

ENGINEERING TRIPOS PART IIA

Monday 7 May 2012 9 to 10.30

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

3C9 datasheet (8 pages).

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Define the energy release rate G and the stress intensity factor K . Briefly describe how these quantities can be used to characterise fracture in engineering materials. [20%]

(b) Show that the energy release rate G in a cracked body of compliance C and subjected to a load P is given by

$$G = \frac{1}{2} \frac{P^2}{B} \frac{\partial C}{\partial a}$$

where a is the crack length and B is the thickness of the body. [30%]

(c) A long plate, of height $2h$ and thickness B (into the page), contains a crack of length $2a$ such that $a \gg h$. The plate is subjected to a uniform stress σ along its upper and lower faces as shown in Fig. 1. Using beam theory, calculate the bending moment distribution in the strips above and below the crack. Hence determine the strain energy release rate G . [50%]

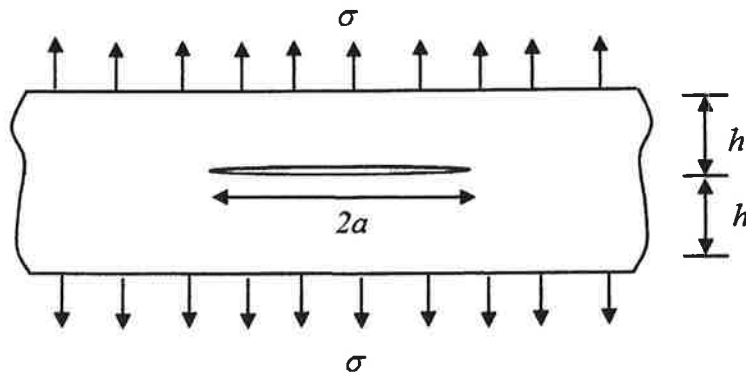


Fig. 1

2 (a) Briefly explain the role of the T -stress in influencing the fracture toughness and crack path in an engineering ceramic. [25%]

(b) Explain the physical basis of the dependence of fracture toughness upon sheet thickness for aluminium alloys. [25%]

(c) A long cylindrical pipe, of radius R and wall thickness t (where $R \gg t$), has a through-thickness crack of length $2a$ at an angle α to the axis of the pipe, as shown in Fig. 2. Calculate the mode-I stress intensity factor if the pipe is subjected to:

(i) An axial load P . [20%]

(ii) A combined axial load P and axial torque Q . [30%]

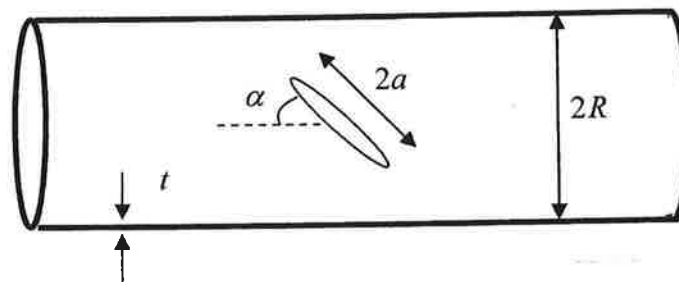


Fig. 2

(TURN OVER)

3 (a) A thin metallic sheet contains an edge-crack of length a_0 , and is loaded by a uniform, in-plane tensile stress σ normal to the crack plane. The R-curve for the sheet is of the form

$$\begin{aligned} K_R &= K_0 \sin\left(\frac{\pi\Delta a}{2\lambda}\right) & 0 \leq \Delta a \leq \lambda \\ &= K_0 & \Delta a > \lambda \end{aligned}$$

where K_R is the crack growth resistance, Δa is the crack extension, and K_0 and λ are material constants. Calculate the failure strength of the sheet for the two cases $a_0 = 0.1\lambda$ and $a_0 = \lambda$. For which case does the material appear to be the more brittle, and why? [60%]

(b) A panel of height H , width $2W$ and thickness B contains a centre-crack of length $2a$. It is made from an annealed steel of yield strength σ_Y , and is subjected to an axial tensile load P . The end displacement u is related to P via

$$P = \sigma_Y B(W - a) \left(\frac{u}{H}\right)^{1/2}$$

- (i) Calculate the potential energy of the panel \mathcal{U} for a given load P_0 . [20%]
(ii) Calculate the value of the J integral, again for a given load P_0 . [20%]

