Module 3B4 2013 Crib

1. (a) The voltage of a three-phase motor is limited by the airgap flux density (saturation, iron losses) and the ability of the winding insulation to withstand the peak electric field. The phase current of a three-phase motor is limited by the heat produced due to I^2R losses in the winding (overheating, reduced efficiency). Hence it is the volt-amps of the motor that is limited, and the volt-amp rating is given by the product of the maximum rms phase current and the maximum rms phase voltage and the number of phases.

Starting from S = 3VI, taking V and E as equal and substituting for I in terms of specific electric loading, J, and V in terms of specific magnetic loading, B, gives the result.

Note that the expression for E involves B_{rms} and so B_{rms} must be expressed in terms of B using $B_{rms}=\pi B/(2\sqrt{2})$.

d: airgap diameter; l: axial length; ω: angular supply frequency; p: pole-pairs; B: specific magnetic loading; J: specific electric loading.

(b) (i) Rated output power = 100 kW and full-load efficiency is 92%, so input power at full load is 100/0.92 = 108.7 kW. The input power factor is 0.8 and so the volt-amp rating of the motor is 108.7/0.8 = 136 kVA = S.

B=0.5 T, J=20 kAm^{-1}, $\omega/p=2\pi\times50/5 = 62.8\ rads^{-1}$ and d=4l. Substituting into the expression for S:

$136000=\pi^2\times4l^3\times62.8\times0.5\times20000/\sqrt{2}$ giving $l^3 = 0.00776$ and so l=0.198 m and d=4l=0.792 m.

(ii) $k_w = \cos(p\alpha/2)\times\sin(mnp\beta/2)/(m\sin p\beta/2)$, number of slots N_S = 60 and so $\beta=360/60 = 6^0$. Single layer winding so it must be fully-pitched and so α=0. 60 slots means 30 coils (for a single layer winding) and so 10 coils per phase, so since p=5, m=2 giving $mnp\beta/2=30^0$ and $p\beta/2=15^0$ giving:

$k_w=\cos(0)\times\sin30/(2\times\sin(15)=0.966$.

$E=V=6.6kV/\sqrt{3}=3.81$ kV $=l\times(\omega/p)\times d\times N_{ph}\times k_w\times B_{rms}$ with $B_{rms}=\pi B/(2\sqrt{2})=0.555$ T. Putting in the numbers and solving for N_{ph} gives N_{ph} = 722.5 and hence 72.3 turns per coil. This should be rounded up to the nearest integer so that the specific magnetic loading is slightly under the desired value, so N_{coil} = 73 and N_{ph} = 730.

(iii) Assuming that all the flux crossing a slot pitch enters the tooth then:

$B_tw_t=(w_s+w_t)B\pi/2$ where w_s and w_t are the slot width and tooth width respectively. Rearranging:

$B_t=(1+w_s/w_t)\ B\pi/2$ and solving for w_s/w_t with B_t=1.57 gives w_s/w_t=1 and so $w_s=w_t$.

Also, $w_s+w_t=\pi d/60$ = 41.4 mm and so $w_s=w_t$=20.7 mm

For the stator core thickness, the principle is that all of the pole flux must split in half, travel round the core and then back through the teeth one pole-pitch later. Thus, the peak core flux, φ_c, is given by:

$B_cyl=0.5\times Bl\pi d/(2p)$ where y is the core thickness and B_c is the peak core flux density which is to be 1.4 T. Solving for y gives 44.4 mm.

(iv) The stator slot must be deep enough so that its area is big enough to accommodate the required total conductor area. First find the rated phase current:

$S=3V_{ph}I_{ph}$ gives $136000=\sqrt{3}\times6600\times I_{ph}$ resulting in $I_{ph}=11.9$ A.

