ENGINEERING TRIPOS PART IIA

Wednesday 9 May 2012 2.30 to 4

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: None

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: FINAL

1 (a) Figure 1(a) shows a propped cantilever consisting of two beams. The two beams lie in one plane and the load vector, with the magnitude W, is orthogonal to that plane. The flexural stiffness of both beams is EI and their torsional stiffness is $GJ = 2EI$.

(i) Indicate the non-zero boundary reactions at the built-in support (without computing the actual values). [20%]

(ii) Determine the bending moment at the built-in support. [50%]

(b) Figure 1(b) shows a grillage structure built in to a rigid wall. The three beam members lie in one plane and the two load vectors shown are orthogonal to that plane. The flexural and torsional stiffness of the beams is EI and $GJ = 2EI$, respectively. Using your result from (a) determine the bending moments at both supports. [30%]

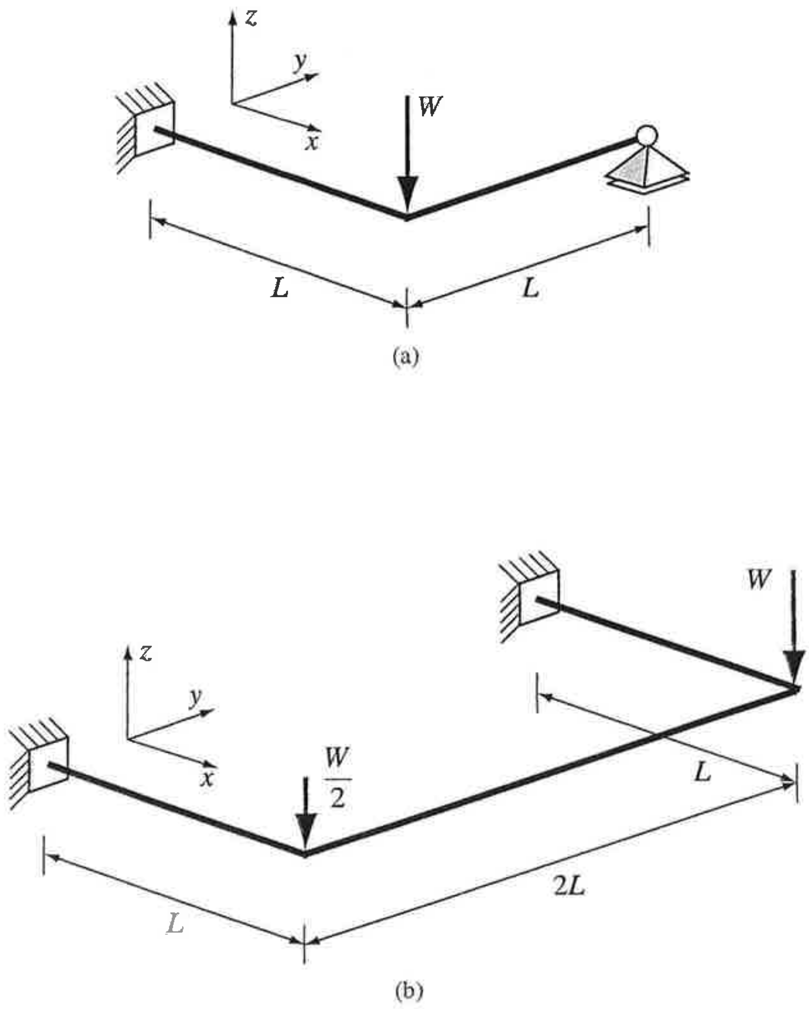

Fig. 1

2 (a) Figure 2(a) shows a thin-walled unsymmetric angle cross-section with constant thickness t.

(i) Determine the St. Venant torsion constant of the cross-section. [20%]

(ii) Determine the principal second moments of area of the cross-section. [40%]

(b) Figure 2(b) shows a thin-walled channel-like cross-section with a constant thickness t. The dash-dotted line in the Figure indicates symmetry axis. Determine the shear centre of the cross-section. [40%]

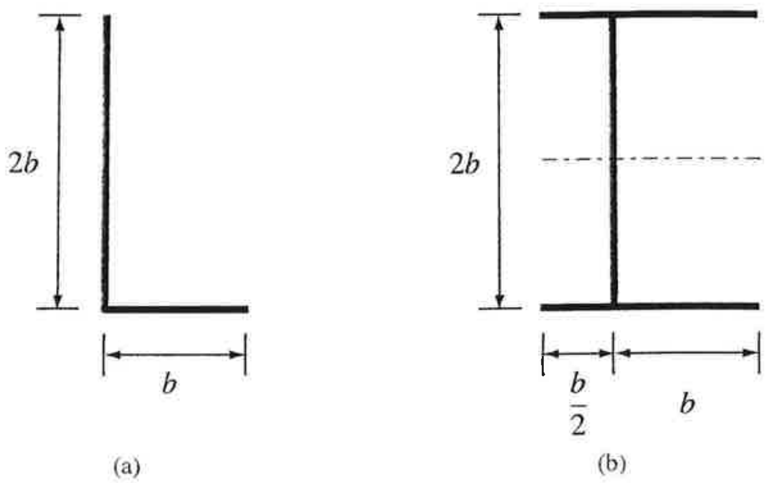

Fig. 2

3 (a) The simply-supported beam on an elastic foundation shown in Fig. 3 has length L and flexural stiffness EI. The foundation has a stiffness k per unit length. The beam carries a compressive axial force P. When the beam has a deflected shape $w(x)$, the total potential energy of the system is given by

$$\Pi(w) = \frac{1}{2}\int_0^L \left(EI\left(\frac{d^2w}{dx^2}\right)^2 - P\left(\frac{dw}{dx}\right)^2 + kw^2 \right) dx$$

