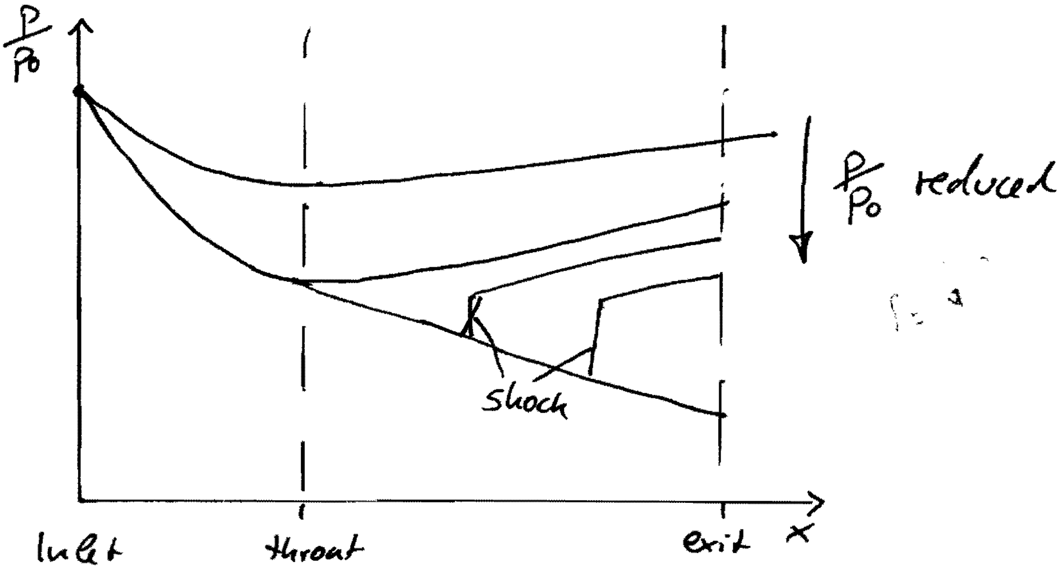
3A3 2013 Solutions

Q1) a) i)



ii) When the throat first chokes $\dfrac{\dot m\sqrt{c_p T_0}}{A_T\, p_0} = 1.281$ $(M=1)$

$$\frac{A_E}{A_T} = 1.2 \Rightarrow \frac{\dot m\sqrt{c_p T_0}}{A_E\, p_0} = \frac{1.281}{1.2} = 1.0675$$

from Tables $M_E = 0.59$ and $\dfrac{p_E}{p_0} = 0.79$

iii) Fully supersonic, no shocks, hence $\dfrac{\dot m\sqrt{c_p T_0}}{A_E\, p_0} = 1.0675$ (also)

Now find supersonic solution from tables: $M_E = 1.535$

and $\dfrac{p_E}{p_0} = 0.258$

b) $\dfrac{\Delta p_0}{p_0} = 1\%$ hence $p_{0E} = 0.99\, p_0$

Find equivalent normal shock for 1% loss. Tables $M = 1.225$

Hence: $\dfrac{\dot m\sqrt{c_p T_0}}{A_s\, p_0} = 1.2338$

$$\Rightarrow \frac{A_s}{A_T} = \frac{1.281}{1.2338} = 1.038$$

Nozzle is linear, full distance from throat to exit is L

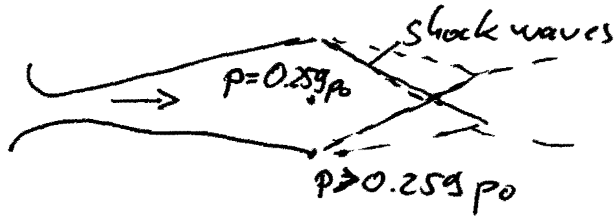
hence shock at $\dfrac{0.038}{0.2} L = 19\%\ L$

Q1c cont.,

When p_E is 67% of p_0 a shock is at the nozzle end. From a) iii) for $M_E = 1.535$, $p_E = 0.259\,p_0$

$$\frac{p_s}{p} = 0.2589 \quad \Rightarrow p_{s,E} = 0.2589 \cdot 0.259\,p_0 = 0.67\,p_0$$

For pressures between $p_E = 0.259\,p_0$ and $0.67\,p_0$ there can not be a shock inside nozzle, yet the exit pressure is above that for clean supersonic outflow. This is an overexpanded nozzle and oblique shock waves form on exit:



Q2 a)

