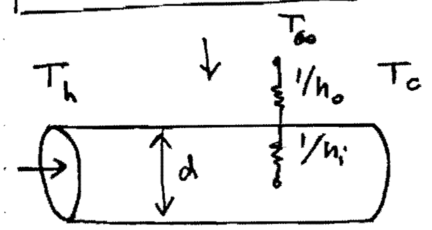
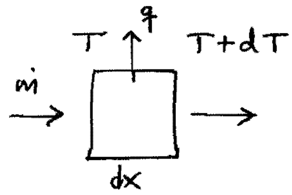
# 3A6 2013 CRIBS





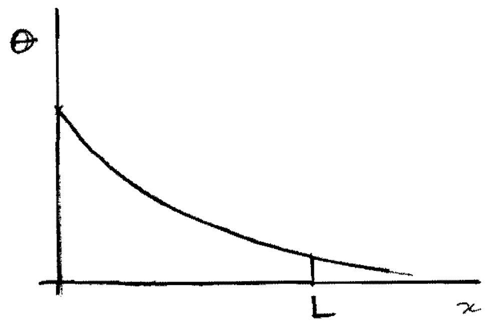

(a) DRY OPERATION

$$\dot m c\, dT = -q \pi d\, dx = -U \pi d (T - T_\infty)\, dx$$

$$U = \left(\frac{1}{h_o} + \frac{1}{h_i}\right)^{-1}$$

$$\frac{dT}{T - T_\infty} = -\frac{U \pi d}{\dot m c}\, dx$$

$$\ln \frac{(T - T_\infty)}{(T_h - T_\infty)} = -\frac{U \pi d}{\dot m c} x$$



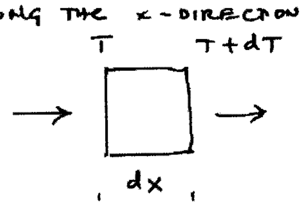

$$\theta = \frac{(T_c - T_\infty)}{(T_h - T_\infty)} = \exp\left(-\frac{U \pi d L}{\dot m c}\right)$$

$$\dot Q_A = \dot m c (T_h - T_c) = \dot m c \left[(T_h - T_\infty) - (T_c - T_\infty)\right]$$

$$= \dot m c (T_h - T_\infty)\left[1 - \exp\left(-\frac{U \pi d L}{\dot m c}\right)\right]$$

$$= \dot m c (T_h - T_\infty)\left[1 - \exp\left(-\frac{\pi d L}{\dot m c} \frac{1}{1/h_o + 1/h_i}\right)\right] \quad //$$

<u>NOTE</u> : A COMMON MISTAKE WAS TO TAKE AN ENERGY BALANCE WITH CONDUCTION ALONG THE x-DIRECTION :




$$-\lambda \frac{\partial T}{\partial x} \qquad \lambda \frac{dT}{dx}\Big|_{x+dx}$$

$$\dot m c T \qquad \dot m c T_{x+dx}$$

$$\dot m c \frac{dT}{dx} + \lambda A \frac{d^2 T}{dx^2} = \frac{dq}{dx}$$

$$\dot m c \frac{\Delta T}{L} \qquad \lambda A \frac{\Delta T}{L^2}$$

CONVECTION DOMINANT ALONG x : $\dot m c \gg \frac{\lambda}{L}$ OR $\frac{\alpha}{UL} \ll 1$

(b) WET CONDITIONS

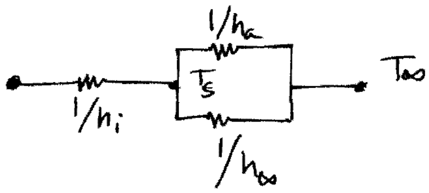
(i) $G_v = h_m(\rho_{v,s} - \rho_{v,\infty})$

$$\frac{h_m d}{\mathcal{D}} = \frac{h_a d}{\lambda_a} \longrightarrow h_m = h_a \frac{\mathcal{D}}{\lambda_a}$$

$$G_v = \frac{h_a \mathcal{D}}{\lambda_a}(\rho_{v,s} - \rho_{v,\infty}) = \frac{h_a \mathcal{D}}{\lambda_a}\beta(T_s - T_\infty)$$

$$q_v = G_v h_{ev} = \frac{h_a}{\lambda_a}\mathcal{D} h_{ev}\beta(T_s - T_\infty)$$

