ENGINEERING TRIPOS PART IIA

Thursday 3 May 2012 9 to 10.30

Module 3D5

WATER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

The values of relevant parameters are listed at the end of the Data Book unless otherwise noted in the question.

Attachment: 3D5 Data Book (5 pages).

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL version

1 (a) Rain falls uniformly on a 20 km^2 drainage basin at an intensity of 10 mm hr^{-1} for 4 hours. The initial and ultimate infiltration capacities are f_0 = 20 mm hr^{-1} and f_c = 2 mm hr^{-1}, respectively, and the time constant K_f is 0.8 hr^{-1}.

(i) Show that the runoff begins 1.44 hours after the rainfall begins. [20%]

(ii) Calculate the volume of the surface runoff produced at the catchment outlet as the result of this rainfall. [20%]

(b) A three-hour uniform excess rainfall produces the following hydrograph at the catchment outlet.

Duration (hr)	0-3	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Discharge (m^3 s^{-1})	10	50	30	10	3	2	2	2

Derive a hydrograph at the catchment outlet after a separate two-hour uniform excess rainfall of the same intensity, and calculate the peak discharge. [30%]

(c) Explain how to construct Theissen polygons and what they are used for in hydrology. [10%]

(d) Briefly answer the following questions concerning open channel flows:

(i) What is the physical meaning of the Froude number? [10%]

(ii) Why is the flow at one section influenced only by upstream flows in supercritical flows, while it is influenced by both upstream and downstream flows in subcritical flows? [10%]

2 (a) A long concrete channel has rectangular cross section with constant width 3 m and uniform bed slope 0.01. The Manning roughness coefficient is 0.013 s $m^{-1/3}$, and the flow rate is 41.6 m^3 s^{-1}.

(i) Show that the water depth along the central portion of the channel is 2 m. [15%]

(ii) Half way along the channel, a cross-channel pipeline is now laid on the bed and protected by a concrete layer, which forms a gradual hump with height 0.1 m. Estimate the water depth above the hump. [15%]

(iii) Near the ends of the channel, the flow is gradually varied. The water depths at two nearby sections are 2.1 m and 2.2 m, respectively. Assuming a linear variation of the water depth between these two sections, estimate the distance between them and explain whether the section with 2.1 m depth is upstream or downstream of the section with 2.2 m depth. [20%]

(b) A long rectangular river with constant width initially has water depth 2 m and flow velocity 0.5 m s^{-1}. From $t = 0$ to $t = 100$ s, the water depth at the river mouth drops as follows:

$$h(t) = \left(\sqrt{2} - 0.002t\right)^2$$

where h is in metres and t in seconds. Neglect the bed slope and bed friction.

(i) Prove that the outflow velocity at the river mouth follows:

$$U(t) = 0.5 + 0.0125t$$ [15%]

(ii) Calculate the water depth 50 m upstream of the river mouth at $t = 100$ s. [35%]

3 (a) Using the Darcy-Weisbach equation and the Colebrook-White formula, derive the following equation

$$U = -8.86\sqrt{S_f D} \times \log_{10}\left(\frac{k_s}{3.7D} + \frac{0.57\nu}{D\sqrt{S_f D}}\right)$$ [20%]

where S_f is the hydraulic gradient. Explain how this equation can be used to plot pipeline design charts, one example of which is shown in Fig. 1. [15%]