Impulse function $\quad F = A(P + \rho V^2)$

Continuity $\quad \rho A V = \text{const.}$

Energy cons. $\quad h + \frac{1}{2}V^2 = \text{const}$

$$c_p T + \frac{1}{2}V^2 = \text{const}$$

Const. Area ∴ $dA = 0$

$$dF = A\,dP + \rho A V\,dV \qquad (\text{note } \rho A V = \text{const.})$$

* $\frac{P}{\rho} = RT \Rightarrow dp = RT\,d\rho + \rho R\,dT$

$$\Rightarrow dF = A(RT\,d\rho + \rho R\,dT + \rho V\,dV)$$

from Energy: $c_p\,dT = -V\,dV \Rightarrow dT = -\frac{V}{c_p}\,dV$

from continuity: $A(\rho\,dV + V\,d\rho) = 0 \Rightarrow d\rho = -\frac{\rho}{V}\,dV$

$$\Rightarrow dF = A\left(-\rho\frac{RT}{V}\,dV - \rho R\frac{V}{c_p}\,dV + \rho V\,dV\right)$$

$$= A\frac{dV}{V}\left[-\rho RT + \rho V^2\left[1 - \frac{R}{c_p}\right]\right] \qquad \frac{R}{c_p} = \frac{\gamma - 1}{\gamma}$$

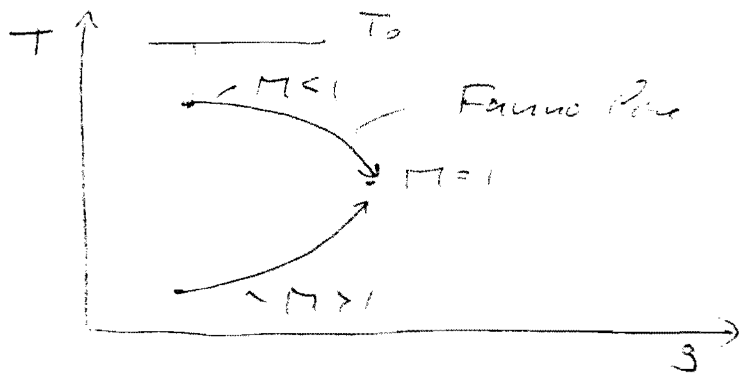
$$= A\frac{dV}{V}\left[-P + \frac{P}{RT}V^2\frac{1}{\gamma}\right] = PA(M^2 - 1)\frac{dV}{V}$$

a^2

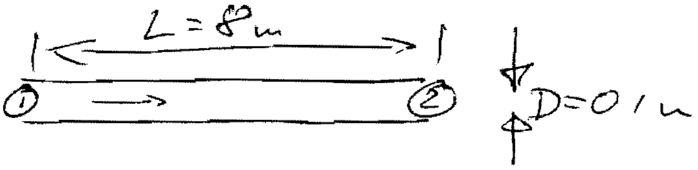
b) i) Friction ⇒ F is reducing ⇒ dV is positive ⇒ accelerate

ii) " " dV is negative ⇒ slow down

⇒ tends towards M=1 in both cases



Q2 cont.)



c) $p_0 = 5\,bar$

$p_1 = 4\,bar$

$p_2 = 2\,bar$

$L = 8m$

At ①: $\left.\frac{p}{p_0}\right)_1 = 0.8 \Rightarrow M = 0.57 \qquad \frac{4c_f L_{max}}{D} = 0.62$

$$\frac{\dot m\sqrt{c_p T_0}}{A\,p_0} = 1.04$$

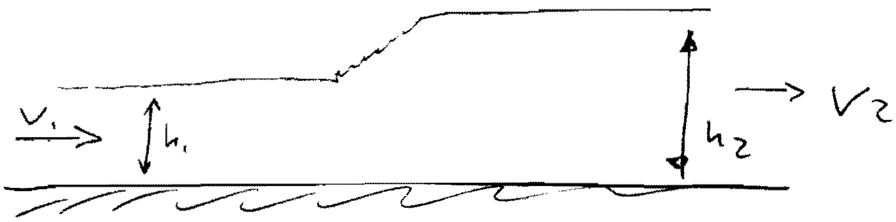
At ②: $\frac{\dot m\sqrt{c_p T_0}}{A\,p_2} = \frac{\dot m\sqrt{c_p T_0}}{A\,p_0}\cdot\frac{p_0}{p_2} = 1.04\cdot\frac{5}{2.5} = 2.08$

$\Rightarrow M_2 = 0.88 \Rightarrow \frac{4c_f L_{2max}}{D} = 0.02$

$\Rightarrow \frac{4c_f L}{D} = \frac{4c_f L_{1max}}{D} - \frac{4c_f L_{2max}}{D} = 0.62 - 0.02 = 0.6$

$\Rightarrow c_f = 0.6\cdot\frac{D}{4L} = \underline{\underline{1.88\cdot 10^{-3}}}$

