ENGINEERING TRIPOS PART IIA

Friday 27 April 2012 9 to 10.30

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL version

SECTION A *Medical Image Acquisition*

Answer ***one*** *question from this section*

1 (a) Using the material properties given in Table 1, calculate the ultrasound intensity reflection coefficients

(i) between fat and water, and

(ii) between fat and liver. [30%]

(b) Ultrasound beams from a 7 MHz transducer are used to image the objects shown in Fig. 1. Consider the two beams shown in the figure. Take the transmitted intensity of these beams to be one unit in the water at points A and B.

(i) For the beam that starts at point A, calculate the proportion that follows path A, is reflected by the water-fat interface and returns to point A.

(ii) For the beam that starts at point B, calculate the proportion that follows path B, is reflected by the liver-fat interface and returns to point B. [40%]

(c) The ultrasound machine is calibrated to assume a sound speed of $1540\,\mathrm{ms}^{-1}$.

(i) Calculate the perceived distance between the face of the probe and the water-fat interface based on ultrasound travelling along path A.

(ii) Calculate the perceived distance between the face of the probe and the liver-fat interface based on ultrasound travelling along path B. [30%]

(cont.

	sound speed	specific acoustic impedance	attenuation factor
fat	1450	1.38	0.63
liver	1604	1.69	0.7
water	1498	1.5	0.0022
units	$\mathrm{m\,s^{-1}}$	$10^6\,\mathrm{kg\,m^{-2}\,s^{-1}}$	$\mathrm{dB\,(cm)^{-1}\,(MHz)^{-1}}$

Table 1

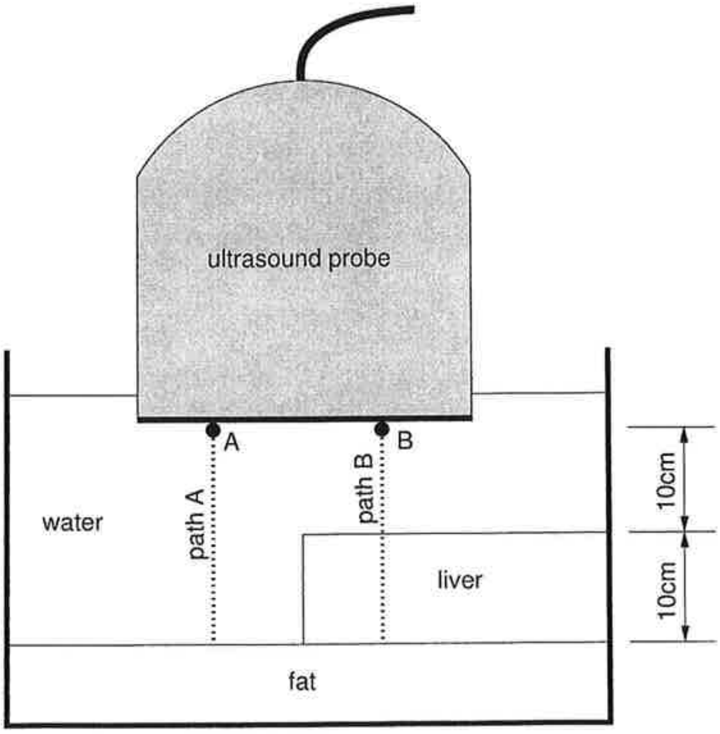

Fig. 1

(TURN OVER

2 (a) Explain, with the help of a detailed labelled diagram, how X-rays are produced by an X-ray tube with a rotating anode. What range of accelerating voltages is typically used between the cathode and the anode? What proportion of the energy in the electron beam is emitted from the tube in the form of X-rays? How is the cathode heated and to what temperature? [30%]

(b) List three properties of X-rays that enable their detection in medical imaging devices and systems. [15%]

(c) Starting from the *projection theorem*, derive the *filtered backprojection* computed tomography reconstruction algorithm. [35%]

(d) Sketch the Ram-Lak filter in the spatial frequency domain. How should the maximum and minimum spatial frequencies in the filter response be determined from the X-ray beam-width of the scanning system? [20%]

SECTION B *Extracting Information from 3D Data*

*Answer **one** question from this section*

3 (a) Comment on the advantages and disadvantages of using Bézier, Catmull-Rom and B-spline cubic parametric representations when designing, refining and displaying single or multi-segment curves. [30%]

(b) For each type of curve in (a), specify what degree of continuity between curve segments is easily achievable, and explain under what conditions this continuity is achieved. [15%]

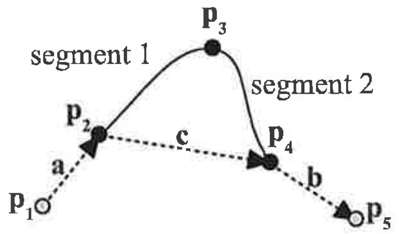

Fig. 2

(c) It is required to generate a smooth curve which interpolates three points $\mathbf{p}_2 \ldots \mathbf{p}_4$. This is accomplished by using two segments of a Catmull-Rom spline defined by the points $\mathbf{p}_1 \ldots \mathbf{p}_5$, as in Fig. 2. The Catmull-Rom basis matrix is

$$\mathbf{M} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

(i) What are the conditions on $\mathbf{p}_1 \ldots \mathbf{p}_5$ for the curve to have parametric continuity in the second derivative (C2 continuity)? [20%]

