 4 000 | 5 300 | 0,67 | **33011** | 2CE |
| | 95 | 30 | 110 000 | 156 000 | 18 000 | 3 800 | 5 000 | 0,86 | **33111** | 3CE |
| | 100 | 22,75 | 89 700 | 106 000 | 12 200 | 3 800 | 5 000 | 0,70 | **30211** | 3DB |
| | 100 | 26,75 | 106 000 | 129 000 | 15 000 | 3 800 | 5 000 | 0,83 | **32211** | 3DC |
| | 100 | 26,75 | 101 000 | 127 000 | 15 300 | 3 600 | 4 800 | 0,87 | **32211 B** | – |
| | 100 | 35 | 138 000 | 190 000 | 22 000 | 3 400 | 4 500 | 1,20 | **33211** | 3DE |
| | 110 | 39 | 179 000 | 232 000 | 26 500 | 3 400 | 4 500 | 1,70 | **T2ED 055** | 2ED |
| | 115 | 34 | 125 000 | 163 000 | 19 600 | 3 000 | 4 000 | 1,60 | **T7FC 055** | 7FC |
| | 120 | 31,5 | 142 000 | 163 000 | 19 600 | 3 200 | 4 300 | 1,55 | **30311** | 2FB |
| | 120 | 31,5 | 121 000 | 137 000 | 17 000 | 2 800 | 3 800 | 1,55 | **31311** | 7FB |
| | 120 | 45,5 | 198 000 | 250 000 | 29 000 | 3 000 | 4 000 | 2,30 | **32311** | 2FD |
| | 120 | 45,5 | 190 000 | 260 000 | 30 000 | 2 600 | 3 800 | 2,50 | **32311 B** | 5FD |
| 60 | 95 | 23 | 82 500 | 122 000 | 13 700 | 3 800 | 5 000 | 0,59 | **32012 X** | 4CC |
| | 95 | 24 | 84 200 | 132 000 | 15 000 | 3 600 | 4 800 | 0,62 | **K-JLM 508748/K-JLM 508710** | – |
| | 95 | 27 | 91 300 | 143 000 | 16 000 | 3 800 | 5 000 | 0,71 | **33012** | 2CE |
| | 100 | 30 | 117 000 | 170 000 | 19 600 | 3 600 | 4 800 | 0,92 | **33112** | 3CE |
| | 110 | 23,75 | 99 000 | 114 000 | 13 400 | 3 400 | 4 500 | 0,88 | **30212** | 3EB |
| | 110 | 29,75 | 125 000 | 160 000 | 19 000 | 3 400 | 4 500 | 1,15 | **32212** | 3EC |
| | 110 | 38 | 168 000 | 236 000 | 27 000 | 3 000 | 4 000 | 1,60 | **33212** | 3EE |
| | 115 | 39 | 168 000 | 250 000 | 27 500 | 3 000 | 4 000 | 1,85 | **T5ED 060** | 5ED |
| | 115 | 40 | 194 000 | 260 000 | 30 000 | 3 200 | 4 300 | 1,85 | **T2EE 060** | 2EE |
| | 125 | 37 | 154 000 | 204 000 | 24 500 | 2 600 | 3 600 | 2,05 | **T7FC 060** | 7FC |
| | 130 | 33,5 | 168 000 | 196 000 | 23 600 | 3 000 | 4 000 | 1,95 | **30312** | 2FB |
| | 130 | 33,5 | 145 000 | 166 000 | 20 400 | 2 600 | 3 600 | 1,90 | **31312** | 7FB |
| | 130 | 48,5 | 229 000 | 290 000 | 34 000 | 2 600 | 3 600 | 2,85 | **32312** | 2FD |
| | 130 | 48,5 | 220 000 | 305 000 | 35 500 | 2 600 | 3 600 | 2,80 | **32312 B** | 5FD |
| 65 | 100 | 23 | 84 200 | 127 000 | 14 300 | 3 400 | 4 500 | 0,63 | **32013 X** | 4CC |
| | 100 | 27 | 96 800 | 156 000 | 17 600 | 3 400 | 4 500 | 0,78 | **33013** | 2CE |
| | 110 | 28 | 123 000 | 183 000 | 21 200 | 3 200 | 4 300 | 1,05 | **K-JM 511946/K-JM 511910** | – |
| | 110 | 34 | 142 000 | 208 000 | 24 500 | 3 200 | 4 300 | 1,30 | **33113** | 3DE |
| | 120 | 24,75 | 114 000 | 134 000 | 16 300 | 3 000 | 4 000 | 1,15 | **30213** | 3EB |
| | 120 | 32,75 | 151 000 | 193 000 | 23 200 | 3 000 | 4 000 | 1,50 | **32213** | 3EC |
| | 120 | 39 | 161 000 | 240 000 | 27 500 | 3 000 | 4 000 | 1,95 | **T5ED 065** | 5ED |