(ii) TWO PATHWAYS FOR HEAT TRANSFER: DIRECT AND EVAPORATION




$$h_v = \frac{q_v}{(T_s - T_\infty)} = \frac{h_a}{\lambda_a}\mathcal{D} h_{ev}\beta$$

$$R_B = \frac{1/h_a \, 1/h_v}{1/h_a + 1/h_v} + \frac{1}{h_i} = \frac{1}{h_a + h_v} + \frac{1}{h_i}$$

$$U_B = \frac{1}{R_B} = \frac{1}{\frac{1}{h_a + h_v} + \frac{1}{h_i}} = \left[\frac{1}{h_a\left(1 + \frac{\mathcal{D} h_{ev}\beta}{\lambda_a}\right)} + \frac{1}{h_i}\right]^{-1}$$

USING THE SAME ANALYSIS AS IN (a):

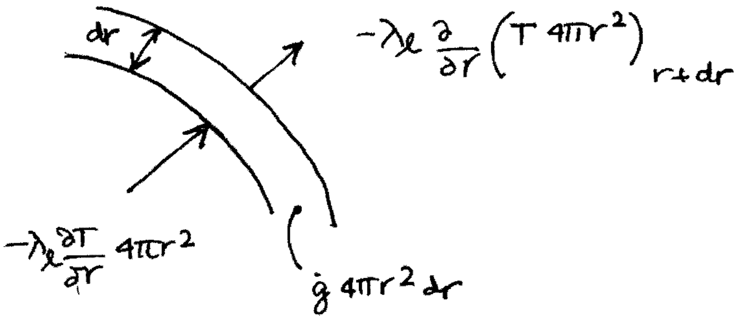
$$\dot{Q}_B = \dot{m}c(T_h - T_c) = \dot{m}c(T_b - T_\infty)\left[1 - \exp\left(-U_B \frac{\pi d L}{\dot{m}c}\right)\right]$$

THE EVAPORATION CREATES A NEW PATH FOR HEAT TRANSFER, NOW LIMITED BY THE RATE OF MASS TRANSFER.

THE SIMPLE FINAL ANSWER IS ONLY POSSIBLE BECAUSE OF THE ASSUMPTION REGARDING THE RELATIONSHIP OF PARTIAL DENSITIES AND TEMPERATURE. IN GENERAL, THE TWO WOULD BE COUPLED NON-LINEARLY.

(2)

(a) SPHERICAL SHELL $dr$




ENERGY CONSERVATION:

$$\rho c \left(4\pi r^2 dr\right) \frac{\partial T}{\partial t} = -\lambda_e \frac{\partial T}{\partial r} 4\pi r^2 + \lambda_e \left(\frac{\partial T}{\partial r} 4\pi r^2 + \frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r} 4\pi r^2\right) dr\right) + \dot{g}\, 4\pi r^2 dr$$

CANCELLING OUT TERMS AND DIVIDING BY $4\pi r^2$, WE HAVE

$$\rho c \frac{\partial T}{\partial t} dr = -\lambda_e \frac{\partial T}{\partial r} + \lambda_e \frac{\partial T}{\partial r} + \lambda_e \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) dr + \dot{g}\, dr$$

$$\boxed{\rho c \frac{\partial T}{\partial t} - \lambda_e \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \dot{g}}$$

QED

(b) NEGLECTING THE UNSTEADY TERM, WE HAVE

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = -\frac{\dot{g}}{\lambda_e}$$

$$r^2 \frac{\partial T}{\partial r} = -\frac{\dot{g}\, r^3}{\lambda_e 3} + C_1$$

$$\frac{\partial T}{\partial r} = -\frac{\dot{g}}{\lambda_e}\frac{r}{3} + \frac{C_1}{r^2}$$

$$T = \frac{-\dot{g}}{\lambda_e}\frac{r^2}{6} - \frac{C_2}{6} + C_2$$

FROM SYMMETRY: $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \rightarrow C_1 = 0$