4 A long steel plate of thickness t and width W contains a surface-breaking, semi-circular thumbnail crack of initial radius a_0 , such that $a_0 \ll t \ll W$. The plate is subjected to a cyclic in-plane tensile stress range $\Delta\sigma$ and load ratio R , and the associated stress intensity range is $\Delta K = \Delta\sigma\sqrt{\pi a}$. The crack grows by fatigue, such that its radius increases from a_0 to a at the rate da/dN given by

$$\frac{da}{dN} = C\Delta K^n$$

where C and n are material constants. The material possesses a fatigue threshold stress intensity range ΔK_{th} and a fracture toughness K_{IC} .

- | | |
|--|-------|
| (a) Determine the endurance limit for the plate. | [20%] |
| (b) Deduce the S-N curve for the plate. | [35%] |
| (c) Describe the effect of mean stress upon the S-N curve. | [20%] |
| (d) Explain the effect of humidity upon the S-N curve. | [25%] |

END OF PAPER

ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \frac{1}{\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane stress} \\ \frac{1}{3\pi} \left(\frac{K_I}{\sigma_y} \right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

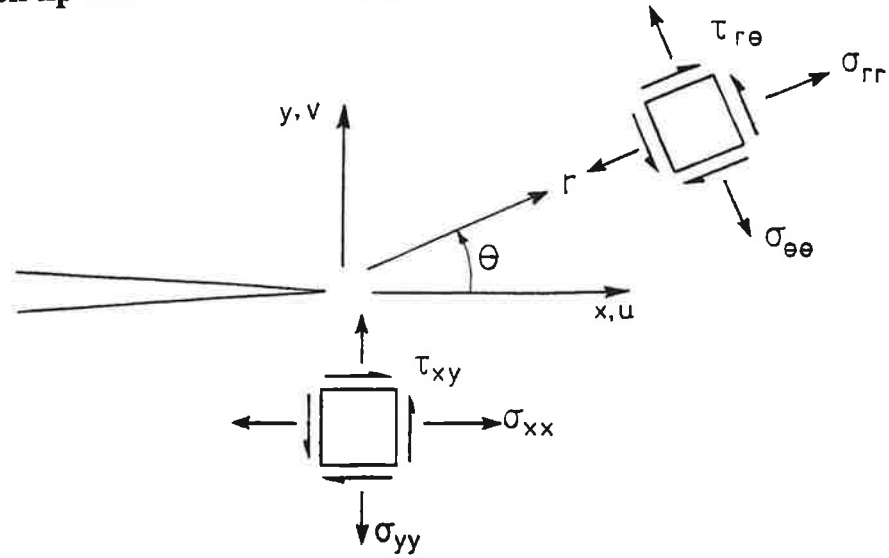
$$\delta = \begin{cases} \frac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \frac{1}{2} \frac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \frac{1}{E} K_I^2 & \text{Plane stress} \\ \frac{1-\nu^2}{E} K_I^2 & \text{Plane strain} \end{cases}$$

$$\text{Related to compliance } C: G = \frac{1}{2} \frac{P^2}{B} \frac{dC}{da}$$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \left(\frac{1}{4} \sin \frac{\theta}{2} + \frac{1}{4} \sin \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{1-\nu}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(1 - 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_I}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu - \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin \frac{3\theta}{2} \right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos \frac{3\theta}{2} \right)$$

$$u = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{2}{1+\nu} + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(2 - 2\nu + \cos^2 \frac{\theta}{2} \right) \sin \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(\frac{\nu-1}{1+\nu} + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane stress} \\ \frac{K_{II}}{G} \sqrt{\frac{r}{2\pi}} \left(-1 + 2\nu + \sin^2 \frac{\theta}{2} \right) \cos \frac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}$$

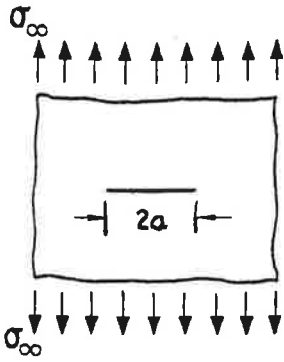
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

