ENGINEERING TRIPOS PART IIA

Tuesday 30 April 2013 2 to 3.30

Module 3B5

SEMICONDUCTOR ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this

1 (a) The particle will behave quantum mechanically when its wavelength is of the same order of magnitude, or larger, than the width of the well. In the case of the parabolic potential well of the form $V(x)=cx^2/2$, the apparent width of the well will depend on the particle's energy. Foe an energy, *E*, the width will appear to be $2\sqrt{2E/c}$.

(b) (i) We need to substitute the given wavefunction into the time-independent Schrodinger equation (TISE), for which we need the second differential with position, so

$$\frac{\partial\psi}{\partial x}=-A_0\alpha^2 x\exp\left(\frac{-\alpha^2x^2}{2}\right)=-\alpha^2x\psi$$

$$\frac{\partial^2\psi}{\partial x^2}=A_0\alpha^2\left(\alpha^2x^2-1\right)\exp\left(\frac{-\alpha^2x^2}{2}\right)=\alpha^2\left(\alpha^2x^2-1\right)\psi$$

Substitution into the TISE gives

$$E\psi=\frac{-\hbar^2}{2m}\alpha^2\left(\alpha^2x^2-1\right)\psi+\frac{cx^2}{2}\psi$$

Cancelling ψ on both sides of the equation, and collecting terms for *x* gives

$$E=\left(\frac{c}{2}-\frac{\hbar^2\alpha^4}{2m}\right)x^2+\frac{\hbar^2\alpha^2}{2m}$$

As *E* is a simple number (not a function of *x*) then this can only be a valid solution if the x^2 coefficient is zero, so

$$\frac{c}{2}=\frac{\hbar^2\alpha^4}{2m}$$

$$\therefore\alpha^2=\frac{\sqrt{cm}}{\hbar}$$

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(ii) Hence, if the x^2 term in the equation for E in part (i) is zero, then the total energy of the electron in this ground state , E_0, is

$$E_0 = \frac{\hbar^2 \alpha^2}{2m}$$

Hence, substituting for α gives

$$\underline{E_0 = \frac{\hbar}{2}\sqrt{\frac{c}{m}}}$$

(iii) The probability that a particle is at a particular position in space and time is given by $|\psi|^2$. Therefore, the probability that the particle is at the centre of the well is

$$P(x=0) = |\psi(x=0)|^2 = A_0^2$$

Elsewhere,

$$P(x) = |\psi(x)|^2 = A_0^2 \exp\left(-\alpha^2 x^2\right)$$

Hence, of the probability of finding the electron will have dropped to 0.1% of that at x=0 at a position $x_{0.1}$, then