$$T(R) = T_s = -\frac{\dot{g} R^2}{\lambda_e 6} + C_2 \rightarrow C_2 = T_s + \frac{\dot{g} R^2}{6\lambda_e}$$

$$\boxed{T(r) = T_s + \frac{\dot{q}}{6\lambda_l}\left(R^2 - r^2\right)}$$

QED

(c) FOR AIR, WE HAVE THE SAME EQUATION FOR CONDUCTION, WITH $\dot{q} = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = 0$$

$$r^2 \frac{\partial T}{\partial r} = D_1$$

$$\frac{\partial T}{\partial r} = \frac{D_1}{r^2}$$

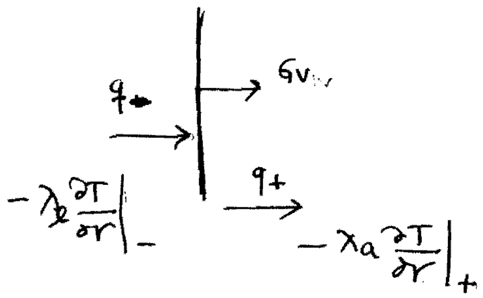
$$T = -\frac{D_1}{r} + D_2$$

$$\left.\begin{aligned} T(R) &= -\frac{D_1}{R} + D_2 = T_s \\ T(\infty) &= D_2 = T_\infty \end{aligned}\right\} \longrightarrow D_1 = (T_\infty - T_s) R$$

$$\boxed{T(r) = T_\infty + (T_s - T_\infty)\frac{R}{r}}$$

(d)

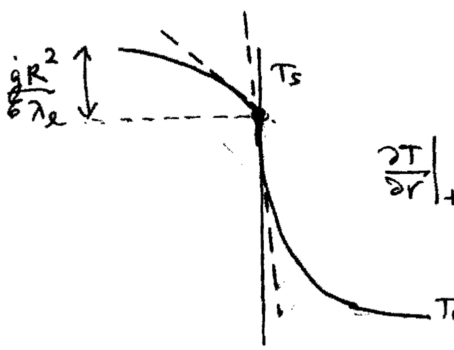
(i)




$$-\lambda_l \frac{\partial T}{\partial r}\Big|_- + \lambda_a \frac{\partial T}{\partial r}\Big|_+ - G_v h_{ev} = 0$$

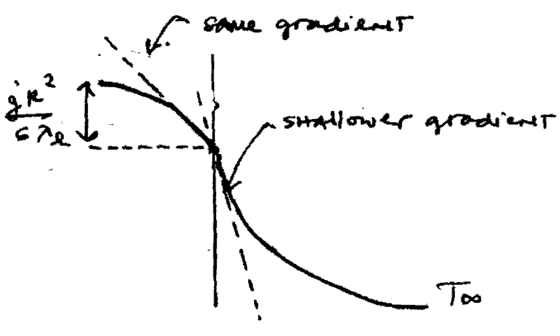
- NOTE THAT $\dot{q}$ DOES NOT APPEAR EXPLICITLY, AS $dV \to 0$

(ii) $G_v = 0$ : $\lambda_l \frac{\partial T}{\partial r}\Big|_- = \lambda_a \frac{\partial T}{\partial r}\Big|_+$ ; $G_v > 0$: $\lambda_a \frac{\partial T}{\partial r}\Big|_+ - \lambda_l \frac{\partial T}{\partial r}\Big|_- = G_v h_{ev}$




$$\frac{\partial T}{\partial r}\Big|_+ = \frac{\lambda_l}{\lambda_a}\frac{\partial T}{\partial r}\Big|_-$$

