ENGINEERING TRIPOS PART IIA

Tuesday 8 May 2012 9.00 to 10.30

Module 3F3

SIGNAL AND PATTERN PROCESSING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL Version

1 (a) A digital filter with impulse response $\{h_n\}$ is to be implemented using fixed point arithmetic. Explain the issues that may arise when compared with a floating point implementation, and methods that may be be employed to reduce these issues. Your explanation should include an explanation of overflow, l_1 or l_2 scaling and frequency response scaling. [30%]

(b) A digital filter has the transfer function

$$H(z) = \frac{(1-0.1z^{-1})}{(1-0.9z^{-1})}$$

Determine the locations of any poles and zeros for the filter and sketch its frequency magnitude response over the range $\omega T = 0$ to 2π, marking on the frequencies and magnitudes where any maxima and minima occur. [20%]

(c) If $H^{(p)}(z)$ is the all-pole part of the transfer function of the above filter, i.e.

$$H^{(p)}(z) = \frac{1}{(1-0.9z^{-1})},$$

and $h_n^{(p)}$ is the corresponding all-pole impulse response, determine the following quantities:

(i) $\sum_{n=0}^{\infty} |h_n^{(p)}|$

(ii) $\sum_{n=0}^{\infty} |h_n^{(p)}|^2$ [20%]

Hence or otherwise determine a Direct Form II implementation of the original filter $H(z)$ that will reduce or eliminate the effects of overflow using l_1 scaling. You should assume that the maximum/minimum signal value which may be stored at input, output and any intermediate stages of the filter is ± 1; your implementation should also make maximum use of any available head-room at any stage of the filter in order to give best quantisation noise performance. Explain whether overflow is completely prevented or just reduced by your implementation. [30%]

2 (a) Define the terms white noise, wide-sense stationarity and ergodicity for a discrete time random process. Explain how ergodicity might enable a scientist to measure practically the autocorrelation function of a stationary random process occurring in an experiment. [30%]

(b) It is proposed to measure the acoustical impulse response of a concert hall by playing zero-mean white noise through a loud-speaker on stage and recording (in digital form, sampled with appropriate anti-aliasing filters) the resulting sound at a key listening point in the hall.

If the white noise process is denoted by $\{W_n\}$ and that of the received sound at a key location is denoted by $\{X_n\}$, show that:

(i) $r_{WX}[k] = h_k \sigma_W^2$;

(ii) $r_{XX}[k] = \sigma_W^2 \sum_{i=0}^{\infty} h_i h_{k+i}$,

where σ_W is the standard deviation of the white noise and h_k, $k = 0, 1, 2, \ldots$, is the response of the hall to a digital impulse emitted by the loud-speaker at time $k = 0$ and recorded at the key location. [30%]

(c) Hence suggest a simple method for estimating the digital impulse response of the hall based purely on the white noise-generated recordings. [20%]

(d) It is suggested that the process could be i) speeded up, or ii) made less intrusive by

(i) setting off a loud percussive noise, such as a gun, on stage and measuring the response at the key location;

(ii) playing recorded music through the loudspeaker and measuring the signal at the key location over a period of time.

Discuss whether recorded data from each of these two approaches might be analysed to obtain estimates of the hall's digital impulse response. What would be the pitfalls/advantages of these two methods compared with the original white noise approach? [20%]

(TURN OVER

3 (a) Briefly describe the circumstances in which i) a matched filter, and ii) a Wiener filter, would be the appropriate tool for detection or estimation of a signal buried in additive white noise. Your description should identify any stationarity requirements for the signals and noise components for each type of filter. You should also identify a typical real-world application in each case. [30%]

(b) In a chemical experiment it is required to detect the presence or otherwise of a pulse waveform $p[t-L]$ buried in additive noise, where L is an unknown time offset for the pulse. The pulse waveform has the formula:

$$p[t] = \begin{cases} \exp(-\lambda t) & 0 \leq t \leq 20 \\ 0, & \text{otherwise} \end{cases}$$

where λ is a known constant.

Determine the coefficients of an optimal FIR filter which can detect the most likely time offset L for the pulse when it is observed in white noise v_t with zero mean and standard deviation σ_v:

$$x_t = p[t-L] + v_t$$

[you do not need to derive the form of this filter from first principles]. [20%]

(c) For the optimal FIR filter in part (b), determine:

(i) the mean-squared value of the output from the filter when just noise is present, i.e. $x_t = v_t$; [15%]

(ii) the maximum output from the filter when just the pulse is present, i.e. $x_t = p[t-L]$. [15%]

Hence determine the maximum expected signal-to-noise ratio at the output of the optimal filter.

An alternative simpler approach to detection of the pulse consists in finding the time location of the maximum value of the signal x_t from part (b) and using this as an estimate of L. Determine how much better the optimal filter will be in terms of maximum expected signal to noise ratio compared with this simple scheme. In particular, comment on performance as $\lambda \to 0$ and $\lambda \to \infty$. [20%]

4 Consider a dataset of observations $\mathscr{D} = \{(\mathbf{x}_n, y_n)\}$ where $n = 1, \dots, N$, and N is the total number of data points. $\mathbf{x}_n$ is a two dimensional vector. A regression model of the following form is to be trained using the following form of regression

$$y_n = \mathbf{a}^{\mathrm{T}}\mathbf{x}_n + \varepsilon_n$$

where ε_n is independent zero-mean Gaussian noise with variance σ^2.

(a) Write down the log-likelihood $\log(p(y_1, \dots, y_N | \mathbf{x}_1, \dots, \mathbf{x}_N, \mathbf{a}, \sigma^2))$ in terms of $y_1, \dots, y_N, \mathbf{x}_1, \dots, \mathbf{x}_N, \mathbf{a}, \sigma^2$. [20%]

(b) Show that the maximum likelihood estimate of the regression parameters, $\hat{\mathbf{a}}$, can be expressed in the following form

$$\hat{\mathbf{a}} = \mathbf{C}^{-1}\mathbf{B}$$

You should clearly state the forms of the two matrices $\mathbf{C}$ and $\mathbf{B}$. The following equality may be useful

$$\frac{\partial \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}}{\partial \mathbf{x}} = \mathbf{x}^{\mathrm{T}}\mathbf{A} + \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$$

for any square matrix $\mathbf{A}$ and vector $\mathbf{x}$. [50%]

(c) A non-linear transformation $\phi(\mathbf{x}_n)$ is applied to the observations $\mathbf{x}_n$. The size of the resulting vector $\phi(\mathbf{x}_n)$ is d. Regression based on these transformed data points is then performed. Now

$$y_n = \mathbf{a}^{\mathrm{T}}\phi(\mathbf{x}_n) + \varepsilon_n$$

where ε_n is again independent zero-mean Gaussian noise with variance σ^2. Briefly discuss how the performance of the regression process and the estimation of the regression parameters may be impacted as the size of the transformed features, d, increases. [30%]

END OF PAPER

3F3 Answers:

1.b) Pole at 0.9, zero at 0.1

2. c)(i) 10, (ii) 5.26