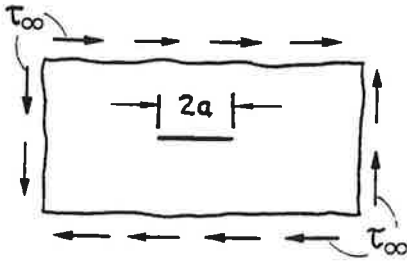
$$w = \frac{K_{III}}{G} \sqrt{\frac{2r}{\pi}} \sin \frac{\theta}{2}$$

$$u = v = 0$$

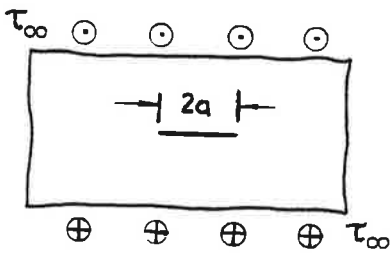
Tables of stress intensity factors



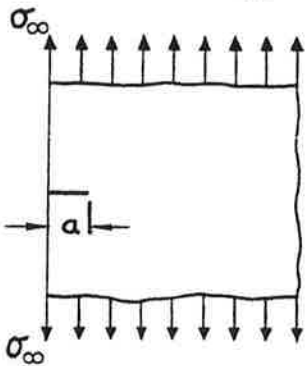
$$K_I = \sigma_\infty \sqrt{\pi a}$$



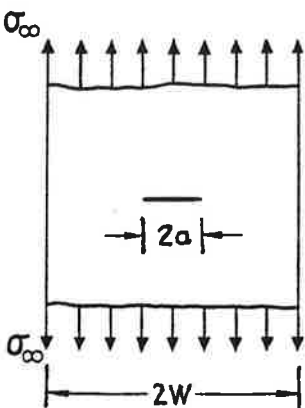
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



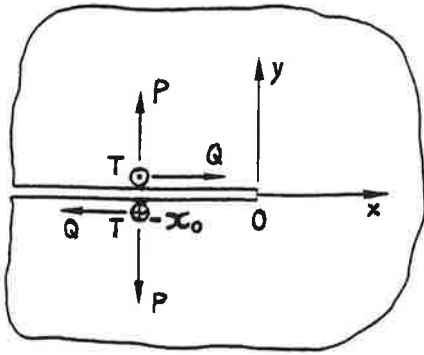
$$K_{III} = \tau_\infty \sqrt{\pi a}$$



$$K_I = 1.12 \sigma_\infty \sqrt{\pi a}$$



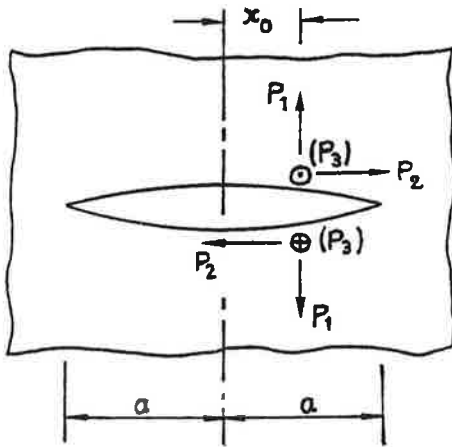
$$K_I = \sigma_\infty \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326a^2/W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_0}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_0}}$$

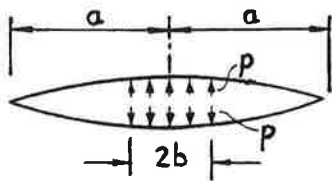
$$K_{III} = \frac{2T}{\sqrt{2\pi x_0}}$$



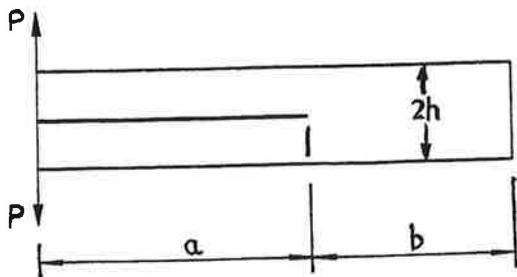
$$K_I = \frac{P_1}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$