NO EVAPORATION

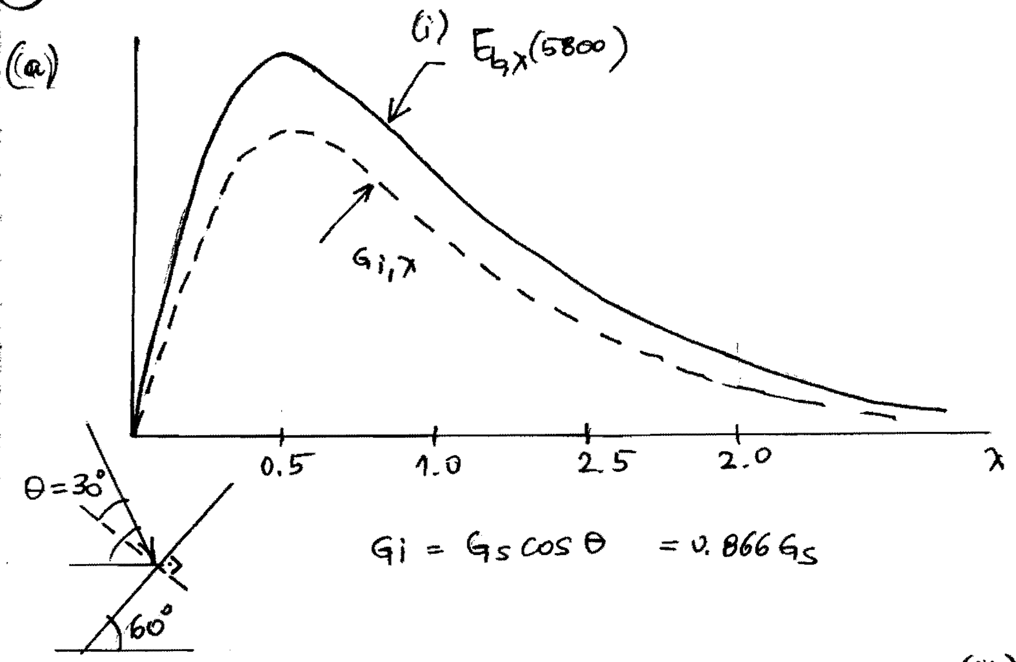



W/ EVAPORATION

BONUS: SHOW THAT $T_s = T_\infty + \dot{q}R^2/3\lambda_a - G_v h_{ev}$

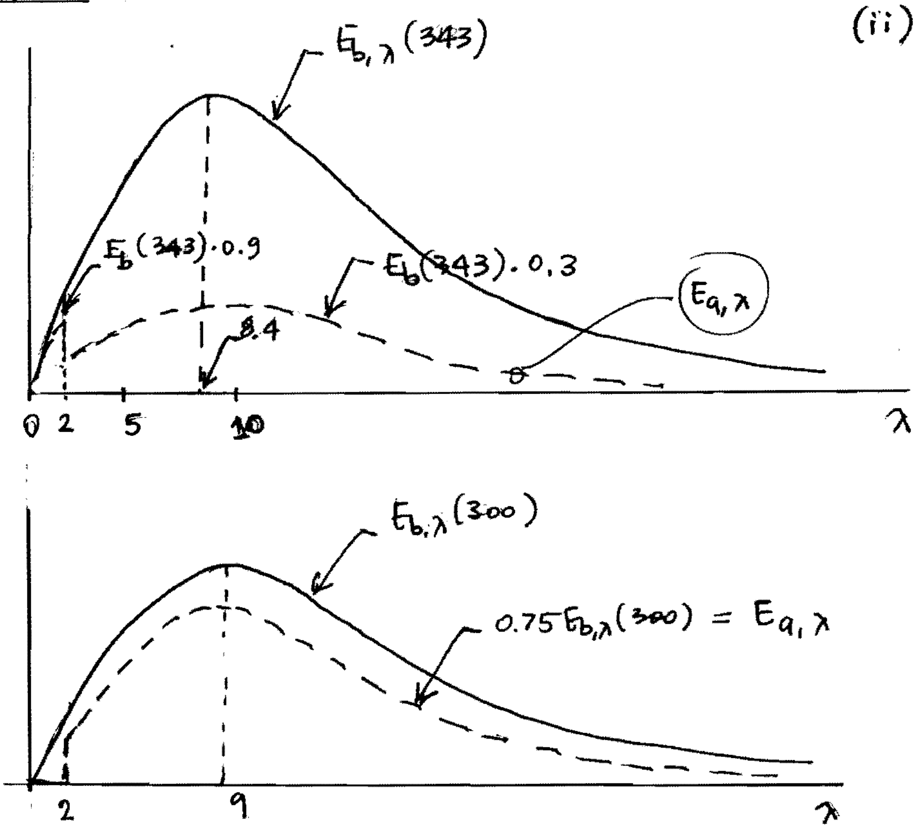
3 

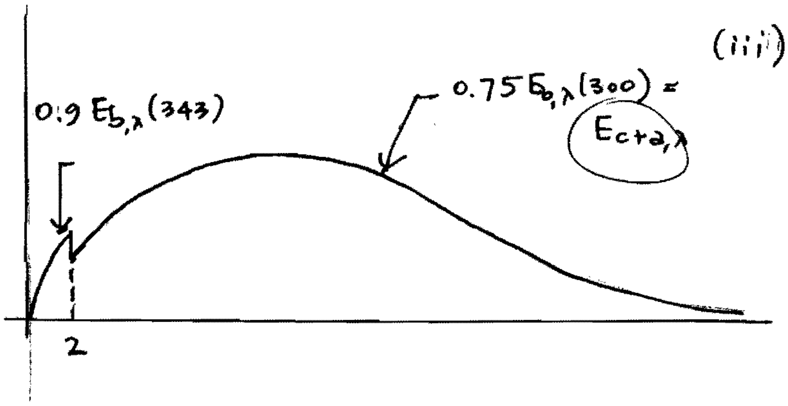
(a)




$G_i = G_s \cos\theta = 0.866\, G_s$

(ii)




(iii) THE ABSORBER COATING IS TRANSPARENT TO THE SHORT WAVELENGTH OF SUNLIGHT, BUT OPAQUE TO THE LONGER WAVELENGTHS EMITTED BY THE ABSORBER, THUS TRAPPING RADIATIVE POWER.

(b) THE GROSS SOLAR ENERGY ABSORBED IS:

$$G_a = \int_0^{\infty} G_{i,\lambda}\,\alpha\,d\lambda = \left[0.9\,F(0-2\mu m)\right] G_s \cos\theta$$

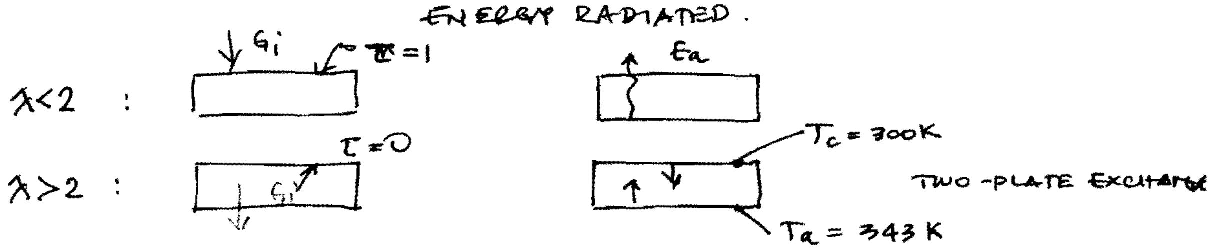
$$\lambda T = (2\mu m)(5800\,K) = 11600\ \mu m{\cdot}K$$

INTERPOLATION: $F_{0-\lambda} = 0.939959 + (0.945098 - 0.939959)\cdot 100/500$
$= 0.9409868$