Q3



a) Continuity: $V_1 h_1 = V_2 h_2$

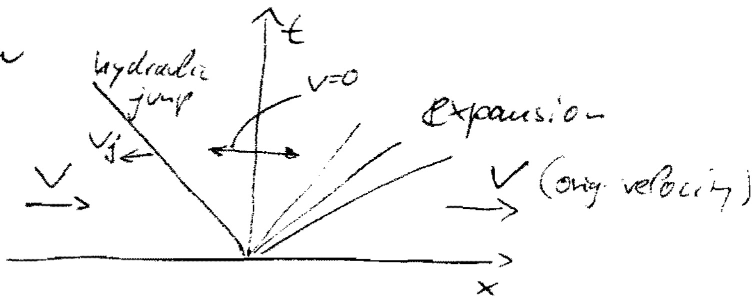
Momentum: $\bar{p}_1 h_1 - \bar{p}_2 h_2 = \dot{m}(v_2 - v_1)$

$$\tfrac{1}{2}\rho g h_1^2 - \tfrac{1}{2}\rho g h_2^2 = \rho h_1 v_1 (v_2 - v_1) = \rho h_2 v_2 v_1 - \rho h_1 v_1 v_1$$

$$\tfrac{1}{2} g (h_1^2 - h_2^2) = h_2 v_2 v_1 - h_1 v_1 v_2$$

for $h_1 \neq h_2$: $\tfrac{1}{2} g (h_1 + h_2) = v_1 v_2$

b) Unsteady flow



Use hydraulic jump frame of ref:

$\Rightarrow$ Incoming flow speed: $V + v_j$ $\hat{=}$ V_1 above

Outgoing " " : v_j $\hat{=}$ V_2 above

$\Rightarrow$ from a) $(V + v_j) v_j = \tfrac{1}{2} g (h_1 + h_2)$

And $(V + v_j) \cdot h_1 = v_j \cdot h_2$ $\Rightarrow$ $(V + v_j) = v_j \dfrac{h_2}{h_1}$

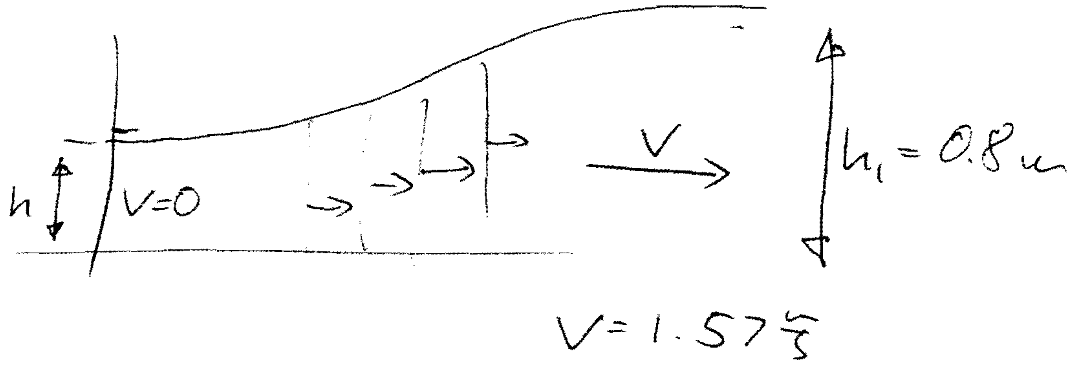
$\Rightarrow$ $v_j^2 = \tfrac{1}{2} g \dfrac{h_1}{h_2} (h_1 + h_2)$

$$v_j = 2.52 \ \tfrac{m}{s}$$

$$\Rightarrow V + v_j = 4.09$$

$$\Rightarrow \underline{V = 1.57 \ \tfrac{m}{s}}$$

c) Expansion wave



Using $V - 2c = \text{const}$ (right running wave)

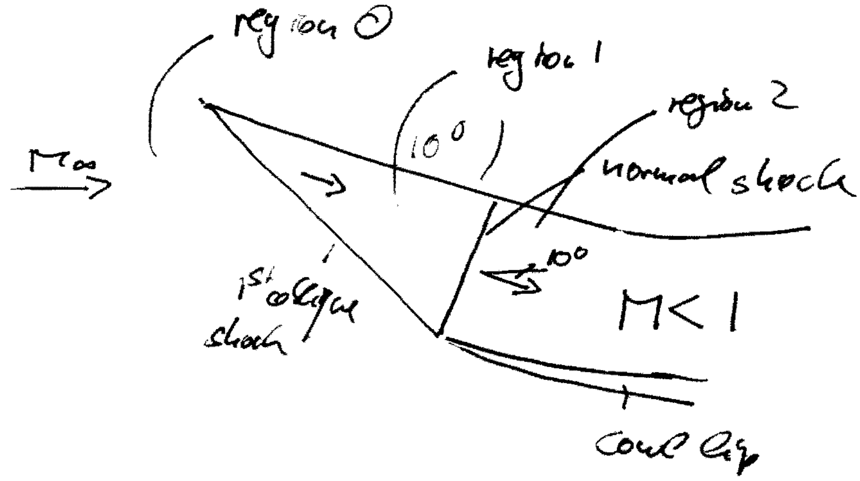
$$0 - 2\sqrt{gh} = V - 2\sqrt{gh_1}$$

$$\Rightarrow 4gh = \left(2\sqrt{gh_1} - V\right)^2$$

$$h = \frac{\left(2\sqrt{gh_1} - V\right)^2}{4g} = \underline{\underline{0.41\ m}}$$