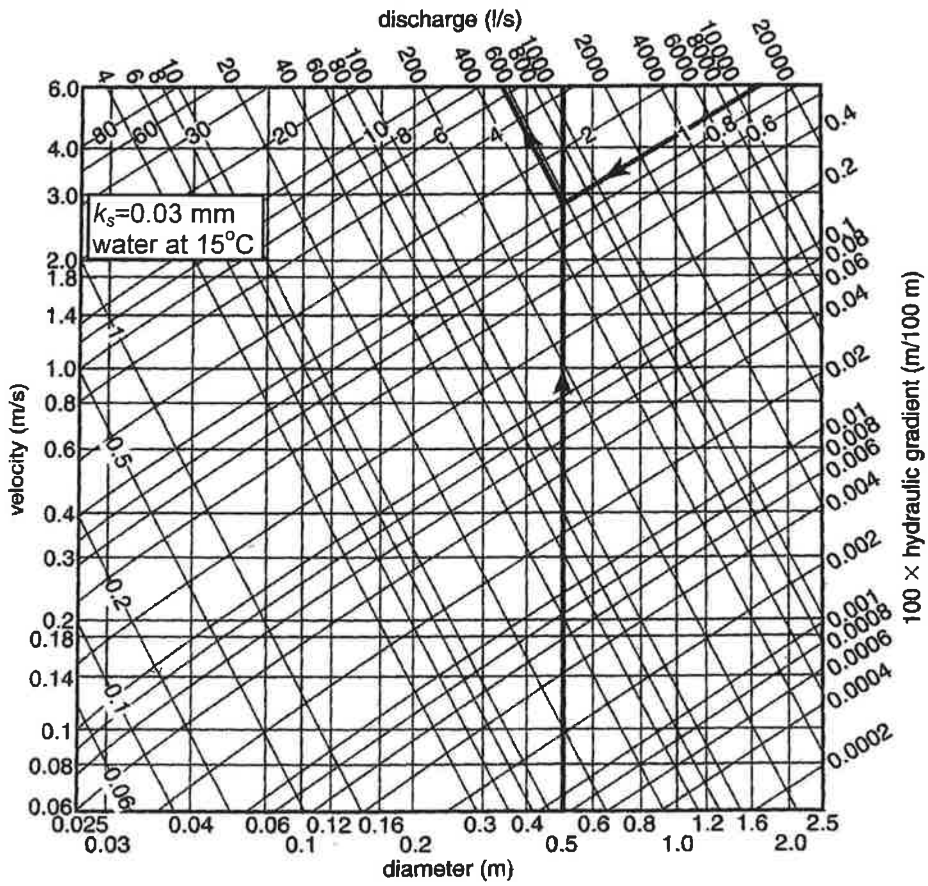


Fig. 1

(b) When running at 1450 rpm, a variable speed pump has the characteristics tabulated below.

Discharge (litre s^{-1})	0	10	20	30	40	50	60	70
Total head (m)	45.0	44.0	42.5	39.5	35.0	29.0	20.0	6.0

This pump is fitted to a pipeline to lift 27 litre s^{-1} water from a lake to a large water tank whose water level is 15 m higher than the water level in the lake. The pipeline has a diameter of 250 mm and a roughness height of 0.06 mm. The total length of the pipeline is 2000 m, within which 100 m lies between the lake and the pump and 1900 m lies between the pump and the water tank. The local head loss due to the filter at the pipe inlet is $10U^2/(2g)$. All other local losses can be ignored.

(i) Calculate the pump speed at this discharge. [40%]

(ii) On the suction side of the pump, the stagnation pressure head, $p/(\rho g)+U^2/(2g)$, should not be more than 3 m lower than the atmospheric pressure head, as specified by the pump manufacturer, to avoid cavitation. Calculate the maximum elevation of the pump above the water surface in the lake. [25%]

4 (a) Briefly explain the origins of turbulent diffusion and longitudinal dispersion. [15%]

(b) In the middle of a large and deep reservoir, 50 kg of neutrally buoyant waste falls off a barge and is dissolved immediately at the water surface. A water quality monitoring point is located 50 m away from the barge and is 2 m below the water surface. The unidirectional component of the velocity is negligible in the lake, but the velocity fluctuations due to surface waves and other small disturbances contribute to an increase in the mixing coefficient. Assume that the mixing coefficient is 1 $m^2 s^{-1}$ in all directions and at all places.