By expressing a general deflected shape in a Fourier basis

$$w(x) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}$$

for integer n, derive an expression for the elastic critical load for buckling into the n-th mode shape. For a given mode number n, determine the length that has the lowest buckling load, and hence determine the lowest possible value of P, for any L and any n, at which such a structure will buckle. [80%]

(b) By considering a thin longitudinal strip of the wall of a thin cylindrical shell, the analysis above can be applied to estimate the compressive load required to buckle such a shell, assuming it buckles into a radially-symmetric mode such as the one sketched in Fig. 3. In such an analysis, the foundation stiffness k represents the resistance provided by stresses in the hoop direction. Comment on any shortcomings you might expect such an analysis to have when applied to estimate the buckling load of real cylindrical shells. [20%]

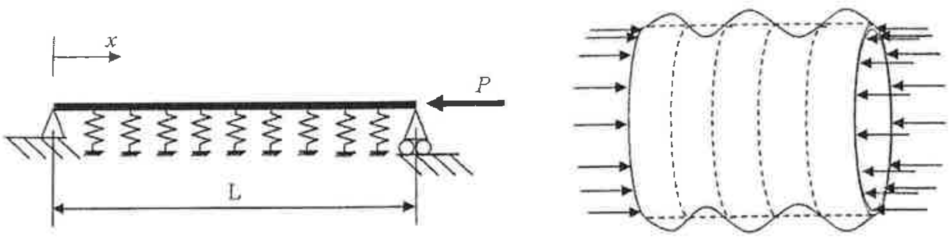

Fig. 3

4 (a) The structure shown in Fig. 4(a) consists of two rigid elements AB and BC, each of length L, connected together by a sliding joint at B. The elements are simply supported at A and C, and at each support there is a rotational spring which provides a resisting moment $G\theta$ in proportion to the rotation θ.

Determine the critical value of tension at which the structure can buckle, and identify the type of bifurcation. Without further calculation, sketch on a bifurcation diagram the behaviour in the presence of imperfections. [50%]

(b) Explain briefly why Shanley's resolution to the Column Paradox presents difficulties for a theory of structural stability based on the Principle of Virtual Displacements. [20%]

(c) Each member of the structure shown in Fig. 4(b) has length L, is axially rigid and has bending stiffness EI for flexure within the plane of the diagram. Each column supports a load P. Any out-of-plane deflection is prevented. Determine the 2×2 stiffness matrix relating couples applied at B and C to the respective rotations there, in terms of EI, L, s and c, where s and c are stability functions. [30%]

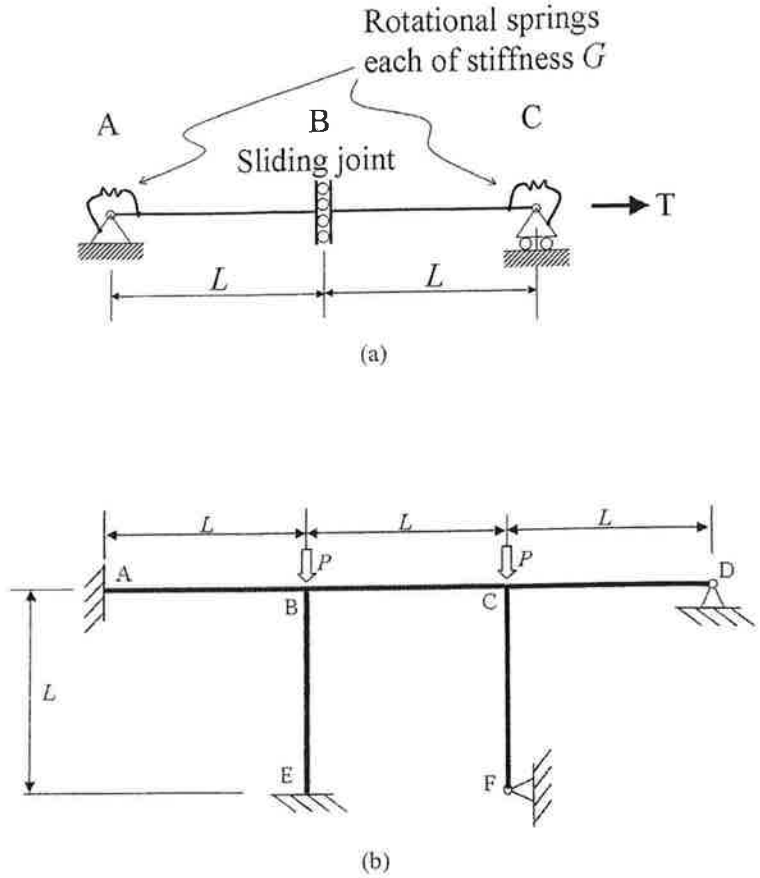

Fig. 4

END OF PAPER

Numerical answers for 3D4

1a) ii) $M = \frac{5}{7} WL$

1b) $M_A = 0.5714\,W$

$M_B = 0.9286\,W$

2a) i) $J = b t^3$

ii) $I_{\xi\xi} = 1.427\, b^3 t$

$I_{\eta\eta} = 0.156\, b^3 t$

2b) $x_s = \frac{9}{44} b$

3a) $P_{cr,min} = 2\sqrt{EIk}$

4a) $T_{min} = \frac{G}{L}$

4c) $$\begin{bmatrix} 8+s & 2 \\ 2 & 7+s(1-c^2) \end{bmatrix} \frac{EI}{L}$$