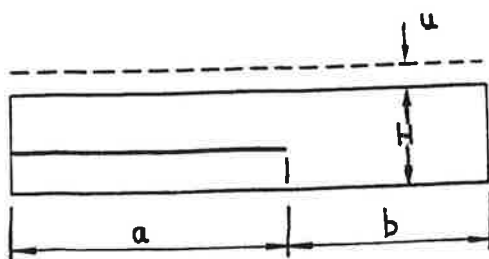
$$K_{III} = \frac{P_3}{\sqrt{\pi a}} \sqrt{\frac{a+x_0}{a-x_0}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}} \frac{a}{b} \arcsin \frac{b}{a}$$

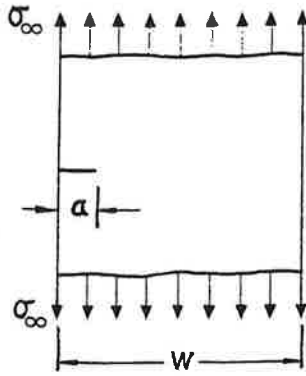


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}} \frac{Pa}{B} \quad h \ll a \text{ and } h \ll b$$



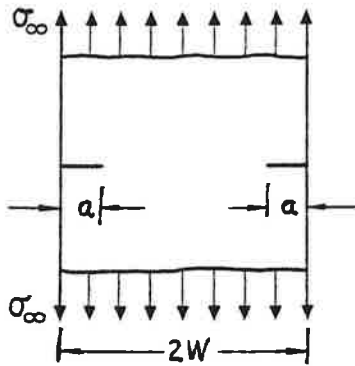
$$K_I = \sqrt{\frac{1}{2\alpha H}} Eu \quad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1 - \nu^2 & \text{Plane stress} \\ 1 - 3\nu^2 - 2\nu^3 & \text{Plane strain} \end{cases}$$

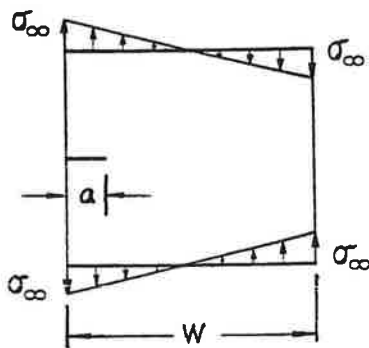


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 0.23 \frac{a}{W} + 10.6 \frac{a^2}{W^2} - 21.7 \frac{a^3}{W^3} + 30.4 \frac{a^4}{W^4} \right)$$

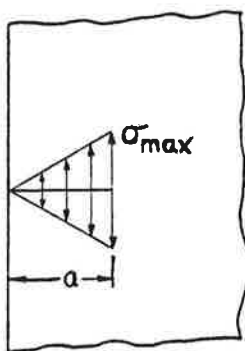


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1 - a/W}} \right)$$

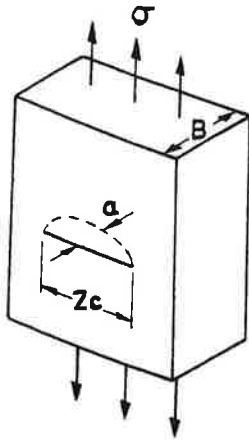


$$a/W < 0.7$$

$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(1.12 - 1.39 \frac{a}{W} + 7.3 \frac{a^2}{W^2} - 13 \frac{a^3}{W^3} + 14 \frac{a^4}{W^4} \right)$$

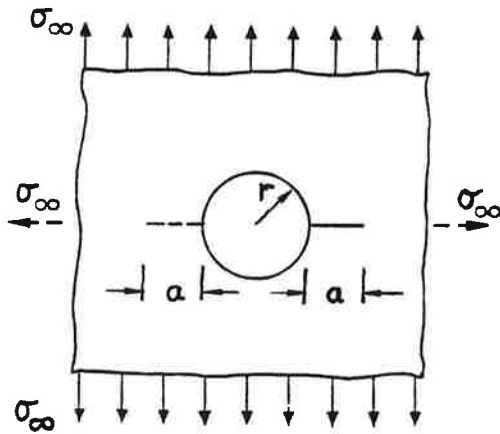
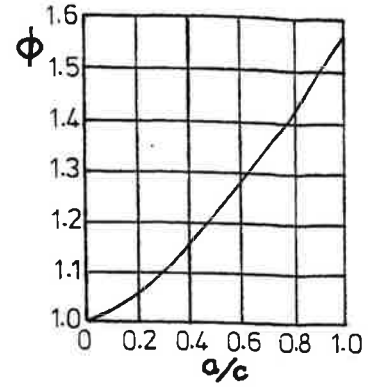