(i) Calculate the waste concentration picked up by the monitor 10 minutes after the incident. [20%]

(ii) Show that the monitoring point records the peak concentration at around 417 s after the incident. [15%]

(c) In a wide estuary, the bed is made up of sand with diameter d = 0.15 mm. The water depth is 10 m and the depth-averaged velocity is 0.65 m s^{-1}. Take the grain-related roughness height to be 2d, and the bedform-related roughness height to be

$$k_s'' = 100d$$

(i) Show that ripples are the dominant bedform according to van Rijn's diagram. [15%]

(ii) Calculate the total shear stress acting on the rippled bed. [15%]

(iii) A power station extracts cooling water from the estuary. The intake pipe is located 1 m above the bed. Using the Zyserman and Fredsøe formula to estimate a reference concentration, calculate the sediment concentration of the intake water in kg m^{-3}. [20%]

END OF PAPER

UNIVERSITY OF CAMBRIDGE

Engineering Tripos Part IIA | Third Year

Module 3D5: Water Engineering
Data Book (SI units [m, kg, s] unless otherwise noted)

Hydrology

Horton's infiltration model (*f*-capacity)

$$f = f_c + (f_0 - f_c)e^{-K_f t}$$

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

Rational method $Q = CiA$

Boundary Layer

For fully developed boundary layer flow with a free surface (uniform flow in a very wide channel):

Eddy viscosity coefficient $\nu_t = \kappa u_* z\left(1 - \frac{z}{h}\right)$

Velocity in hydraulically smooth regime ($u_* k_s/\nu < 5$) $\frac{\overline{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(9.0\frac{z u_*}{\nu}\right)$

Velocity in hydraulically rough regime ($u_* k_s/\nu > 70$) $\frac{\overline{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.0 z}{k_s}\right)$

Open Channel Flow

Chézy coefficient in large Reynolds-number flows $C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$

Froude number for rectangular channels $Fr = \frac{U}{\sqrt{gh}}$

Steady flow momentum equation $\sum F = \rho Q(U_{out} - U_{in})$

Bed shear stress $\tau_b = \rho g R_h S_f = \frac{C_f}{2}\rho \cdot U^2 = \frac{\lambda}{8}\rho \cdot U^2 = \frac{g}{C^2}\rho \cdot U^2 = \frac{g \cdot n^2}{R_h^{1/3}}\rho \cdot U^2 = \rho \cdot u_*^2$

Uniform flows:

Chézy formula $U = C\sqrt{R_h S_b}$

Manning formula $U = \frac{1}{n} \cdot R_h^{1/6}\sqrt{R_h S_b} = \frac{1}{n} \cdot R_h^{2/3} \cdot S_b^{1/2}$

Gradually varied flows:

$$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

or

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{U^2}{C^2 \cdot R_h}}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$$

Characteristics for unsteady flows in rectangular channels:

$$\frac{d}{dt}\left(U+2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U+\sqrt{gh}$$

$$\frac{d}{dt}\left(U-2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U-\sqrt{gh}$$

Pollutant Transport

Analytical values of the mixing coefficients: $D_{tx}=D_{ty}=0.15hu_*$, $D_{tz}=0.067hu_*$, $D_L=5.86hu_*$

For instantaneous release from origin at $t=0$ in uniform flows along x direction:

One-dimensional $$\bar{c}(x,t)=\frac{M/A}{\sqrt{4\pi t D_x}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}\right)$$

Two-dimensional $$\bar{c}(x,y,t)=\frac{M/h}{4\pi t\sqrt{D_x D_y}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}-\frac{y^2}{4D_y t}\right)$$

Three-dimensional $$\bar{c}(x,y,z,t)=\frac{M}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}-\frac{y^2}{4D_y t}-\frac{z^2}{4D_z t}\right)$$

For continuous release from origin in uniform flows along x direction:

Two-dimensional $$\bar{c}(x,y)=\frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}}\exp\left(-\frac{y^2}{4D_y\, x/U}\right)$$

Three-dimensional $$\bar{c}(x,y,z)=\frac{\dot{M}}{4\pi x\sqrt{D_y D_z}}\exp\left(-\frac{y^2}{4D_y\, x/U}-\frac{z^2}{4D_z\, x/U}\right)$$