Q4) Draft solution

a)



i) Across oblique shock: (M=2, 10 deg)

shock angle : 39.3

$\frac{p_1}{p_\infty} = 1.707$

$\frac{p_{01}}{p_{0\infty}} = 0.985$

$M_2 = 1.64$

Across normal shock: $M=1.64$: $\frac{p_2}{p_1} = 2.971$

$\frac{p_{02}}{p_{01}} = 0.88$

=> Total pressure recovery : $\frac{p_2}{p_\infty} = 1.707 \cdot 2.971 = \underline{\underline{5.07}}$

stag p loss $\frac{p_{02}}{p_{0\infty}} = 0.985 \cdot 0.88 = 0.867$

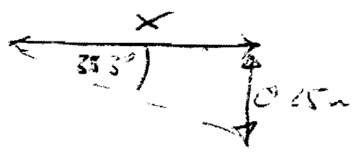
=> loss : 13.3 %

b) At $M = 1.8$ and $10°$ shock deflection :

OS : $\frac{p_1}{p_\infty}$ 1.661 $\frac{p_{01}}{p_{0\infty}} = 0.987$ Shock angle β = 44.06°

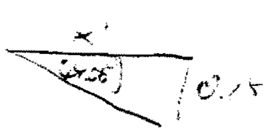
$M_2 = 1.45$

Geometry : at $M=2$



$\Rightarrow x = \frac{0.25m}{\tan 39.3} = 0.305m$

At $M=1.8$



$x' = \frac{0.25}{\tan 44.06°} = 0.258$

4b) cont.

static p recovery: Normal shock $\frac{p_c}{p_i} = 2.286$

~~Oblique shock~~: $\frac{p_{02}}{p_{01}} = 0.945$

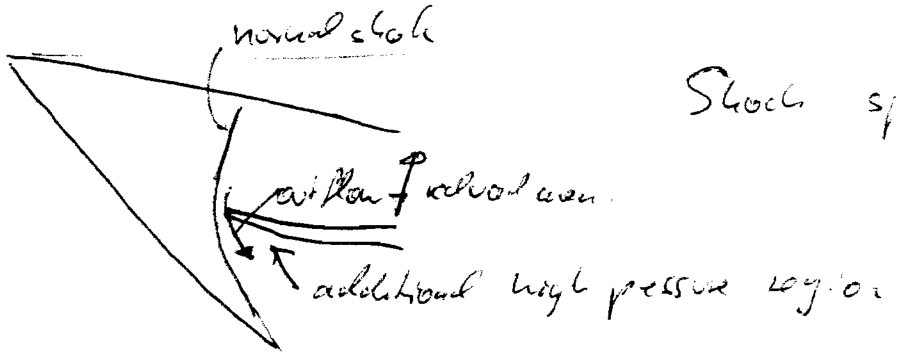
$\Rightarrow$ Recovery $\frac{p_2}{p_\infty} = 2.286 \cdot 1.661 = 3.797$

Stag p loss: $\frac{p_{02}}{p_{0\infty}} = 0.945 \cdot 0.987 = \cancel{93}\, 0.933$

$\Rightarrow$ loss $= 6.7\%$

Losses are reduced because of the lower flight M (shocks are weaker). However the weak shocks also achieve less compression.

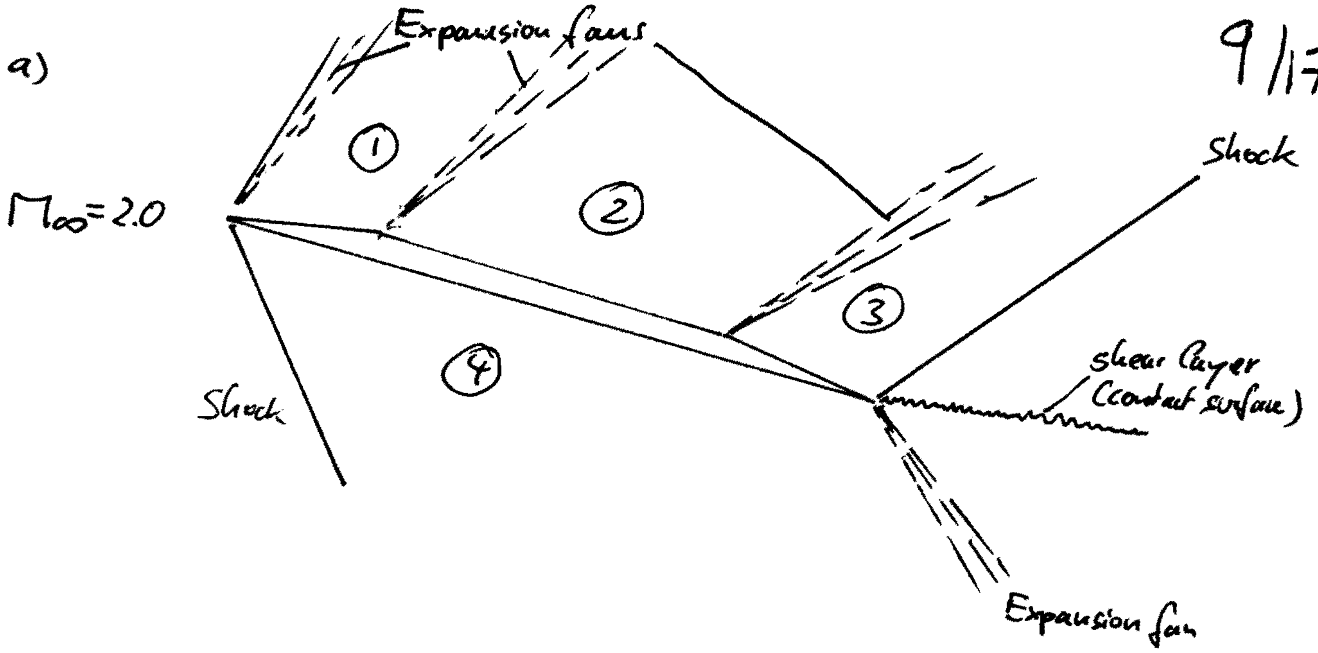
4c)



