3B1 CRIB 2013

1(a) Butterworth : compromise between freq. and time domain responses – approx. 10% overshoot for step response and medium sharp roll off in frequency domain.

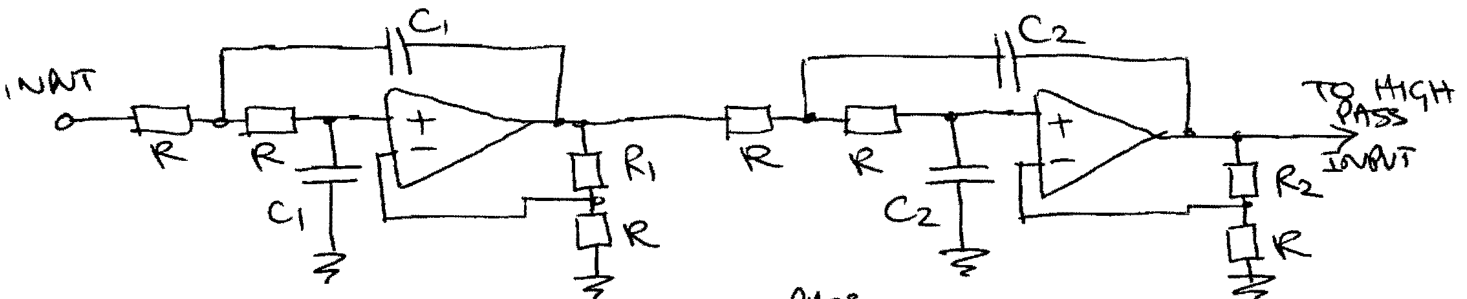
Bessel : constant time delay filter across freq. band, hence good step response (no overshoot, fast response) but freq. roll off is very slow.

Chebyshev : best freq. domain sharp cut-off but time domain 'rings' on after step for several cycles.

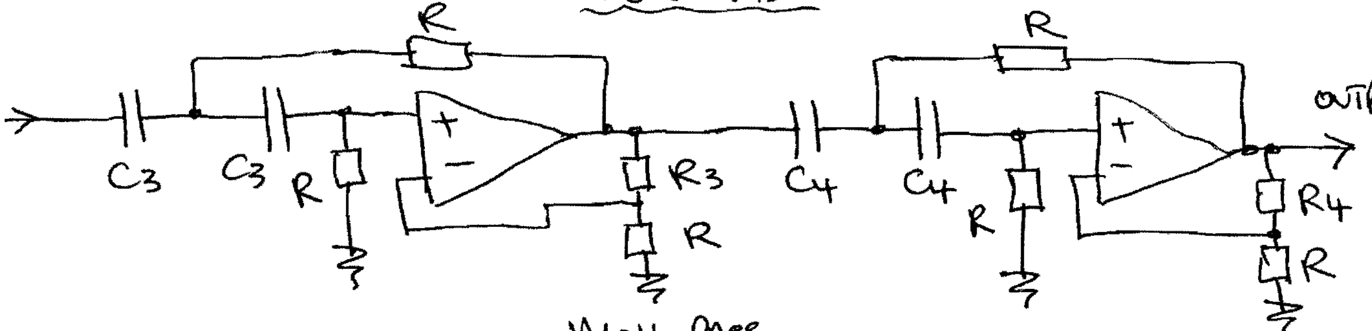
For 4th order filters (low pass) @ twice cut-off freq :–
Bessel = –10 dB, Butterworth = –30 dB, Chebyshev = –45 dB.

[20%]

1(b) For bandpass, cascade low-pass (to 3kHz) and high-pass (over 250Hz) sections :–



LOW PASS



HIGH PASS

use Chebyshev filter for sharp cut-off.