Sediment Transport

Definitions of Shields parameter, non-dimensional grain size and transport stage parameter:

$$\theta=\frac{\tau_b}{g(\rho_s-\rho)d},\quad d_*=d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3},\quad T=\frac{\tau_b'-\tau_{bc}}{\tau_{bc}}=\frac{\theta'-\theta_c}{\theta_c}$$

Critical Shields parameter $$\theta_c=\frac{0.30}{1+1.2d_*}+0.055\left[1-\exp(-0.02d_*)\right]$$

Fall velocity $$w_s=\frac{\nu}{d}\left[\sqrt{10.36^2+1.049\cdot d_*^3}-10.36\right]$$

Shear stress partition: $$C'=7.8\ln\left(\frac{12.0\cdot R_h}{k_s'}\right),\quad \tau_b'=\rho g\frac{U^2}{C'^2}$$

$$C=7.8\ln\left(\frac{12.0\cdot R_h}{k_s}\right),\quad \tau_b=\rho g\frac{U^2}{C^2}$$

Volumetric bedload transport rate per unit width:

Meyer-Peter and Müller $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}}=8\left[\left(\frac{C}{C'}\right)^{1.5}\theta-0.047\right]^{1.5}$$

van Rijn $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}}=0.053\frac{T^{2.1}}{d_*^{0.3}}$$

Rouse profile of suspended sediment concentration $$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

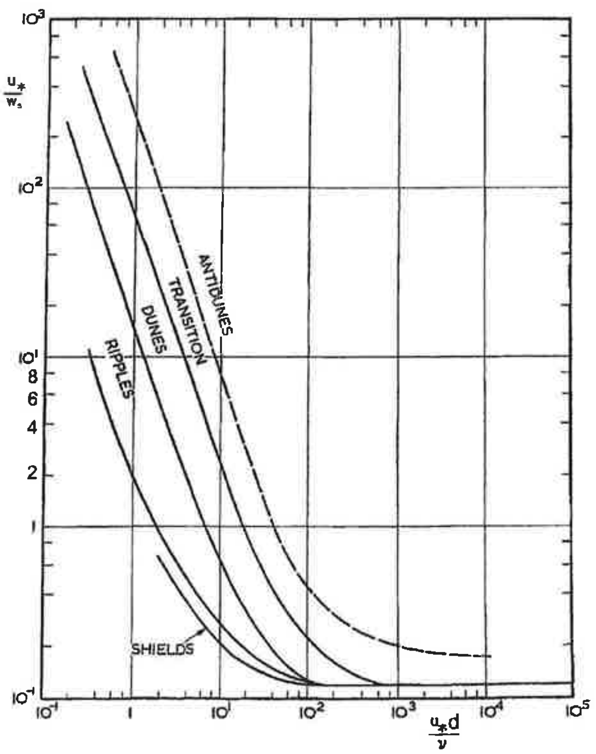
Reference volumetric concentration close to the bed:

Zyserman and Fredsøe $$\bar{c}(2d) = \frac{0.331 \cdot (\theta' - 0.045)^{1.75}}{1 + 0.72 \cdot (\theta' - 0.045)^{1.75}}$$

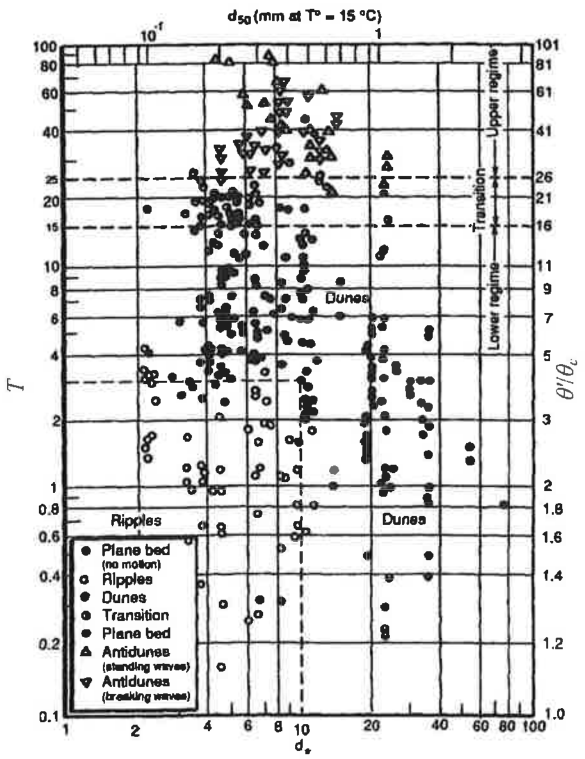
van Rijn $$\bar{c}(a) = 0.015 \frac{d \cdot T^{1.5}}{a \cdot d_*^{0.3}}$$