**Needle roller bearings with flanges with inner ring**
d **40–65** mm




Series NKI(S), NA 49

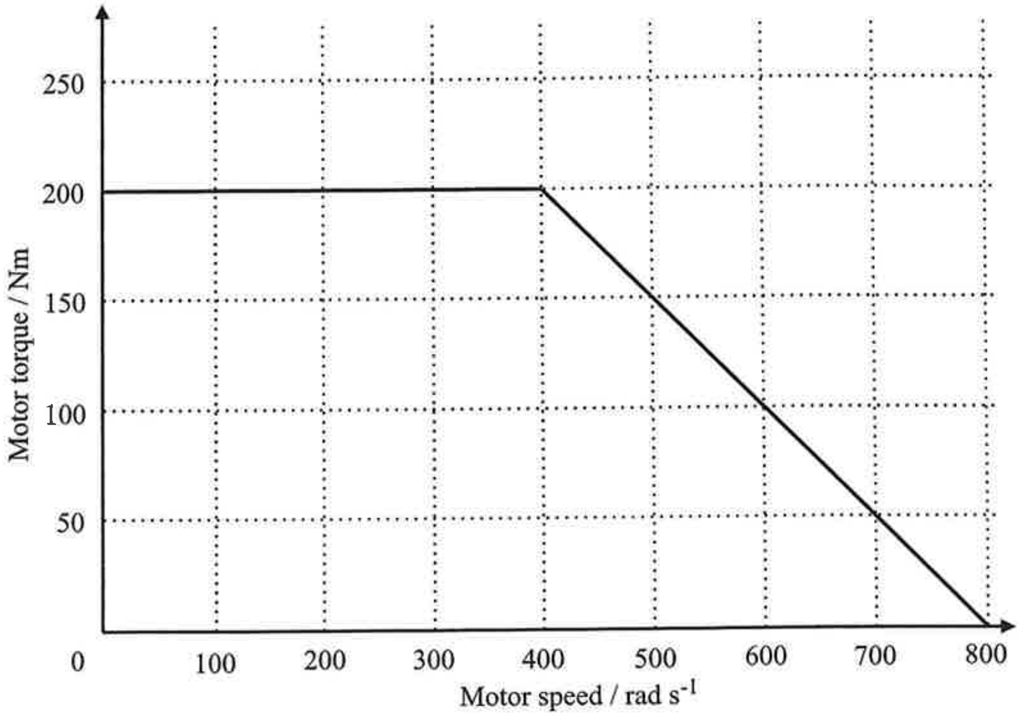



Series NA 69

| Principal dimensions d | D | B | Basic load ratings dynamic C | static $C_0$ | Fatigue load limit $P_u$ | Speed ratings Lubrication grease | oil | Mass | Designation |
|---|---|---|---|---|---|---|---|---|---|
| mm | | | N | | N | r/min | | kg | – |
| 40 | 55 | 20 | 27 500 | 57 000 | 7 200 | 6 300 | 9 000 | 0,14 | NKI 40/20 |
| | 55 | 30 | 40 200 | 93 000 | 12 000 | 6 300 | 9 000 | 0,22 | NKI 40/30 |
| | 62 | 22 | 42 900 | 71 000 | 9 150 | 5 600 | 8 000 | 0,23 | NA 4908 |
| | 62 | 40 | 67 100 | 125 000 | 16 000 | 5 600 | 8 000 | 0,43 | NA 6908 |
| | 65 | 22 | 42 900 | 72 000 | 9 150 | 5 600 | 8 000 | 0,28 | NKIS 40 |
| 42 | 57 | 20 | 29 200 | 61 000 | 7 650 | 6 000 | 8 500 | 0,15 | NKI 42/20 |
| | 57 | 30 | 41 800 | 98 000 | 12 900 | 6 000 | 8 500 | 0,22 | NKI 42/30 |
| 45 | 62 | 25 | 38 000 | 78 000 | 10 000 | 5 600 | 8 000 | 0,23 | NKI 45/25 |
| | 62 | 35 | 49 500 | 110 000 | 14 300 | 5 600 | 8 000 | 0,32 | NKI 45/35 |
| | 68 | 22 | 45 700 | 78 000 | 10 000 | 5 300 | 7 500 | 0,27 | NA 4909 |
| | 68 | 40 | 70 400 | 137 000 | 17 300 | 5 300 | 7 500 | 0,50 | NA 6909 |
| | 72 | 22 | 44 600 | 78 000 | 10 000 | 5 000 | 7 000 | 0,34 | NKIS 45 |
| 50 | 68 | 25 | 40 200 | 88 000 | 11 200 | 5 300 | 7 500 | 0,27 | NKI 50/25 |
| | 68 | 35 | 52 300 | 122 000 | 16 000 | 5 300 | 7 500 | 0,38 | NKI 50/35 |
| | 72 | 22 | 47 300 | 85 000 | 11 000 | 5 000 | 7 000 | 0,27 | NA 4910 |
| | 72 | 40 | 73 700 | 150 000 | 19 000 | 5 000 | 7 000 | 0,52 | NA 6910 |
| | 80 | 28 | 62 700 | 104 000 | 13 700 | 4 500 | 6 300 | 0,52 | NKIS 50 |
| 55 | 72 | 25 | 41 800 | 96 500 | 12 200 | 4 800 | 6 700 | 0,27 | NKI 55/25 |