(b) contd.

f_n	A
0.597	1.582
1.031	2.660

R = 10kΩ in all stages

stage 1

$g = 1.582 \quad \therefore \quad R_1 = (1.582 - 1) \times 10k\Omega = 5.8 k\Omega$ (or 5.6kΩ std)

$f_c = 3000 = \frac{1}{2\pi R C_1 f_n}$ ← 0.597

$\therefore C_1 = 8.9 nF$ (10nF std)

stage 2

$R_2 = 16.6 k\Omega$ (18kΩ)

$C_2 = 5.1 nF$ (4.7nF std)

stage 3

$g = 1.582 \quad \therefore \quad R_3 = R_1 = 5.8 k\Omega$

$f_c = 250 = \frac{1}{2\pi R C_3 \, 1/f_n} \Rightarrow C_3 = 38 nF$ (39nF)

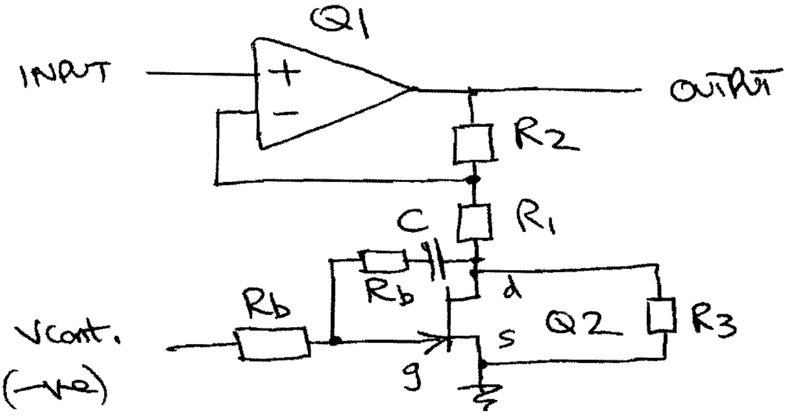
stage 4

$R_4 = R_2 = 16.6 k\Omega$ (18kΩ)

$C_4 = 66 nF$ (68nF std.)

[40%]

1(c)



Q1: operational amplifier to provide gain

Q2: JFET used as voltage controlled resistor

R_2: feedback resistor

R_1: gain limit resistor, when Q2 fully on (low resistance) then max. gain $= 1 + R_2/R_1$

Rb: bias resistors for gate, to improve linearity of amplifier by cancelling non-linear term

C: d.c. block capacitor.

[20%]

R_3: gain limit resistor for min. gain when Q2 is off $= 1 + \frac{R_2}{(R_1 + R_3)}$

1(d) Need to get amplitude control, so demodulate with diode to give -ve voltage for amplifier:-

∴ choose $R_2 = 100\,k\Omega$

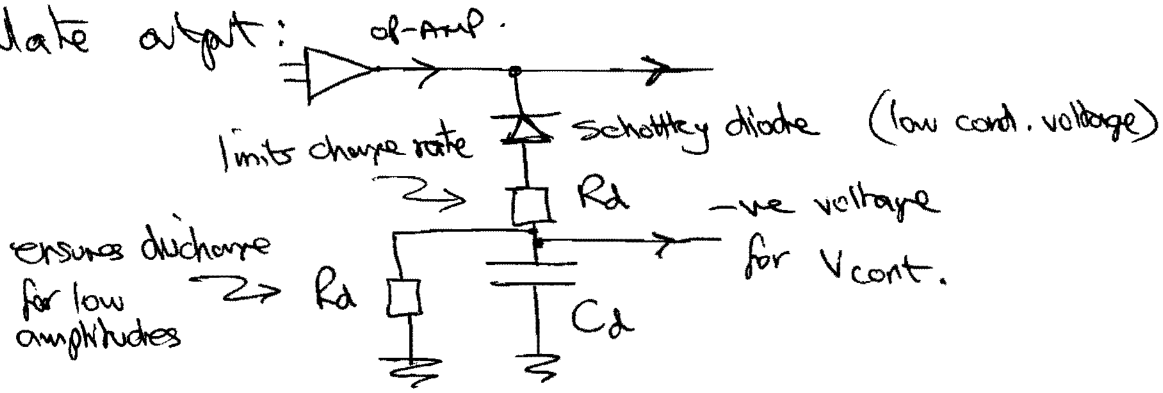
10dB ≡ × 3·2 min.
30dB ≡ × 32 max.