$$G_a = \left[(0.9)(0.9409868)\right](900\,W/m^2)\cos 30^\circ$$

$$G_a = 660\ W/m^2$$

(c) NET FLUX ABSORBED = GROSS ENERGY ABSORBED − ENERGY RADIATED.




$$\lambda<2:\quad E_a = \int_0^2 \alpha_{a,\lambda} E_{a,\lambda}\,d\lambda = (0.9)\,F_{0-2}\,E_{b,a}$$

$$\lambda T = (2\mu m)(343K) = 686\ \mu m{\cdot}K \rightarrow F_{0-2} \cong 0$$

$$E_a \simeq 0$$

$\lambda>2$: $E_{b,c}$ —[$1/\epsilon_c$]— $J_c$ —[$1/F_{ac}$]— $J_a$ —[$1/\epsilon_a$]— $E_{b,a}$ $\quad\quad \epsilon=\alpha$, $F_{ac}=1$

NET RADIATIVE LOSS : $q_L = \dfrac{E_{b,a} - E_{b,c}}{1/\alpha_c + 1/\alpha_a + 1}$

$$q_L = \frac{\sigma(T_a^4 - T_c^4)}{1/\alpha_c + 1/\alpha_a + 1} = 5.670\times10^{-8}\,W/m^2K^4\,\frac{(343^4 - 300^4)\,K^4}{1/0.3 + 1/0.75 + 1}$$

$$q_L = 64\ W/m^2$$

$\eta = 1 - 64/660 \approx 0.903 \rightarrow$ HIGH EFFICIENCY DUE TO COATING.