Suspended load per unit width $$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2 \right]$$

a/h	$w_s/(\kappa u_*) = 0.2$		$w_s/(\kappa u_*) = 0.6$		$w_s/(\kappa u_*) = 1.0$		$w_s/(\kappa u_*) = 1.5$	
	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113



Liu (1957)



Van Rijn (1984)

Pipeline and Pump

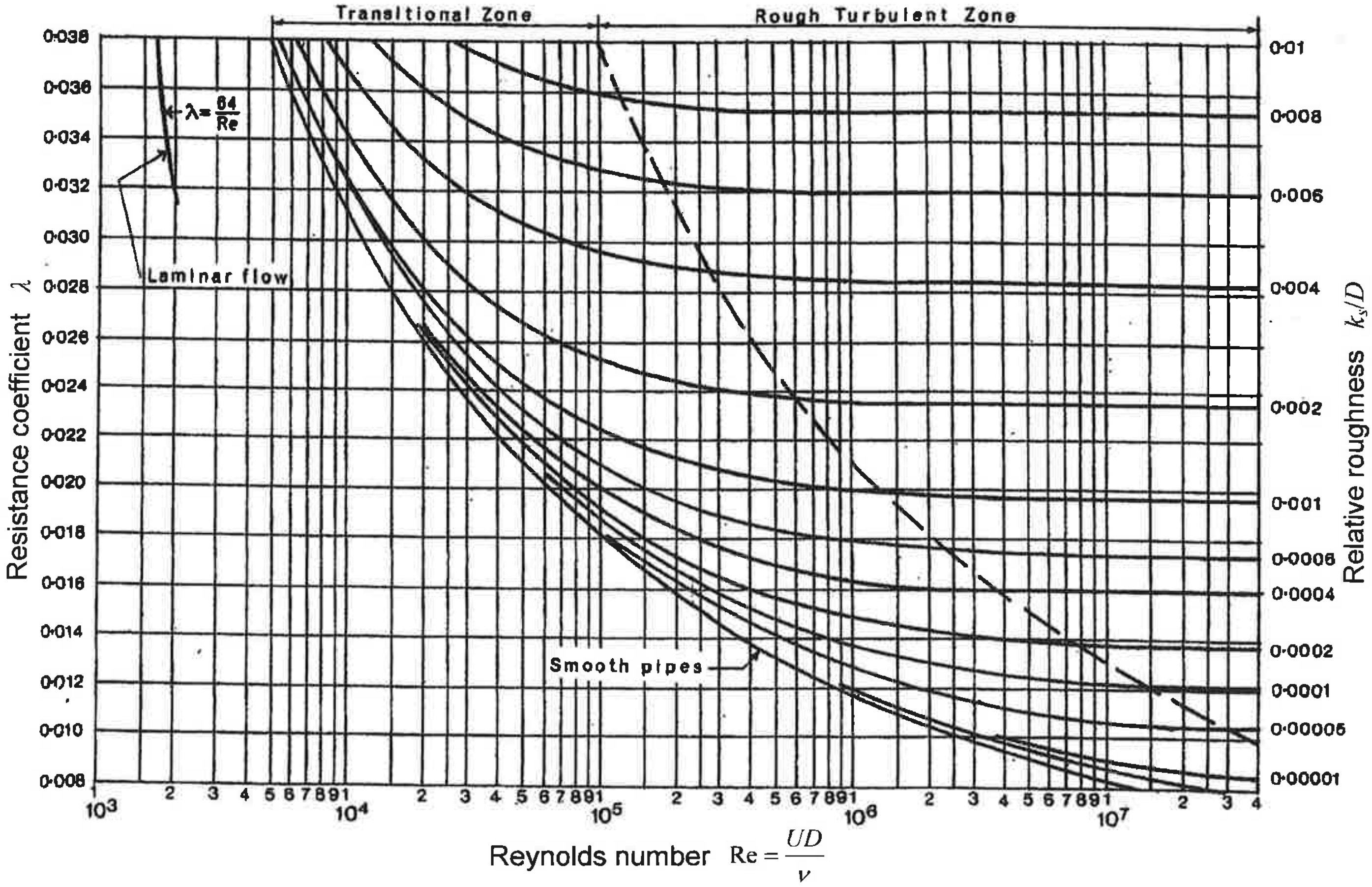
Darcy-Weisbach Equation $H_f = \lambda \frac{L}{D} \frac{U^2}{2g}$