∴ $R_1 = 3 \cdot 2\,k\Omega$ (3.3kΩ std.)

∴ $R_3 = 42 \cdot 3\,k\Omega$ taking R_1 into account
(43k std)

$R_b = 1M\Omega$
$C = 100\,nF$
} large; not much current flow to gate.

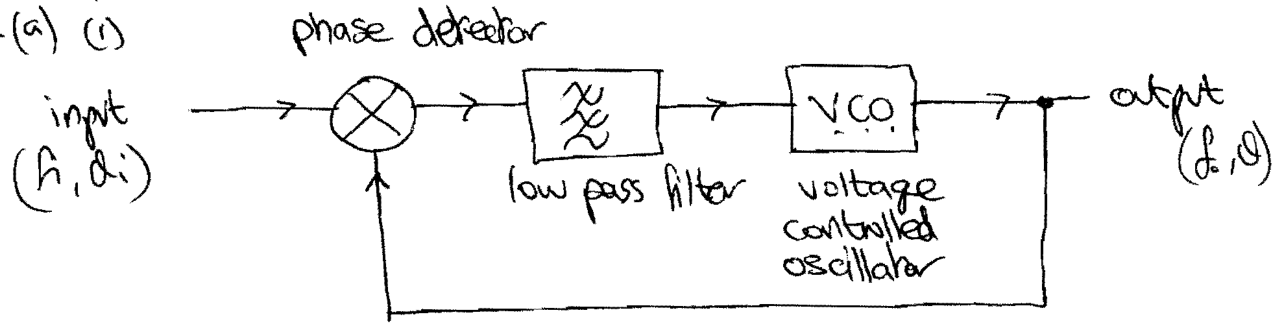
To demodulate output:



Choose $0 \cdot 1s = 2 \cdot 2\, C_d \frac{R_d}{2}$ [time ≃ 2·2 CR]

∴ $C_d = 10\mu F$ ∴ $R_d = 9 \cdot 1 k\Omega$ or $10k\Omega$ [20%]

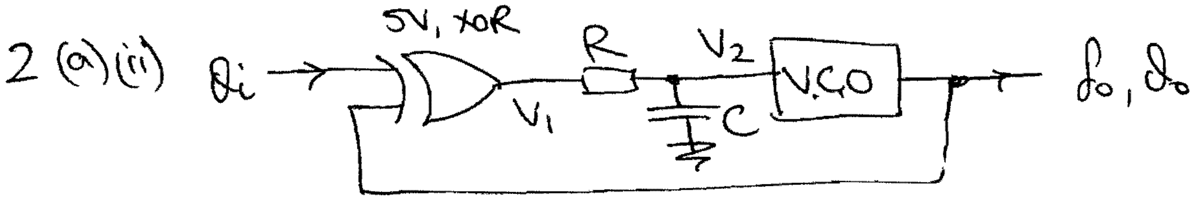
2 (a) (i)



An oscillator in the PLL has its output compared in phase (and freq.) to an input signal by the phase detector (mixer). The output voltage from the phase detector is proportional to the phase diff. between the input and oscillator. The low pass filter passes the dc component of this to the VCO, which increases the output frequency (advances phase) until the phases match. The low pass filter gives the output freq. "inertia" i.e. it can free-wheel for a while if the input disappears. Putting digital freq. dividers at the input and/or in the feedback path enables frequency multiplication & division to be achieved.

[20%]

2 (a) (ii)



$V_1 = K_p(\theta_o - \theta_i)$ phase det.

$2\pi f_o = \frac{d\theta_o}{dt} = j\omega\theta_o = K_o V_2$ V.C.O.

$\frac{V_2}{V_1} = \frac{1}{1+j\omega CR}$ RC low pass filter

$\therefore j\omega\theta_o = \frac{V_1 K_o}{(1+j\omega CR)} \Rightarrow j\omega\theta_o = \frac{K_o K_p(\theta_o - \theta_i)}{(1+j\omega CR)}$

$j\omega\theta_o - \omega^2 CR\theta_o - K_o K_p \theta_o = -K_o K_p \theta_i$

$\Rightarrow \omega^2 \frac{CR}{K_o K_p}\theta_o - j\omega\frac{1}{K_o K_p}\theta_o + \theta_o = \theta_i$

2(a)(ii) contd.