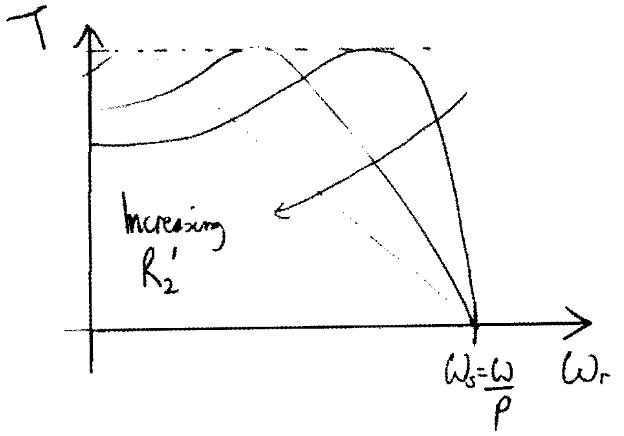
With the rms current density limited to $6Amm^{-2}$ the conductor cross-sectional area is $11.9/6=1.98$ mm^2 and so the total conductor area is $N_{coil}\times1.98=144.5$ mm^2. The slot area required to accommodate this is 144.5/0.7 because of the 70% slot fill factor giving a total slot area of 206 mm^2.

Assuming rectangular stator slots, this area is given by $w_s d_s$ where d_s is the slot depth and w_s was found earlier to be 20.7 mm. Thus $d_s = 9.98$ mm.

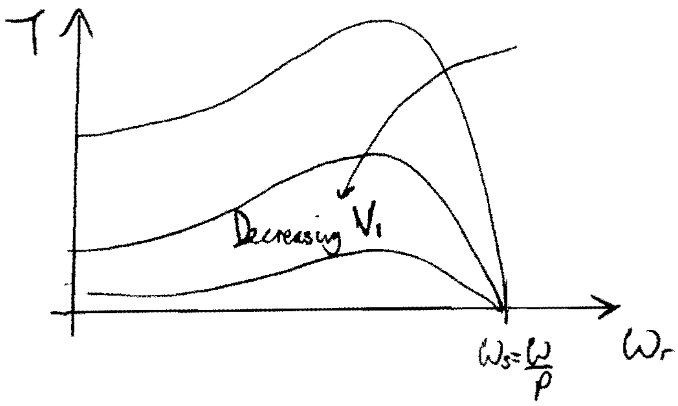
(v) Airgap radius $=d/2=0.792/2=0.396$ m=396 mm. Total motor radius $r_m=d/2+ds+y=396+9.98+44.4=450$mm.

Motor volume is $=\pi r_m^2 l=0.126$ m^3

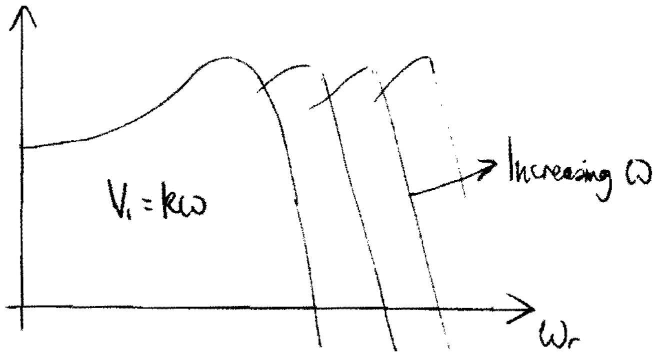
2(a) (i) Torque-speed characteristics for varying R_2.



(ii) Torque-speed characteristics for varying V_1.



(iii) Torque-speed characteristics VVVF speed control.



Advantages of VVVF control: the motor is always operating on the steep part of the torque-speed characteristic, so efficient; wide speed range possible at high efficiency; maximum available torque is unaffected unlike variable voltage control; motor always fully-fluxed which means it is operating efficiently and making good use of the magnetic materials.

(b) Start from the databook expression for torque:

$T = 3I_2^2R_2/s\omega_s$

$I_2^2 = V_1^2/((R_2/s)^2 + \omega^2L_2^2) = s^2V_1^2/(R_2^2 + s^2\omega^2L_2^2)$ assuming that the voltage across the rotor branch, E, is equal to the stator applied voltage, V_1. For small slip, $s\omega L_2 \ll R_2$ and may be ignored (this corresponds to the steep part of the torque-speed characteristic as required) giving $I_2 = sV_1/R_2$

Substituting into the torque equation gives:

$T = 3V_1^2 s/(\omega_s R_2)$ as required.