④ 

(a) (i) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

DEFINE $\theta = \frac{(T - T_s)}{(T_\infty - T_s)}$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2 \theta}{\partial y^2}$$

$$\frac{\partial}{\partial x}(u\theta) - \theta\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}(v\theta) - \theta\frac{\partial v}{\partial y} = \alpha\frac{\partial^2\theta}{\partial y^2}$$

$= 0$ (CONT.)

$$\frac{\partial}{\partial x}(u\theta) + \frac{\partial}{\partial y}(v\theta) = \alpha\frac{\partial^2\theta}{\partial y^2}$$

$$\int_0^\delta \frac{\partial}{\partial x}(u\theta)\,dy + \underbrace{\cancel{v\theta\Big|_0^\delta}}_{0} = \alpha\left[\underbrace{\cancel{\frac{\partial\theta}{\partial y}\Big|_\delta}}_{0} - \frac{\partial\theta}{\partial y}\Big|_0\right]$$

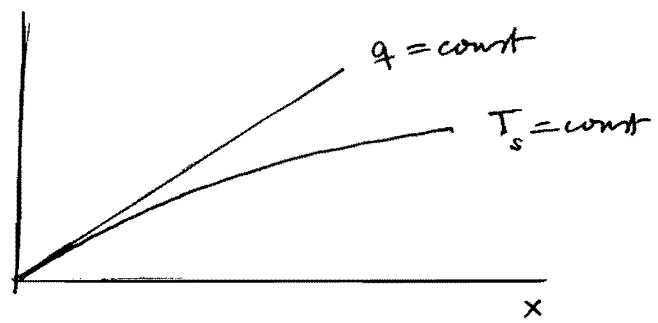
$$\frac{\partial}{\partial x}\int_0^\delta u\theta\,dy = -\frac{\lambda}{\rho c_p}\frac{1}{(T_\infty - T_s)}\frac{\partial T}{\partial y}\Big|_0 = \frac{q}{\rho c_p (T_\infty - T_s)}$$

$\div u_\infty$, $\delta \to \infty$

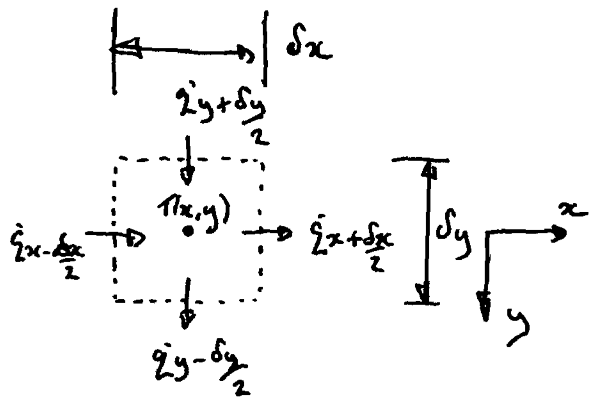
$$\frac{d\delta_\theta}{dx} = \frac{\partial}{\partial x}\int_0^\infty \frac{u\,(T - T_s)}{u_\infty (T_\infty - T_s)}\,dy = \frac{h}{\rho c_p u_\infty}$$

QED

(ii)



b) i)