If $\omega_n^2 = \frac{-K_o K_p}{CR}$ and $c = \frac{\omega_n}{-2 K_o K_p}$

Then. $-\frac{\omega^2}{\omega_n^2}\theta_o + \frac{2c}{\omega_n} j\omega\theta_o + \theta_o = \theta_i$

Since $\theta_o \equiv e^{j(\omega t + \theta)}$ then $\omega^2\theta_o = \ddot{\theta}_o$

$j\omega\theta_o = \dot{\theta}_o$

$\therefore \frac{\ddot{\theta}_o}{\omega_n^2} + \frac{2c}{\omega_n}\dot{\theta}_o + \theta_o = \theta_i$ pgs. 8 & 9 of Mechanics d/book

For 10% overshoot, $c \approx 0.55$ from graph on pg 9

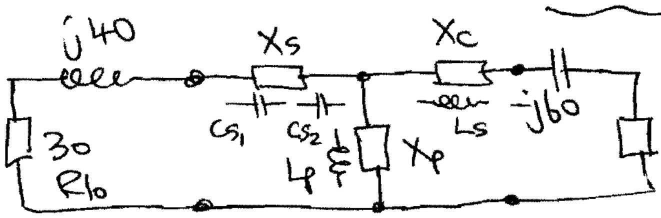
with $K_o = 1\text{MHz}/\text{V} = 2\pi \times 10^6$ rads/s/V

$K_p = 5\text{V}$ for $\pm 180° = \frac{5\text{V}}{\pi \text{ rads}} = \pm 1.59$ V/rad. (−ve in this case for stability)

$\therefore 0.55 = \frac{\omega_n}{2\pi \times 10^6 . 1.59 . 2}$ and $\omega_n = \sqrt{\frac{2\pi \times 10^6 . 1.59}{CR}} = 11 \times 10^6$ rad/s

$\therefore CR = 82.7\text{ ns}$ [55%]

2(b)



Need to cancel reactive components with series L or C, and combine these for minimum parts where possible.

$Q = \frac{R_{hi}}{X_p} = \frac{X_s}{R_{lo}} = \sqrt{\frac{R_{hi}}{R_{lo}} - 1} = \sqrt{\frac{80}{30} - 1} = 1.291$

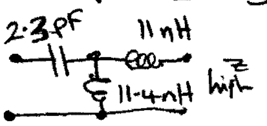
$\therefore X_p = j62\,\Omega$ use inductor $\therefore \omega L = 2\pi f L_p = 62 \quad \therefore L_p = 11.4\text{ nH}$

$X_c = j60\,\Omega$ = inductor $L_s = 11.0\text{nH}$

$C_{s1} = -j40\,\Omega \quad \therefore \frac{1}{2\pi f C_{s1}} = 40 \quad \therefore C_{s1} = 4.58\text{pF}$ [25%]

$C_{s2} = -j30 \times 1.291 = -j38.7\,\Omega \Rightarrow C_{s2} = 4.73\text{ pF}$

$\therefore$ in series = 1 capacitor $C_s = 2.33\text{pF}$



3(a) Gain, G = $\frac{\text{Max power radiated per unit area}}{\text{power per unit area from isotropic antenna}}$

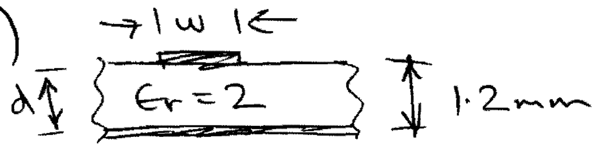
Effective Aperture, Ae (Area) : power delivered to a matched load by an antenna = Ae × power density in incident radio wave

Radiation Resistance, Rr : power radiated = $\frac{1}{2} I^2 R_r$, where I is current feed to antenna

[Radiation Efficiency : proportion of power radiated by an antenna relative to power input $= \frac{R_r}{(R_r + R_{ohmic})}$