The slope of the steep part of the torque-speed characteristic is found by differentiating the above torque expression wrt ω_r. The easiest way to do this is to use the chain rule, so:
$dT/d\omega_r = dT/ds \times ds/d\omega_r$ where $s = (\omega_s - \omega_r)/\omega_s$ and so $ds/d\omega_r = -1/\omega_s$

$dT/ds = 3V_1^2/(\omega_s R_2)$ and so $dT/d\omega_r = -3V_1^2/(\omega_s^2 R_2)$

Substituting in $\omega_s = \omega/p$ gives $dT/d\omega_r = -3p^2 V_1^2/(\omega_s^2 R_2)$

For VVVF control with $V1 = k\omega$ this slope becomes $-3k^2p^2/R_2$ and is therefore fixed.

(c) (i) The unloaded speed at 50 Hz of an 8 pole motor is $60f/p = 3000/4 = 750$ rpm. Therefore the maximum unloaded speed of the drive corresponds to the inverter operating at its maximum frequency of 150 Hz, giving $N_{max} = 750 \times 150/50 = 2250$ rpm.

To find the rated torque, ignore $R_1 + jX_1$ and jX_2. This is justified because the motor will be operating at a small value of slip.

$I_1 = 415\sqrt{3}/jX_m + I_2 = -j2.40 + I_2$

Since $R_2/s >> X_2$, I_2 is approximately in phase with V_1 and so at the stator current limit of 15 A:

$I_2 = \sqrt{(15^2 - 2.4^2)} = 14.8$ A (which shows that $I_1 = I_2$ is a reasonable approximation for an induction motor under full-load conditions)

$k = V_b/\omega_b = 415/\sqrt{3}/(2\pi \times 50) = 0.764$

$I_2 = V_1/(R_2/s) = V_1 s\omega/(\omega R_2) = s\omega k/R_2 = 14.8$ so $s\omega = 14.8 \times 1.2/0.764 = 23.3$ and this is fixed for rated torque and rated rotor current.

$T_{rated} = 3pk^2 s\omega/R_2 = 3 \times 4 \times 0.764^2 \times 23.3/1.2 = 136$ Nm

V_1 is at its maximum when f = 50 Hz and delivering rated torque. The slip at this frequency is given by $2\pi \times 50s = 23.3$ so $s = 0.0742$.

Maximum speed at which rated torque can be delivered is $(1 - s) \times 60f/p = (1 - 0.0742) \times 750 =$ 694 rpm.

(ii) f = 1 Hz so voltage across magnetizing reactance for rated flux is $(1/50) \times 240 = 4.8$ V. At rated magnetizing current torque is proportional to I_2, and so for 50% of rated torque, $I_2 = 14.8/2 = 7.4$ A.

Thus, total stator current is $(7.4 - j2.4)$ A and this will produce a voltage drop across R_1 (no need to include X_1 since its value will be negligible compared to R_1 at f = 1 Hz) of:

$V_{R1} = 1.5 \times (7.4 - j2.4) = 11.1 - j3.6$

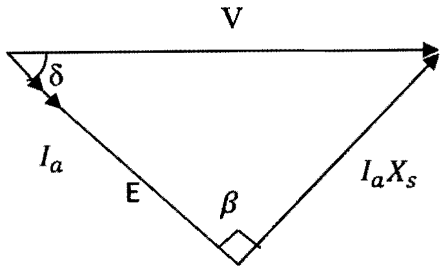
and so $V_1 = 4.8 + 11.1 - j3.6 = 15.9 - j3.6$ and the magnitude of V_1 is 16.3 V. Voltage boost is the difference between V_1 and the voltage required to produce rated flux ie the voltage across the magnetizing reactance, which is 4.8 V. Therefore $V_{boost} = 16.3 - 4.8 = 11.5$ V.

3 (a) Key features are:

A rotor with permanent magnets giving a rotor field of an appropriate number of poles. The magnets can be surface mounted or interior magnets according to the design. The magnetic field should have a circumferential distribution which is as close to sinusoidal as possible - the magnets can be shaped to assist with this. The stator normally has a conventional three-phase winding with a pole number matching that of the rotor.