| | 72 | 35 | 55 000 | 134 000 | 17 600 | 4 800 | 6 700 | 0,38 | NKI 55/35 |
| | 80 | 25 | 57 200 | 106 000 | 13 700 | 4 500 | 6 300 | 0,40 | NA 4911 |
| | 80 | 45 | 89 700 | 190 000 | 24 000 | 4 500 | 6 300 | 0,78 | NA 6911 |
| | 85 | 28 | 66 000 | 114 000 | 15 000 | 4 300 | 6 000 | 0,56 | NKIS 55 |
| 60 | 82 | 25 | 44 000 | 95 000 | 12 000 | 4 300 | 6 000 | 0,40 | NKI 60/25 |
| | 82 | 35 | 60 500 | 146 000 | 19 000 | 4 300 | 6 000 | 0,55 | NKI 60/35 |
| | 85 | 25 | 60 600 | 114 000 | 14 600 | 4 300 | 6 000 | 0,43 | NA 4912 |
| | 85 | 45 | 93 500 | 204 000 | 26 000 | 4 300 | 6 000 | 0,81 | NA 6912 |
| | 90 | 28 | 68 200 | 120 000 | 15 600 | 4 000 | 5 600 | 0,56 | NKIS 60 |
| 65 | 90 | 25 | 61 600 | 120 000 | 15 300 | 4 000 | 5 600 | 0,46 | NA 4913 |
| | 90 | 25 | 52 800 | 106 000 | 13 700 | 4 000 | 5 600 | 0,47 | NKI 65/25 |
| | 90 | 35 | 73 700 | 163 000 | 21 600 | 4 000 | 5 600 | 0,66 | NKI 65/35 |
| | 90 | 45 | 95 200 | 212 000 | 27 000 | 4 000 | 5 600 | 0,83 | NA 6913 |
| | 95 | 28 | 70 400 | 132 000 | 17 000 | 3 800 | 5 300 | 0,64 | NKIS 65 |

Candidate Number:

ENGINEERING TRIPOS PART IIA
Thursday 3 May 2012, Module 3C8, Question 1




Extra copy of Fig. 1: Output characteristic for Question 1.

FINAL version

3C8 2012 Answers

1 (a) 35 m/s
(b) $G = 6/\sqrt{7}$ maximum speed 52.9 m/s
(c) 49.4 s
(d) approximately 33 m/s

2 (b) (ii) $P = 2F_1$
(iv) 1.3% reduction

3 (b) (ii) $\theta = \pi/2$ for bearing A, $\theta = 3\pi/2$ for bearing B.
(iii) $C = 1029$ N

4 (a) (i) $P' = \dfrac{T}{4rw\cos\phi}$ at all eight pressure lines

(ii) $p_0 = 1.35\sqrt{\dfrac{TE^*}{wr^2}}$ at a single contact on the sun/planet pressure line at the addendum circle.

(b) (i) $T_a = 3T$ $T_c = -4T$
(ii) $T_a = 2.85T$ $T_c = -3.85T$
(iii) $T_a = 2.81T$ $T_c = -3.81T$