Colebrook-White formula $\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\text{Re}\sqrt{\lambda}}\right)$ with $\text{Re} = \frac{UD}{\nu}$

Power consumption $P_p = \rho g Q_p H_p / \eta$

Non-dimensional groups $\frac{Q_p}{N_p \cdot D_p^{\;3}}, \frac{gH_p}{N_p^{\;2} \cdot D_p^{\;2}}, \frac{P_p}{\rho \cdot N_p^{\;3} \cdot D_p^{\;5}}$

Specific speed $N_S = \frac{N_p \cdot Q_p^{\;1/2}}{H_p^{\;3/4}}$



Symbols

A	area
C	runoff coefficient or Chézy coefficient
C_f	shear stress coefficient
c	concentration

D	pipeline or pump diameter
D_L	longitudinal dispersion coefficient
D_x,D_y,D_z	diffusion coefficients in x, y and z directions respectively
D_{tx},D_{ty},D_{tz}	turbulent diffusion coefficients in x, y, and z directions respectively
d	particle diameter
d_*	dimensionless particle diameter
F	force
Fr	Froude number
f	infiltration capacity
f_0	initial infiltration capacity
f_c	equilibrium infiltration capacity
g	gravitational acceleration (= 9.81 m s^{-2})
H	head
h	water depth
i	rainfall intensity
K_f	rate of decrease of f capacity
k_s	roughness height, also called equivalent or Nikuradse's sand roughness height
M	amount of the pollutant released
$\dot{M}$	rate of the pollutant release
N	rotational speed
P	power
Q	discharge
q_b	bedload sediment transport rate
R_h	hydraulic radius
S_b	bed slope
S_f	slope of the total energy line
s	specific gravity, also called relative density or density ratio (= 2.65)
T	transport-stage parameter
t	time
U	mean velocity
u_*	shear velocity
w_s	fall velocity
x,y,z	spatial coordinates
θ	Shields parameter
θ_c	critical Shields parameter
η	efficiency
κ	von Karman constant (= 0.4)
λ	Darcy-Weisbach friction factor
ν	kinematic viscosity coefficient of water (= 10^{-6} m^2 s^{-1})
ν_t	eddy viscosity coefficient
ρ	density of water (= 1000 kg m^{-3})
ρ_s	density of sediment (= 2650 kg m^{-3})
τ_b	bed shear stress
τ_{bc}	threshold bed shear stress for particle motion
$\overline{}$	Reynolds-averaged value
$'$	effective shear-stress component, also called grain-related shear-stress component
p	pump

D Liang, Oct 2011

2012 IIA 3D5 - Water Engineering Dr D. Liang

Answers

1. (a.ii) 2.35×10^5 m^3
   (b) 31.76 m^3/s

2. (a.ii) 2.076 m
   (a.iii) $\Delta x \approx 55$ m; 2.1 m section is downstream of 2.2 m section.
   (b.ii) 1.576 m

3. (b.i) 1006 rpm
   (b.ii) 2.744 m

4. (b.i) 5.4×10^{-5} kg/m^3
   (c.ii) 0.85 Pa
   (c.iii) 0.0045 kg/m^3