Antenna eqn. $G = \frac{4\pi A_e}{\lambda^2}$ [15%]

3(b)



speed of light in PTFE, $\epsilon_r = 2$

$v = \frac{C_0}{\sqrt{2}} = 2.12 \times 10^8$ m/s

$v = f\lambda = 2.12 \times 10^8 = 868 \times 10^6 \lambda \quad \therefore \lambda = 0.244$ m

For efficient antenna patch, length = $\lambda/2$ = 0.122 m

For microstrip track, C/unit length = $\frac{(w+2d)\epsilon_0\epsilon_r}{d}$

$v = \frac{1}{\sqrt{LC}}$ and $Z_0 = \sqrt{\frac{L}{C}} \Rightarrow \sqrt{L} = \frac{1}{v\sqrt{C}}$ and $Z_0 = \frac{1}{vC}$

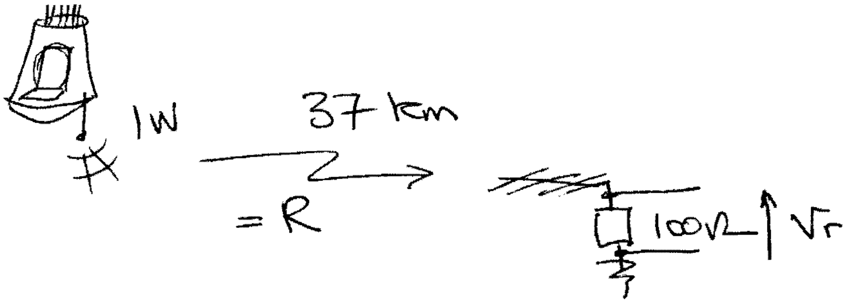
$\therefore Z_0 = \frac{\sqrt{2}}{C_0} \cdot \frac{d}{(w+2d)\epsilon_0\epsilon_r} = 73 = \frac{1.2}{(w+2.4)\epsilon_0\epsilon_r} \frac{\sqrt{2}}{3\times10^8}$

($\epsilon_0 = 8.854 \times 10^{-12}$, $\epsilon_r = 2$)

$\therefore w = 1.98$ mm

[30%]

3(c)



$v = f\lambda$

$\lambda = \frac{v}{f}$

Power density at Rx antenna $= \frac{1\text{W}}{4\pi R^2} = 58.1\ \text{pW/m}^2$
(assuming isotropic G = 1 @ Tx)
or G = 1.5 dipole, G = 3 dipole w/o ½ sphere

Gain of antenna = 20dB = ×100 $= G = \frac{4\pi A_e}{\lambda^2}$

λ @ 868 MHz in air $= \frac{3\times10^8}{868\times10^6} = 0.346\ \text{m}$

$\therefore A_e = \frac{100 \cdot 0.346^2}{4\pi} = 0.95\ \text{m}^2$

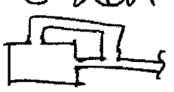
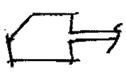
$\therefore$ Power from antenna $= 0.95 \times 58.1\times10^{-12}\ \text{W} = \frac{V_r^2}{100}$

$\therefore$ Signal, V_r = 74.3 μV into matched 100Ω resistor
(or 148.6 μV into open cct) [25%]

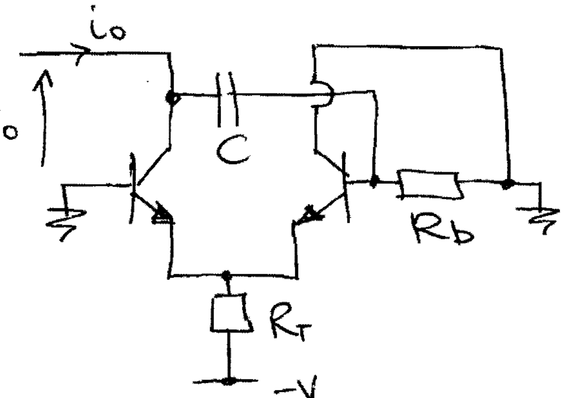
3(d) Skin depth, $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2\rho}{2\pi f \mu_0}}$ @ 868 MHz in Cu
$\delta = 2.96\ \mu\text{m}$

