ENGINEERING TRIPOS PART IIA

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Thursday 3 May 2012 2.30 to 4

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Module 3F4

DATA TRANSMISSION

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

FINAL version

1 A baseband pulse amplitude modulated (PAM) system transmits a signal in the form of a weighted impulse train

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \, \delta(t - nT_S)$$

where $a_n$ are the data symbols and $T_S$ is the symbol period.

(a) Show that the power spectral density (PSD) of the transmitted signal is

$$S_x(\omega) = \frac{1}{T_S} \sum_{m=-\infty}^{\infty} R(m) \exp(jm\omega T_S)$$

where $R(m) = E[a_n \, a_{n+m}]$ is the discrete autocorrelation function (ACF).

[40%]

(b) If the symbols $a_n$ are binary polar encoded and are the output from a maximal length $N$ pseudo-random sequence generator, then the discrete ACF of the transmitted PAM signal is

$$R(m) = \begin{cases} 1 & , \quad m = kN \quad , \quad k = 0, \pm 1, \pm 2, \ldots \\ -1/N & , \quad m \neq kN \end{cases}$$

Hence determine an expression for the PSD of the transmitted PAM signal. [40%]

Note:

$$\sum_{m=-\infty}^{\infty} \exp(jmT_S\omega) = \frac{2\pi}{T_S} \sum_{m=-\infty}^{\infty} \delta\left(\omega - \frac{m2\pi}{T_S}\right)$$

(c) The PAM signal having the discrete ACF in part (b) is passed through a filter having the following impulse response

$$h(t) = \begin{cases} 1 & , \quad -T_S/2 \leq t \leq T_S/2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Determine the PSD of the signal at the output of the filter and sketch the result. [20%]

2 A (6,3) systematic linear binary block code C is constructed as follows:

Code bit 2 is an even parity check on code bits 5 and 4;
Code bit 1 is an even parity check on code bits 4 and 3;
Code bit 0 is an even parity check on code bits 5 and 3

where the data bits are located in code bits 5 to 3. Note that the left-most code bit is labelled code bit 5 and the right-most code bit is labelled code bit 0 .

(a) Find the generator and parity check matrices for the code C . [30%]

(b) Find the minimum distance for code C hence the maximum number of correctable errors. [30%]

(c) Construct a standard array for the code C including the syndromes for all the correctable error patterns. [25%]

(d) Explain how the standard array constructed in part (c) can be used to implement an efficient decoder for code C . [15%]

3 (a) Show how a modulated signal waveform $s(t)$ with carrier frequency $\omega_C$ may be defined in terms of a modulated phasor waveform $p(t)$; and explain how phase, frequency and amplitude modulation may all be represented in this way. [20%]

(b) Derive an expression for the spectrum $S(\omega)$ of the modulated signal in terms of the spectrum $P(\omega)$ of the phasor waveform. [20%]

(c) For a phase-shift keying (PSK) or quadrature amplitude modulation (QAM) scheme, with $m$ bits per transmitted symbol and a bit period of $T_b$, indicate how the power spectral density $|P(\omega)|^2$ relates to $m$, assuming conventional rectangularly shaped symbols are used in all cases. Sketch $|P(\omega)|^2$ and $|S(\omega)|^2$, for the two cases, $m = 2$ and $m = 6$, assuming a carrier frequency (in Hz) of three times the bit rate (in bit $s^{-1}$). [30%]

(d) Sketch the constellation of modulation states for QPSK and for 64-QAM (there is no need to show all 64 states, a small representative set will be adequate). Based on scaling which produces unit mean squared voltage per transmitted bit in each constellation, estimate the minimum amplitude of noise that will just cause an error, both for QPSK and for 64-QAM.

Hence discuss the tradeoffs between these two modulation formats, particularly with regard to their use in digital broadcasting of audio and video. [30%]

Note:

$$\sum_{i=0}^{M-1} (2i+1-M)^2 = \frac{M(M^2-1)}{3}$$

4 (a) Discuss the relative merits of Digital versus Analogue communication systems and explain the recent trend towards digital systems. [25%]

(b) Signals can reach a receiver via multiple transmission paths of different path lengths, especially in urban surroundings. Explain the problem that this causes for high bit-rate digital systems (typically up to 30 Mbit $s^{-1}$) and show how orthogonal frequency division multiplexing (OFDM) can largely overcome this problem. Show also how OFDM can allow 'single-frequency' coverage of the whole of the UK if certain assumptions are made about the maximum spacing between adjacent transmitters. [25%]

(c) A digital audio broadcasting (DAB) system is to be designed to handle differential path delays of up to 300 $\mu$s. If the guard period is to be one quarter of the total period for each transmitted symbol, determine the maximum frequency spacing of adjacent OFDM carriers. Hence estimate the radio frequency bandwidth that would be required to carry a 2 Mbit $s^{-1}$ data stream of multiplexed audio data, assuming that a rate 1:2 error correcting code is employed and that QPSK modulation is used on each OFDM carrier. [25%]

(d) Estimate the spectral efficiency, in Hz per radio channel, achieved by the scheme in (c) above, assuming that audio source compression limits the bit rate to 128 kbit $s^{-1}$ per stereo audio channel. Compare this to the spectral efficiency for analogue stereo FM broadcasts, which typically employ a peak frequency deviation of 100 kHz on each side of the carrier frequency. What additional gains would the 'single-frequency' coverage property of OFDM be likely to provide for national radio channels? [25%]

**END OF PAPER**