Shock spillage

Now, more air is compressed through shocks than reaches the inlet $\rightarrow$ this causes additional wave drag (e.g. through high p acting on cowl). Also, there is now spillage of high p air ahead of the cowl lip because the intake can not 'swallow' all the flow (flow area compared to case 1b).

Q5 a)



b) Flow directions: $\theta_1 = -2^\circ$

$\theta_2 = -6^\circ$

$\theta_3 = -8^\circ$

$\theta_4 = -6^\circ$

At $M = 2$: $\dfrac{p_\infty}{p_{0\infty}} = 0.1278$

$\nu = 26.38$

1 $\nu_\infty + \theta_\infty = \nu_1 + \theta_1 \Rightarrow \nu_1 = 28.38 \Rightarrow M_1 = 2.073 \quad \dfrac{p_1}{p_\infty} = 0.892$

(from $\dfrac{p_2}{p_0} = 0.114$)

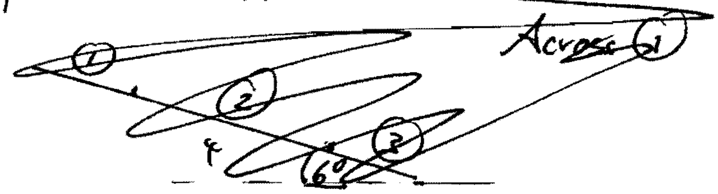
2 $\nu_\infty + \theta_\infty = \nu_2 + \theta_2$ (no shocks in between)

$\Rightarrow \nu_2 = 32.38 \Rightarrow M_2 = 2.225 \quad \dfrac{p_2}{p_\infty} = 0.704$

3 $\nu_\infty + \theta_\infty = \nu_3 + \theta_3 \Rightarrow \nu_3 = 34.38 \quad M_3 = 2.304 \quad \dfrac{p_3}{p_\infty} = 0.622$

4 Oblique shock, 6° turning: $M_4 = 1.7856 \quad \dfrac{p_4}{p_\infty} = 1.387$

~~Calculate forces per unit width~~



5b cont

Calculate force contributions per ~~[illegible]~~ unit width on each surface (use correct angles).

① $R_{\uparrow} = 0.892 \cdot \cos 2^{\circ} \cdot 0.5\,m \cdot 2\,\frac{N}{m^2} = -0.89\,\frac{N}{m}$

$R_{\rightarrow} = 0.892 \cdot \sin 2^{\circ} \cdot 0.5\,m \cdot 2\,\frac{N}{m^2} = -0.03\,\frac{N}{m}$

② $R_{\uparrow} = 0.704 \cdot \cos 6^{\circ} \cdot 2 = -1.4\,\frac{N}{m}$

$R_{\rightarrow} = 0.704 \cdot \sin 6^{\circ} \cdot 2 = -0.147\,\frac{N}{m}$

③ $R_{\uparrow} = 0.622 \cdot \cos 8^{\circ} = -0.616\,\frac{N}{m}$

$R_{\rightarrow} = \quad = -0.087\,\frac{N}{m}$

④ $R_{\uparrow} = 1.387 \cdot \cos 6^{\circ} \cdot 2\,m \cdot 2\,\frac{N}{m^2} = 5.52\,\frac{N}{m}$