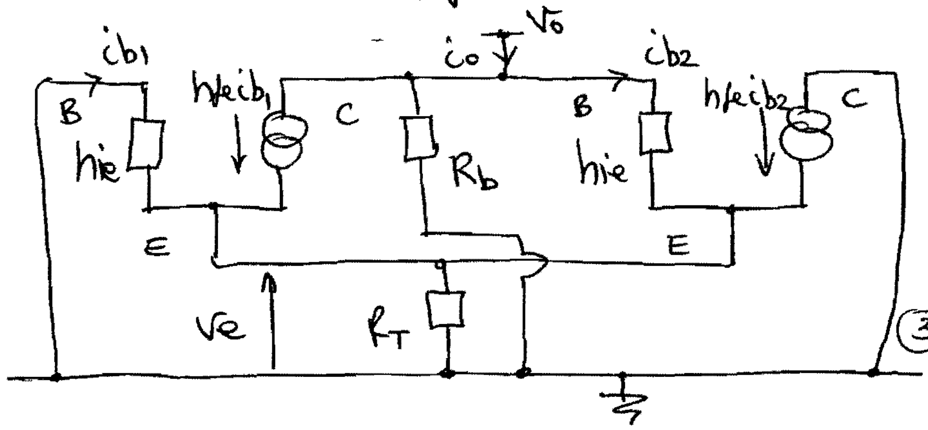
$\therefore$ ohmic resistance of patch $R_{ohm} = \frac{\rho L}{A} = \frac{3\times10^{-8} \cdot 0.122}{2.96\times10^{-6} \cdot 10\times10^{-3}} = 0.124\ \Omega$

$\therefore$ Radiation Effic. $= \frac{R_r}{R_r + R_{ohm}} = \frac{100}{100.124} = 99.88\%$. [20%]

3(e) With linear polarisation, the antennas can be orthogonal giving rise to poor coupling. A pair of antennas at 90° at the basestation, with outputs coupled via a 90° phase shift (by posn or cable length) will create a circular polarisation reception (could also do for Tx) – so no orientation null is seen. Can also be achieved with twin fed or notched patch.  OR  [10%]

4(a)





① $i_{b1} = \frac{-V_e}{h_{ie}}$

② $i_{b2} = \frac{V_o - V_e}{h_{ie}}$

③ $V_e \simeq R_T\, h_{fe}(i_{b1} + i_{b2})$

④ $i_o = h_{fe} i_{b1} + i_{b2} + V_o / R_b$ subst ① and ② into ③ :-

$$V_e = -\frac{R_T h_{fe} V_e}{h_{ie}} + R_T h_{fe}\frac{V_o}{h_{ie}} - R_T h_{fe}\frac{V_e}{h_{ie}}$$

$$\therefore\ V_e\left(\cancel{1+}\ \frac{2R_T h_{fe}}{h_{ie}}\right) = \frac{R_T h_{fe} V_o}{h_{ie}} \qquad \therefore V_e \simeq \frac{V_o}{2} \quad ⑤$$

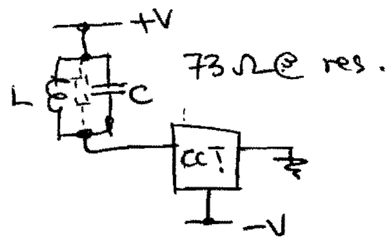
subs. ① and ② into ④ and subs. for V_e using ⑤

$$\therefore\ i_o = -\frac{h_{fe} V_o}{2h_{ie}} + \underbrace{\cancel{\frac{V_o}{2h_{ie}}}}_{\text{small}} + \frac{V_o}{R_b} = V_o\left(\frac{1}{R_b} - \frac{h_{fe}}{2h_{ie}}\right)$$

with $r_e = h_{ie}/h_{fe}$ $\Rightarrow Z_o = \frac{V_o}{i_o} = \left(\frac{1}{R_b} + \frac{1}{-2r_e}\right)^{-1} = R_b \| -2r_e$