Rare earth magnets, as opposed to fernite materials, are usually preferred as they are stronger, giving a higher magnetic loading and hence a higher torque density for the machine.

(b) The important thing in achieving maximum torque for Amp is to have the torque angle $\beta 90°$



E	excitation voltage
I_a	machine current
X_s	synchronous reactance
V	terminal voltage
β	torque angle
δ	load angle

(c) (i) The thermal capacity represents heat stored and the rate of loss of heat is determined by the dissipation coefficient. A heat balance requires that

$$P = \frac{Cd\Theta}{dt} + k\Theta$$

Where Θ is the temperature rise above ambient. This equation has a solution

$$\Theta = \frac{P}{k}\left(I - e^{-t/t}\right)$$

Where τ is the thermal time constant, $C/_{k}$.

(ii) For the motors in question $\hat{\tau} = \frac{5000}{10} = 500\ s$. Losses are 3.6 kW for 100 s so the temperature rise is

$$\theta = \frac{3600}{10}\left(I - e - \frac{100}{500}\right) = 65\ K$$

With an ambient temperature of 40°C, this gives a motor temperature of 115 °C.

Under duty cycle operation when the motor is initially at Θ_i °C,

$$\Theta = \Theta_o + (\Theta_m - \Theta)\left(I - e^{-{}^{t}/_{\tau}}\right)$$

Where $\Theta_m = {}^{P}/_{K} = 360K$

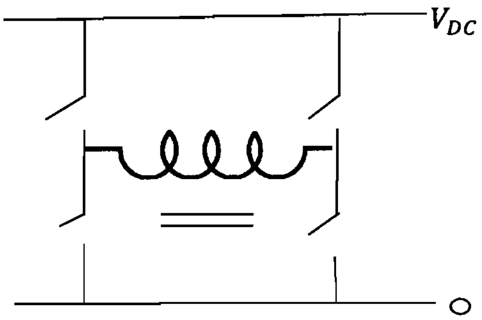
So $\Theta = 90 + (360 - 90)(I - e^{-0.2})$

$= 90 + 49 = 139\ °C$

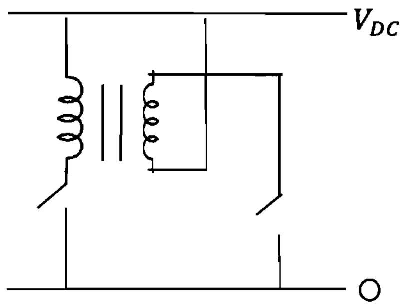
This exceeds the rating of the insulation so a designer could either use a higher grade of insulation or fit a cooling fan to increase the dissipation co-efficient.

4 (a) The brushless dc motor (BLDC) will have overall the better performance in terms of torque density and dynamic performance. However, its control is more complex: commutation requires position sensors and it is likely that knowledge of position will be needed; this implies a position encoder. In contrast, a stepper motor's position will always be determined by the number of steps executed, provided it does not pull out.

(b) The current in the windings in a normally configured hybrid stepper motor has to be reversed during operation, requiring relatively complicated circuitry. Bifilar, that is two independent wires, windings allow current to be directed in either sense with fewer switches, as shown below.



Normal



Bifilar

(c) The restoring torque T is

$$T = -\hat{T}sin(N_t\theta)$$

linearizing gives

$$T = -\hat{T}N_t\theta$$

If the system is lightly damped this torque balances the inertia torque.

$$T = J\frac{d^2\theta}{dt^2}$$

Equating gives a second order differential equation with $f = \frac{1}{2\pi}\sqrt{\frac{\hat{T}N_t}{J}}$

Microstepping gives a stepped waveform so the resonances are less likely to be excited.

(d) Knowing the current drawn by the motor and its drive electronics at a fixed voltage gives the power consumed, P. If there are no losses, then this is converted into mechanical power $T\omega$ where T is the torque ω is known in a stepper motor from the stepping rate, so the current drawn can be related to torque.

However this breaks down at zero speed and also is inaccurate at low speeds as the losses will be significant compared to power converted.