ENGINEERING TRIPOS PART IIA

Friday 4 May 2012 2.30 to 4

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL Version

1 (a) Customers arrive at a fast food restaurant with one server according to a Poisson process at a mean rate of 30 per hour. The server has just resigned, and the two candidates for the server's replacement are X (fast but expensive) and Y (slow but inexpensive). Both candidates would have an exponential distribution for service times with X having a mean of 1.2 minutes and Y having a mean of 1.5 minutes. Restaurant revenue per month is given by £6,000/W where W is the expected waiting time (in minutes) of a customer in the system. How much more would you be willing to pay X compared to Y? [20%]

(b) You are given an M/M/1 queuing system in which the expected waiting time and expected number in the system are 120 minutes and 8 customers, respectively. Determine the probability that a customer's service time exceeds 20 minutes. [20%]

Note: pdf of the exponential $f(x) = me^{-mx}$.

(c) The victims of a new deadly disease being treated at a Medical Center are classified annually as follows: cured, in temporary remission, sick, or dead from the disease. Once a victim is cured, that victim is permanently immune. Each year, those in remission get sick again or are cured with probabilities 1/2 each, while those who are sick get cured, go into remission, or die of the disease with probabilities of 1/3 each. If Jack gets sick with this disease,

(i) What is the probability he is eventually cured? [20%]

(ii) How many years are expected to pass until Jack is either cured or dies from the disease? [20%]

(d) Briefly explain what the omitted variable bias is and how it can be addressed. [20%]

2 (a) You are given the opportunity to guess whether a coin is fair or two-headed (i.e., both sides are heads), where the prior probabilities are 0.5 for each of these possibilities. If you are correct, you win £5; otherwise, you lose £5. You are also given the option of seeing the outcome of a demonstration flip of the coin before making your guess. You wish to use Bayes' decision rule to maximize expected profit.

(i) Develop a decision analysis formulation of this problem by identifying the decision alternatives, states of nature and payoff table. [10%]

(ii) What is the optimal decision without considering the option of seeing a demonstration flip? [10%]

(iii) Find the Expected Value of Perfect Information (EVPI). [15%]

(iv) Determine your optimal policy with access to the demonstration flip. What is the most that you should be willing to pay to see the demonstration flip. [25%]

Hint: recall the Bayes' theorem: $P(A_i/B) = P(A_i) \times P(B/A_i) / (\Sigma P(A_i) \times P(B/A_i))$

(b) The main findings of Prospect Theory can be summarized in Fig. 1 which depicts the utility function of an individual. Based solely on Fig.1, discuss in what sense prospect theory departs from expected value theory according to which individuals make decisions based on the expected value. [40%]

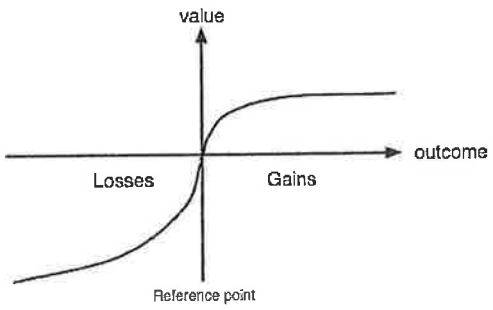

Fig. 1 Utility curve

3 (a) You have collected data on the monthly returns for the London FT Index (the market), for Marks and Spencer (M&S) shares and for Cambridge Telecom (CT) shares. Tables 1 and 2 give the results of carrying out linear regressions with the market returns as the independent variable and the M&S and CT returns as the dependent variables, respectively.

Parameter	Estimate	Std.Error	T Statistic	P-Value
Constant	0.014592	0.006701	2.1775	0.0319
FTINDEX	0.796007	0.11776	6.7596	0.0000

R-SQ.=0.3199; St.Error of Est =0.062396

Table 1 M&S on the market

Parameter	Estimate	Std.Error	T Statistic	P-Value
Constant	-0.003494	0.007037	-0.4965	0.6207
FTINDEX	1.319081	0.123663	10.6667	0.0000

R-SQ. =0.5428 St Error of Est = 0.065524

Table 2 CT on the market

(i) Write the regression equation for M&S and briefly explain its meaning. [10%]

(ii) If you were risk-averse, which company would you choose to invest in on the evidence of this data? Explain your choice. [15%]

(iii) A colleague said that she used data from an earlier time period (e.g., from the last decade) and found for CT the coefficient of FTINDEX to be 0.65. Suggest an explanation. [15%]

(b) BigChip Ltd is a computer chip manufacturer. It is considering investing in a production plant for a new product. Due to rapid changes in both consumer trends and silicon fabrication technology, this plant must pay for itself on the basis of a single run of production. A back-of-the-envelope description of this project follows:

The plant will cost £40 million for a production capacity of 100,000 chips, which is also the expected demand. The production cost per chip is zero. The price per chip is £450. Therefore,

$$\text{Profit} = \text{Revenue} - \text{Cost} = 450 \times 100{,}000 - 40 = £5 \text{ million.}$$

(i) Suppose that cost and price data are fixed (£40 million and £450, as above), e.g., due to contracts already signed by BigChip, but demand is uncertain and varies around its expected value of 100,000. Is the value of the plant subject to the Flaw of Averages? Why or why not? If so, is the back-of the-envelope calculation of profit too small or too big, or is it not possible to say without doing some simulations? [20%]

(ii) Suppose instead that cost and demand data are fixed (£40 million and 100,000 units, as above) but price is uncertain and varies around its expected value of £450 per chip. Is the value of the plant subject to the Flaw of Averages? Why or why not? If so, is the back-of-the-envelope calculation of profit too small or too big, or is it not possible to say without doing some simulations? [20%]

(c) Stock Y has a beta of 1.50 and an expected return of 17%. Stock Z has a beta of 0.80 and an expected return of 10.5%. The risk-free rate is 5.5% and the market risk premium is 7.5% . Are these stocks correctly priced? What would the risk-free rate have to be for the two stocks to be correctly priced? [20%]

4 (a) You are contemplating an investment project that has two phases. As currently planned, the first phase of the project requires an investment of £100,000 today. One year from now, the project will pay out either £120,000 or £80,000, with equal probabilities. When these Phase I payouts occur, you will have an option to invest an additional £100,000 in Phase II. One year later, Phase II will pay out either 20 percent more than Phase I actually delivered, or 20 percent less, again with equal probabilities.

You may commit to both phases at the start, or you may commit to Phase I (and postpone a decision on Phase II), or you may invest in neither. If you commit to both phases at the start, there is no reason to delay. Suppose, that you can choose, in that case, to implement both phases simultaneously, so that all payouts occur one year from now (note, however, that still the size of the Phase II payout depends on the size of the Phase I payout).

(i) Using the expected value criterion, and discounting at 10 percent per year, what should be your optimal investment strategy? [35%]

(ii) Suppose you have access to an additional investment that resembles the original but is more volatile: for the same initial investment, this project will pay out either £140,000 or £60,000, with equal probabilities. Similarly, it delivers a Phase II return of either 40 percent more or 40 percent less than the Phase I payouts. Determine the optimal investment strategy for the new alternative and explain how it compares to that of the original investment. [30%]

(b) Briefly explain what the tangent portfolio is and why it is such an important concept in portfolio management theory. [35%]

END OF PAPER