[40%]

4(b)



For 1W power across 73Ω :

$1 = \frac{V^2}{R} \Rightarrow V = 8.5\,V_{rms}$

$= 24\,V_{p\text{-}p}$

hence supply must be 36V say

∴ put +V @ 24V and −V at −12V

4(b) contd. for 73 Ω impedance, we need a −ve resistance of say −35 Ω to be sure it is unstable ∴ $r_e = 18\,\Omega$ say

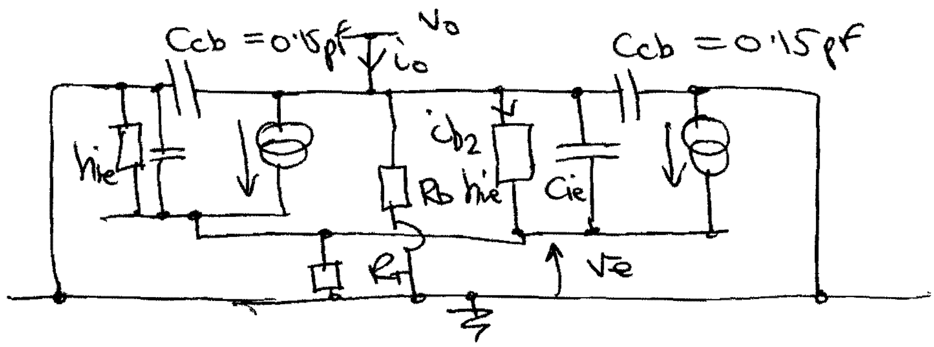
$R_b = 1\,k\Omega$ choice arb. $C = 100\,nF$ choice arb.

$r_e = 18 = \frac{V_T}{I_c}$ ← 0.025 ∴ $I_c = 1.4\,mA$

∴ current thro' $R_T \simeq 2.8\,mA$ with p.d. = (12 − 0.6)V

∴ $R_T = 4.1\,k\Omega$ (3.9 kΩ say) [20%]

4(c) Consider small sig. model @ i_o/v_o node:



$r_e = 18\,\Omega$ $f_t = 22\times10^9 = \frac{1}{2\pi r_e C_{ie}}$ ∴ $C_{ie} = 0.40\,pF$

as $v_e = v_o/2$ then effective $C_{ie}' = \frac{0.40}{2} = 0.20\,pF$ to gnd.

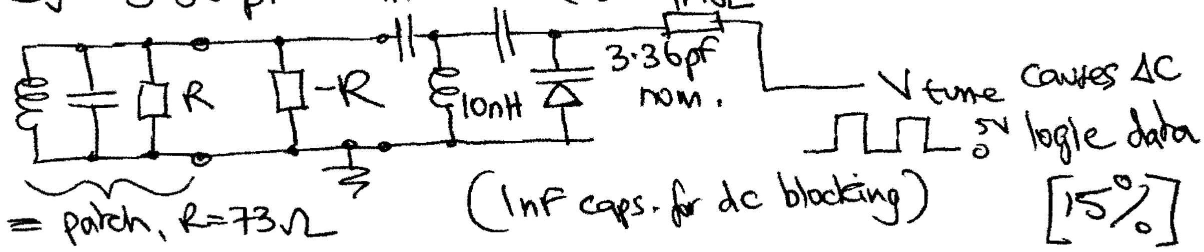
∴ total capacitance @ input = 0.15 + 0.15 + 0.20 = 0.50 pF

$R_b = 1\,k\Omega$, $h_{ie} = h_{fe} \times r_e = 4500\,\Omega$ ∴ dominating low impedance = 73 Ω from antenna

$f_{-3dB} = \frac{1}{2\pi\, 73 \cdot 0.5\times10^{-12}} = 4.36\,GHz$ [25%]

4(d) Use a varactor with nominal $C_v = \frac{1}{(2\pi f)^2 L}$ ← $10\times10^{-9}\,H$, ← $868\times10^6\,Hz$

$C_v = 3.36\,pF$



(1nF caps. for dc blocking) [15%]