$R_{\rightarrow} = 1.387 \cdot \sin 6^{\circ} \cdot 4 = 0.58\,\frac{N}{m}$

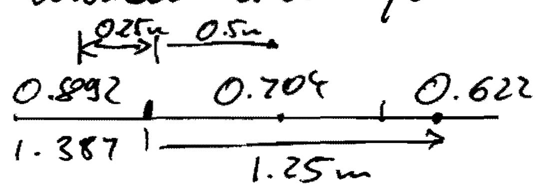
At $M = 2$ $\quad \frac{\frac{1}{2}\rho V^2}{p_0} = 0.358 \Rightarrow \frac{1}{2}\rho V^2 = 0.358 \cdot \frac{p_0}{p_\infty} \cdot p_\infty$

i) $C_L = \frac{L}{\frac{1}{2}\rho V^2 \cdot c} = \frac{0.1278 \cdot [5.52 - 0.89 - 1.4 - 0.616]}{0.358 \cdot 2\frac{N}{m^2} \cdot 2m} = \underline{\underline{0.23}}$

ii) $C_D = \frac{0.1278\,[0.58 - 0.03 - 0.147 - 0.087]}{0.358 \cdot 2 \cdot 2} = 0.03$

iii) Pitching moment about quarter chord:

Pressures:



$C_M = \frac{M}{\frac{1}{2}\rho V^2 c^2} = \ldots = \frac{0.1278}{0.358 \cdot (2m)^2} \cdot \Big[(1.387 - 0.892) \cdot 0.25m - (1.387 - 0.704) \cdot 0.5m - (1.387 - 0.622)\,1.25m$

$= -0.10$ (nose down)

5c) At subsonic speed the centre of pressure is near the quarter chord point, further forward than at supersonic speed. This can cause pitch up when slowing down which can lead to dangerous stall.

Q6

12/17 (7)

$$V_i^{n+1} = V_i^n + \frac{\nu \Delta t}{\Delta y^2}\left(V_{i+1}^n - 2V_i^n + V_{i-1}^n\right)$$

(a) time: taylor series
$$V_i^{n+1} = V_i^n + \left.\frac{\partial V}{\partial t}\right|_n \Delta t + \left.\frac{\partial^2 V}{\partial t^2}\right|_n \frac{\Delta t^2}{2!} + \ldots$$

$$\therefore \left.\frac{\partial V}{\partial t}\right|_n = \frac{V_i^{n+1} - V_i^n}{\Delta t} - \left.\frac{\partial^2 V}{\partial t^2}\right|_n \frac{\Delta t}{2!} + \ldots$$

$$= \frac{V_i^{n+1} - V_i^n}{\Delta t} + O(\Delta t)$$ first order

In space:
$$V_{i+1}^n = V_i^n + \left.\frac{\partial V}{\partial y}\right|_i \Delta y + \left.\frac{\partial^2 V}{\partial y^2}\right|_i \frac{\Delta y^2}{2!} + \left.\frac{\partial^3 V}{\partial y^3}\right|_i \frac{\Delta y^3}{3!} + \left.\frac{\partial^4 V}{\partial y^4}\right|_i \frac{\Delta y^4}{4!} + \ldots$$

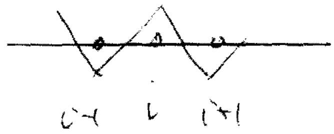
$$V_{i-1}^n = V_i^n - \left.\frac{\partial V}{\partial y}\right|_i \Delta y + \left.\frac{\partial^2 V}{\partial y^2}\right|_i \frac{\Delta y^2}{2!} - \left.\frac{\partial^3 V}{\partial y^3}\right|_i \frac{\Delta y^3}{3!} + \left.\frac{\partial^4 V}{\partial y^4}\right|_i \frac{\Delta y^4}{4!} + \ldots$$

Add:
$$V_{i+1}^n + V_{i-1}^n = 2V_i^n + 2\left.\frac{\partial^2 V}{\partial y^2}\right|_i \frac{\Delta y^2}{2!} + 2\left.\frac{\partial^4 V}{\partial y^4}\right|_i \frac{\Delta y^4}{4!}$$

$$\therefore \left.\frac{\partial^2 V}{\partial y^2}\right|_i = \frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{\Delta y^2} + O(\Delta y^2)$$

second order

6(b) use a sawtooth disturbance



$$V_i^{n+1} = \varepsilon + \left(\frac{\nu \Delta t}{\Delta y^2}\right)(-\varepsilon - 2\varepsilon - \varepsilon)$$

$$V_i^n = \varepsilon$$
$$V_{i+1}^n = -\varepsilon = V_{i-1}^n$$

let $\alpha = \dfrac{\nu \Delta t}{\Delta y^2}$;

$$\frac{V_i^{n+1}}{\varepsilon} = 1 - 4\alpha \; ; \quad \left|\frac{V_i^{n+1}}{\varepsilon}\right| = |1-4\alpha| \le 1$$

for stability

$$1 - 4\alpha \le 1 \quad \text{or} \quad 1 - 4\alpha \le -1$$

hence $\alpha \le \frac{1}{2}$ for stability

$$\therefore \; \Delta t \le \frac{\Delta y^2}{2\nu}$$

6(c) Stability can be improved by using an implicit method, or a Runge-Kutta method, or by artificial smoothing.

Q7a) hyperbolic $\frac{\partial V}{\partial t} + A\frac{\partial V}{\partial x} = 0$

Initial value problem, signals propagate along characteristics
Use upwinding to mimic the directionality of signals.