$$K_I = 0.683 \sigma_{\max} \sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi} \sigma \sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2} \left(1 - \frac{c^2 - a^2}{c^2} \sin^2 \theta \right)^{\frac{1}{2}} d\theta$$

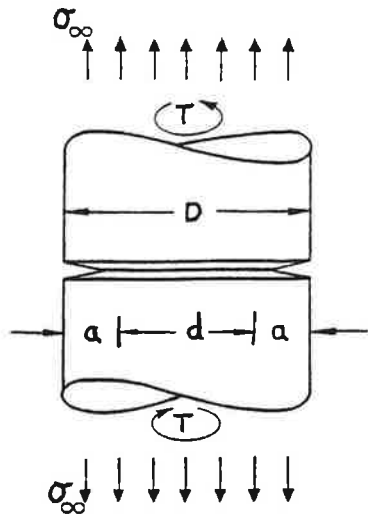


$$K_I = \sigma_{\infty} \sqrt{\pi a} F\left(\frac{a}{r}\right)$$

value of $F(a/r)^\dagger$

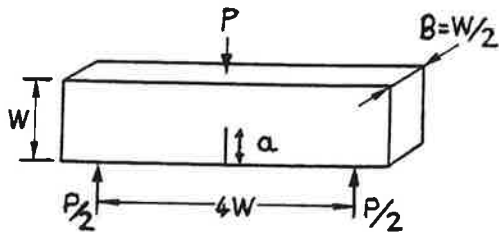
$\frac{a}{r}$	One crack		Two cracks	
	U	B	U	B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

$\dagger U = \text{uniaxial } \sigma_{\infty} \quad B = \text{biaxial } \sigma_{\infty}.$

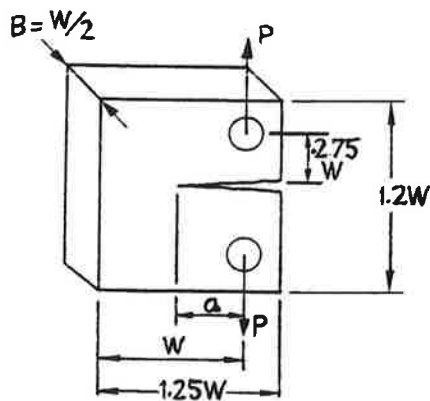


$$K_I = \sigma_{\infty} \sqrt{\pi a} \left(\frac{D}{d} + \frac{1}{2} + \frac{3d}{8D} - 0.36 \frac{d^2}{D^2} + 0.73 \frac{d^3}{D^3} \right) \frac{1}{2} \sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3} \sqrt{\pi a} \left(\frac{D^2}{d^2} + \frac{1}{2} \frac{D}{d} + \frac{3}{8} + \frac{5}{16} \frac{d}{D} + \frac{35}{128} \frac{d^2}{D^2} + 0.21 \frac{d^3}{D^3} \right) \frac{3}{8} \sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B} \sqrt{\frac{\pi}{W}} \left\{ 1.6 \left(\frac{a}{W} \right)^{1/2} - 2.6 \left(\frac{a}{W} \right)^{3/2} + 12.3 \left(\frac{a}{W} \right)^{5/2} - 21.2 \left(\frac{a}{W} \right)^{7/2} + 21.8 \left(\frac{a}{W} \right)^{9/2} \right\}$$



$$K_I = \frac{P}{B} \sqrt{\frac{\pi}{W}} \left\{ 16.7 \left(\frac{a}{W} \right)^{1/2} - 104.7 \left(\frac{a}{W} \right)^{3/2} + 369.9 \left(\frac{a}{W} \right)^{5/2} - 573.8 \left(\frac{a}{W} \right)^{7/2} + 360.5 \left(\frac{a}{W} \right)^{9/2} \right\}$$