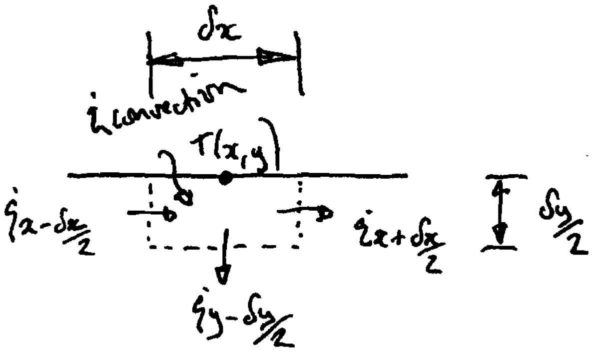

Energy balance: $\dot{q}_{x-\frac{\delta x}{2}} - \dot{q}_{x+\frac{\delta x}{2}} + \dot{q}_{y+\frac{\delta y}{2}} - \dot{q}_{y-\frac{\delta y}{2}} = 0$

$$-\lambda \frac{\partial T}{\partial x}\bigg|_{x-\frac{\delta x}{2}} \delta y - \left(-\lambda \frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}} \delta y\right) + -\lambda \frac{\partial T}{\partial y}\bigg|_{y+\frac{\delta y}{2}} \delta x - \left(-\lambda \frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}} \delta x\right) = 0$$

$$-\frac{\partial T}{\partial x}\bigg|_{x-\frac{\delta x}{2}} \delta y + \frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}} \delta y - \frac{\partial T}{\partial y}\bigg|_{y+\frac{\delta y}{2}} \delta x + \frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}} \delta x = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

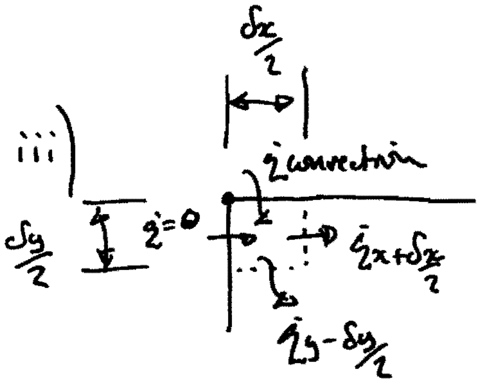
ii)




Energy balance: $\cancel{\lambda}\frac{\partial T}{\partial x}\bigg|_{x-\frac{\delta x}{2}} \frac{\delta y}{2} - \left[-\cancel{\lambda}\frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}} \frac{\delta y}{2}\right] - \left(-\cancel{\lambda}\frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}} \delta x\right) + \frac{h\delta x}{\lambda}(T_\infty - T_s) = 0$

$$-\left[\frac{\partial T}{\partial x}\bigg|_{x-\frac{\delta x}{2}} - \frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}}\right]\frac{\delta y}{2} + \frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}} \delta x + \frac{h\,\delta x}{\lambda}(T_\infty - T_s) = 0$$

iii)

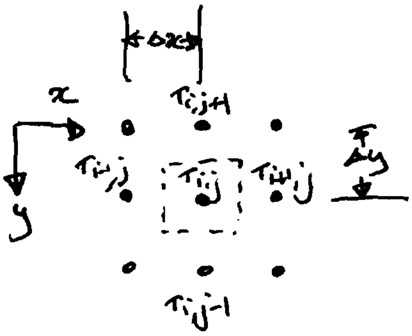



$$0-\left(-\cancel{\lambda}\frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}}\frac{\delta y}{2}\right)-\left(-\cancel{\lambda}\frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}}\frac{\delta x}{2}\right)+\frac{h\,\delta x}{2\lambda}(T_\infty-T)=$$

$$\frac{\delta y}{\cancel{2}}\frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}}+\frac{\delta x}{\cancel{2}}\frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}}+\frac{h\,\delta x}{\cancel{2}\lambda}(T_\infty-T)=0$$

$$\delta y\,\frac{\partial T}{\partial x}\bigg|_{x+\frac{\delta x}{2}}+\delta x\,\frac{\partial T}{\partial y}\bigg|_{y-\frac{\delta y}{2}}+\frac{h\,\delta x}{\lambda}(T_\infty-T)=0.$$

c) $\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} = 0$




i)

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_x \approx \frac{\left.\frac{\partial T}{\partial x}\right|_{x+\frac{\Delta x}{2}} - \left.\frac{\partial T}{\partial x}\right|_{x-\Delta/2}}{\Delta x}$$

$$\approx \frac{\frac{T_{i,j} - T_{i-1,j}}{\Delta x} - \frac{(T_{i+1,j} - T_{i,j})}{\Delta x}}{\Delta x}$$