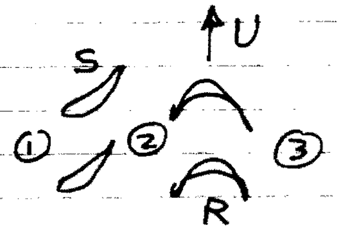
elliptic $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

Boundary value problem – every point in the domain affects every other point. Use a Poisson solver and iterate to find a solution.

parabolic $\frac{\partial V}{\partial t} - \frac{\partial^2 V}{\partial x^2} = 0$

initial and boundary value problem. Signals travel downstream and diffuse throughout the domain. Use a marching scheme to advance the solution in time/downstream.

Q7 b) HALF QUESTION



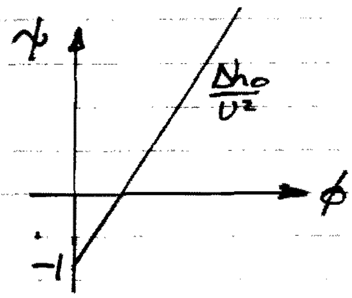
(i) $\frac{\Delta h_o}{U^2} = \frac{h_{o2} - h_{o3}}{U^2} = \frac{U(v_{\theta 2} - v_{\theta 3})}{U^2}$

$= \frac{1}{U}(v_{\theta 2} - v_{\theta 3})$

$= \frac{1}{U}(v_{\theta 2} - [v_{\theta 3}^{REL} + U])$

$= \frac{1}{U}(v_x \tan\alpha_2 - v_x \tan\alpha_3^{REL} - U)$

$$\frac{\Delta h_o}{U^2} = \phi(\tan\alpha_2 - \tan\alpha_3^{REL}) - 1$$



(ii) $\alpha_2 = 60^\circ$ $\alpha_3 = -30^\circ$ $\tan\alpha_3^{REL} = \tan\alpha_3 - \frac{1}{\phi}$

$\phi = 250/300$ $\tan\alpha_3^{REL} = \tan(-30^\circ) - 300/250 = -1.777$

$\psi = \frac{\Delta h_o}{U^2} = \phi(\tan\alpha_2 - \tan\alpha_3^{REL}) - 1$

$= \frac{250}{300}\left(\tan(60^\circ) - -1.777\right) - 1$

$$\psi = 1.924$$

Stage loading ≈ 2 is a reasonable value for a turbine stage.

(iii) Aerodynamic limiting no-load $\Rightarrow \frac{\Delta h_o}{U^2} = 0$

Hence $\phi(\tan\alpha_2 - \tan\alpha_3^{REL}) = 1$

$\Rightarrow U_{LIMIT} = v_x(\tan\alpha_2 - \tan\alpha_3^{REL})$

$= 250(\tan 60^\circ - -1.777)$

$$U_{LIMIT} = 877 \text{ m/s}$$

Note: Aerodynamic limit is approximately 3.5 times the normal operating speed – disc burst will probably occur before this limit is reached.

FULL QUESTION

8a). $U = r\Omega = 0.5 \times \frac{4800}{60} \times 2\pi = \underline{251.33\ m/s}$

$$T_1 = \frac{T_{01}}{(1+\frac{\gamma-1}{2}M_1^2)} = \frac{280}{1+0.2\times0.4^2} = \underline{271.32\ K}$$

$$V_{x1} = M_1\sqrt{\gamma R T_1} = 0.4\sqrt{1.4\times287\times271.32} = \underline{132.07\ m/s}$$

$$V_{\theta1}^{REL} = V_{\theta1} - U = 0 - 251.33 = \underline{-251.33\ m/s}$$

$$V_1^{REL} = \sqrt{V_{x1}^2 + V_{\theta1}^{REL\,2}} = \sqrt{132.07^2 + 251.33^2} = \underline{283.92\ m/s}$$

$$\alpha_1^{REL} = \tan^{-1}\left(\frac{V_{\theta1}^{REL}}{V_{x1}}\right) = \tan^{-1}\left(\frac{-251.33}{132.07}\right) = \underline{\underline{-62.28^\circ}}$$

$$M_1^{REL} = \frac{V_1^{REL}}{\sqrt{\gamma R T_1}} = \frac{283.92}{\sqrt{1.4\times287\times271.32}} = \underline{\underline{0.860}}$$

$$T_{01}^{REL} = T_1 + \frac{V_1^{REL\,2}}{2C_p} = 271.32 + \frac{283.92^2}{2\times1005} = \underline{\underline{311.42\ K}}$$

$$P_{01}^{REL} = P_{01}\left(\frac{T_{01}^{REL}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 1.0\left(\frac{311.42}{280}\right)^{3.5} = \underline{\underline{1.4510\ bar}}$$