(ii) By expressing the answer to (i) in terms of the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ defined in Fig. 2, or otherwise, suggest how the points $\mathbf{p}_1$ and $\mathbf{p}_5$ can be manipulated whilst preserving C2 continuity. [20%]

(iii) Discuss whether this scheme could be extended to a three-segment Catmull-Rom spline interpolating four points with C2 continuity. [15%]

(TURN OVER

4 (a) Describe how the *marching cubes* algorithm can be used to extract a triangulated iso-surface from a 3D scalar data set. [25%]

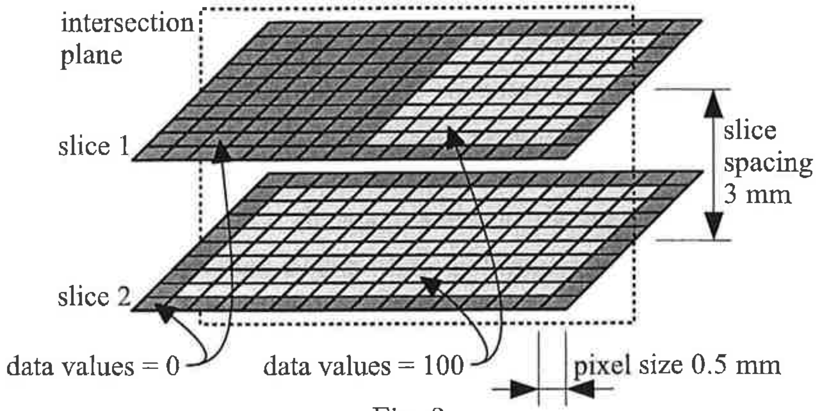

Fig. 3

(b) An iso-surface at a value of 50 is to be extracted from the two-slice data set in Fig. 3. To this end, marching cubes is applied:

(i) directly to the data by extending one of the 'cube' dimensions to the slice width of 3 mm;

(ii) to new data formed by the addition of five intermediate, equally spaced slices created by linear interpolation of the existing slices;

(iii) as (i), but thresholding and using a distance transform on each slice;

(iv) as (ii), but thresholding and using a distance transform on each original slice *before* data interpolation to give the intermediate slices.

For each case, sketch the intersection of the resulting iso-surface with the orthogonal plane shown dashed in Fig. 3. Comment on the size and shape of the triangles over each part of the surface in the vicinity of the intersection. [40%]

(c) How might the surface created by marching cubes be post-processed and why would this be useful? [15%]

(d) In practice, sharp data discontinuities as in Fig. 3 are blurred by the imaging process. How would such blurring affect your answers to (b)(i) and (b)(iii)? [20%]

SECTION C *3D Graphical Rendering*

Answer ***one*** *question from this section*

5 (a) The basic Phong reflection model can be written as

$$I_\lambda = c_\lambda(I_a k_a + I_p k_d \mathbf{L}.\mathbf{N}) + I_p k_s (\mathbf{R}.\mathbf{V})^n$$

Explain the meanings of the various terms. Show how the model can be extended to account for multiple light sources, depth cueing and shadows. [40%]

(b) Why might it be advantageous to replace **R.V** with **N.H**, where **H** is half way between **L** and **V**? [10%]

(c) Figure 4 shows three OpenGL surface renderings of a 3D letter 'O'. In each case, there is a single light source positioned at infinity, behind the viewer, along the line defined by the centres of the two specular highlights.

(i) Explain the differences between the three renderings in terms of the rendering parameters. [20%]

(ii) There are several artefacts common to all three renderings. Identify these and suggest how they might be suppressed. [30%]

Fig. 4

(TURN OVER

6 (a) Draw a block diagram of the surface rendering pipeline. Show in each block the operations performed at that stage, and the coordinate systems used. [25%]

(b) The mapping from view coordinates to homogeneous 3D screen coordinates can be written as

$$\begin{bmatrix} wx_s \\ wy_s \\ wz_s \\ w \end{bmatrix} = \begin{bmatrix} d/x_{\max} & 0 & 0 & 0 \\ 0 & d/y_{\max} & 0 & 0 \\ 0 & 0 & -f/(f-n) & -fn/(f-n) \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$

(i) Why is it important that the mapping can be written in this form? [10%]

(ii) Explain why it may not be necessary to convert every vertex's homogeneous 3D screen coordinates into Cartesian equivalents. [10%]

(iii) Sketch a typical relationship between z_v (on the x-axis) and z_s (on the y-axis). [10%]

(c) A hardware z-buffer represents the depth values as 16-bit integers (z_s is multiplied by 2^{16} and rounded down to the nearest integer). Consider a view volume with $n = 1$ and $f = 100$. In the rendering, it is observed that one vertex fails to obscure another, even though it maps to the same pixel and its z_v value is smaller in magnitude. What is the maximum difference between the two z_v values consistent with this observation? [30%]

(d) Discuss how this error might be corrected with little risk of affecting the appearance of other objects in the scene. [15%]

END OF PAPER

Part IIA 2012

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (a) (i) 1.763×10^{-3}, (ii) 10.196×10^{-3}

   (b) (i) 1.5×10^{-3}, (ii) 1.495×10^{-12}

   (c) (i) 20.56 cm, (ii) 19.88 cm

6. (c) 0.151