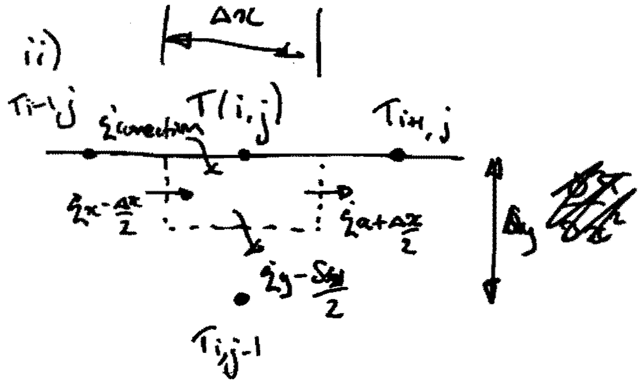
$$\approx \frac{2T_{i,j} - T_{i-1,j} - T_{i+1,j}}{\Delta x^2}$$

similarly $\left.\dfrac{\partial^2 T}{\partial y^2}\right|_y \approx \dfrac{2T_{i,j} - T_{i,j-1} - T_{i,j+1}}{\Delta y^2}$

$\Rightarrow \quad 4T_{i,j} - T_{i-1,j} - T_{i+1,j} - T_{i,j-1} - T_{i,j+1} = 0$

$\Rightarrow \quad T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j} = 0$

c) ii)




from part b) ii) $$-\left[\frac{(T_{i,j}-T_{i-1,j})}{\Delta x}-\frac{(T_{i+1,j}-T_{i,j})}{\Delta x}\right]\frac{\Delta y}{2}+\frac{(T_{i,j-1}-T_{i,j})\Delta x}{\Delta y}+\frac{h\Delta x}{\lambda}(T_\infty-T_{i,j})=0$$

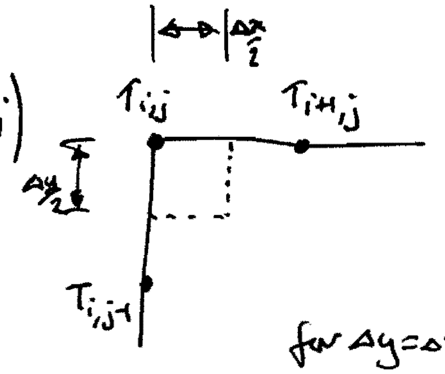
$$\Rightarrow\left[-2T_{i,j}+T_{i-1,j}+T_{i+1,j}\right]\frac{\Delta y}{\cancel{\Delta x 2}}+2(T_{i,j-1}-T_{i,j})\frac{\Delta x}{\Delta y}\ \frac{2h\Delta x}{\lambda}(T_\infty-T_{i,j})=0$$

for $\Delta x=\Delta y$

$$\text{so}\ -4T_{i,j}+T_{i-1,j}+T_{i+1,j}+2T_{i,j-1}+\frac{2h\Delta x}{\lambda}T_\infty-\frac{2h\Delta x}{\lambda}T_{i,j}=0$$

$$T_{i-1,j}+T_{i+1,j}+2T_{i,j-1}-2\left(2+\frac{h\Delta x}{\lambda}\right)T_{i,j}+\frac{2h\Delta x}{\lambda}T_\infty=0$$

iii)




$$\Delta y\frac{(T_{i+1,j}-T_{i,j})}{\Delta x}+\Delta x\frac{(T_{i,j-1}-T_{i,j})}{\Delta y}+\frac{h\Delta x}{\lambda}(T_\infty-T_{i,j})=0$$

for $\Delta y=\Delta x$

$$T_{i+1,j}-2T_{i,j}+T_{i,j-1}+\frac{h\Delta x}{\lambda}T_\infty-\frac{h\Delta x}{\lambda}T_{i,j}=0$$

$$T_{i+1,j}+T_{i,j-1}-2\left(1+\frac{h\Delta x}{2\lambda}\right)T_{i,j}+\frac{h\Delta x}{\lambda}T_\infty=0$$

Set up $T_6$ as a variable in iterative scheme, when RADIATION PRESENT.