8b) $Y_p = \frac{P_{02}^{REL,ISEN} - P_{02}}{P_{01} - P_1} = 0.05$

$T_{02}^{REL} = T_{01}^{REL}$ (FIXED RADIUS) $\Rightarrow$ $\underline{P_{02}^{REL,ISEN} = P_{01}^{REL} = 1.4510\ bar}$

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 1.0\left(\frac{271.32}{280}\right)^{3.5} = \underline{0.8956\ bar}$$

$$P_{02}^{REL} = P_{01}^{REL} - Y_p(P_{01}^{REL} - P_1) = 1.4510 - 0.05(1.4510 - 0.8956) = \underline{\underline{1.4232\ bar}}$$

$$\frac{\dot m\sqrt{C_p T_{02}^{REL}}}{P_{02}^{REL} A_2\cos\alpha_2^{REL}} = \frac{\dot m\sqrt{C_p T_{01}^{REL}}}{P_{01}^{REL} A_1\cos\alpha_1^{REL}} \times \sqrt{\frac{T_{02}^{REL}}{T_{01}^{REL}}} \times \frac{P_{01}^{REL}}{P_{02}^{REL}} \times \frac{A_1}{A_2} \times \frac{\cos\alpha_1^{REL}}{\cos\alpha_2^{REL}}$$

$$M_1^{REL} = 0.860 \Rightarrow \frac{\dot m\sqrt{C_p T_{01}^{REL}}}{P_{01}^{REL} A_1\cos\alpha_1^{REL}} = 1.2585$$

$$\Rightarrow \frac{\dot m\sqrt{C_p T_{02}^{REL}}}{P_{02}^{REL} A_2\cos\alpha_2^{REL}} = 1.2585 \times \sqrt{1} \times \frac{1.4510}{1.4232} \times \frac{1}{0.896} \times \frac{\cos(-62.28^\circ)}{\cos(-48^\circ)} = 0.9955$$

TABLES $\Rightarrow$ $\underline{\underline{M_2^{REL} = 0.530}}$

FULL QUESTION cont.

8c) $$P_2 = \frac{P_{02}^{REL}}{\left(1+\frac{\gamma-1}{2}M_2^{REL\,2}\right)^{\frac{\gamma}{\gamma-1}}} = \frac{1.4232}{(1+0.2\times 0.53^2)^{3.5}} = \underline{1.1754\ bar}$$

$$\left.\frac{P_2}{P_{01}}\right|_{TIP} = \frac{1.1754}{1.0} = \underline{\underline{1.1754}}$$

D) $$T_2 = \frac{T_{02}^{REL}}{\left(1+\frac{\gamma-1}{2}M_2^{REL\,2}\right)} = \frac{311.42}{(1+0.2\times 0.53^2)} = \underline{294.86\ K}$$

$$V_2^{REL} = M_2^{REL}\sqrt{\gamma R T_2} = 0.53\sqrt{1.4\times 287\times 294.86} = \underline{182.43\ m/s}$$

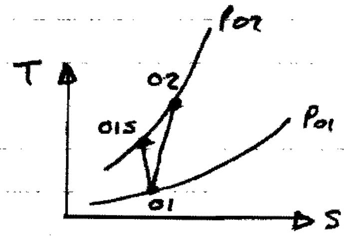
$$V_{\theta 2}^{REL} = V_2^{REL}\sin\alpha_2^{REL} = 182.43\sin(-48^\circ) = \underline{-135.57\ m/s}$$

$$V_{\theta 2} = V_{\theta 2}^{REL} + U = -135.57 + 251.33 = \underline{115.76\ m/s}$$

$$T_{02} = T_{01} + \frac{U(V_{\theta 2} - V_{\theta 1})}{c_p} = 280 + \frac{251.33\times(115.76-0)}{1005} = \underline{\underline{308.95\ K}}$$

$$P_{02} = P_{02}^{REL}\left(\frac{T_{02}}{T_{02}^{REL}}\right)^{\frac{\gamma}{\gamma-1}} = 1.4232\left(\frac{308.95}{311.42}\right)^{3.5} = \underline{\underline{1.3841\ bar}}$$

$$T_{02}^{ISEN} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 280\left(\frac{1.3841}{1.0}\right)^{\frac{1}{3.5}} = \underline{307.25\ K}$$



$$\eta_{tt} = \frac{T_{02}^{ISEN} - T_{01}}{T_{02} - T_{01}} = \frac{307.25-280}{308.95-280} = \underline{\underline{0.941}}$$

$\eta_{tt} \approx 94\%$ because no stator loss has been included.

[NOTE: $V_{x2} = V_2^{REL}\cos\alpha_2^{REL} = 182.43\cos(-48^\circ) = 122.07\ m/s$
$V_{x1} = 132.07$ $\Rightarrow$ NOT CONSTANT AXIAL VELOCITY!]

e) Because of the non-zero swirl at the rotor exit ($V_{\theta 2} > 0$) there must be a pressure gradient associated with radial equilibrium. Hence $P_{HUB} < P_{TIP}$. So it would be expected that the hub pressure ratio would be lower than the rotor tip pressure ratio.