$$\frac{P(x_{0.1})}{P(x=0)} = \exp\left(-\alpha^2 x_{0.1}^2\right) = 0.001$$

Hence,

$$x_{0.1}^2 = \frac{-\ln(0.001)}{\alpha^2}$$

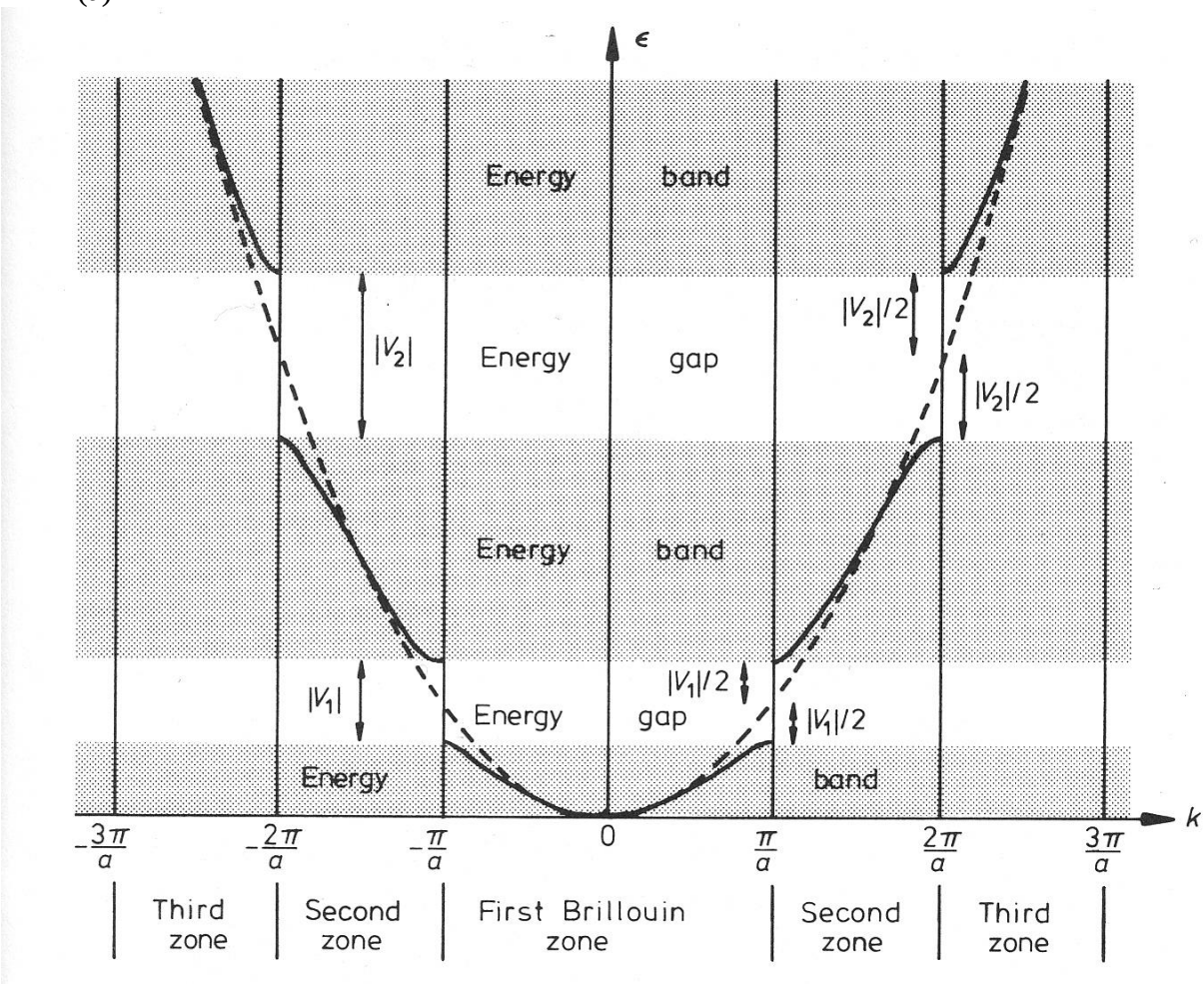
$$x_{0.1}^2 = \frac{-\ln(0.001)\hbar}{\sqrt{cm}}$$

$$x_{0.1}^2 = \frac{-\ln(0.001)\times 1.055\times 10^{-34}}{\sqrt{3.054\times 9.109\times 10^{-31}}}$$

$$\therefore \underline{x_{0.1} = 0.66 \text{ nm}}$$

(c) If the wells overlap, then the higher energy levels would extend across the wells. As the Pauli Exclusion Principal prevents two Fermions from being in the same state, the energy levels must split to form energy bands.

2 (a)



(Free electron theory is the dashed line, and the nearly-free electron theory is the solid line).

The gaps emerge because when $\lambda/2$ is close to the atomic spacing a, then the electron no longer sees the lattice as an average potential well, but as a potential raised or lowered by the lattice depending of whether the electron is likely to be close to the nuclei or in the region between the nuclei.

(b) If the total energy is both kinetic and potential, then

$$\omega = \frac{E}{\hbar} = \frac{mv^2/2 + V(x)}{\hbar}$$
$$\therefore \frac{\partial \omega}{\partial v} = \frac{mv}{\hbar} \Rightarrow v = \frac{\hbar}{m}\frac{\partial \omega}{\partial v}$$

From de Broglie,

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{2\pi}{\lambda}$$
$$\therefore v = \frac{\hbar k}{m} \Rightarrow \partial v = \frac{\hbar}{m}\partial k$$

Therefore,

$$v = \frac{\hbar}{m}\frac{\partial \omega}{\partial v} = \frac{\hbar}{m}\frac{m}{\hbar}\frac{\partial \omega}{\partial k} = \frac{\partial \omega}{\partial k}$$

However,

$$E = \hbar\omega \Rightarrow \partial E = \hbar \partial \omega$$

So, by substitution,

$$\underline{v = \frac{1}{\hbar}\frac{\partial E}{\partial k}}$$

(c)

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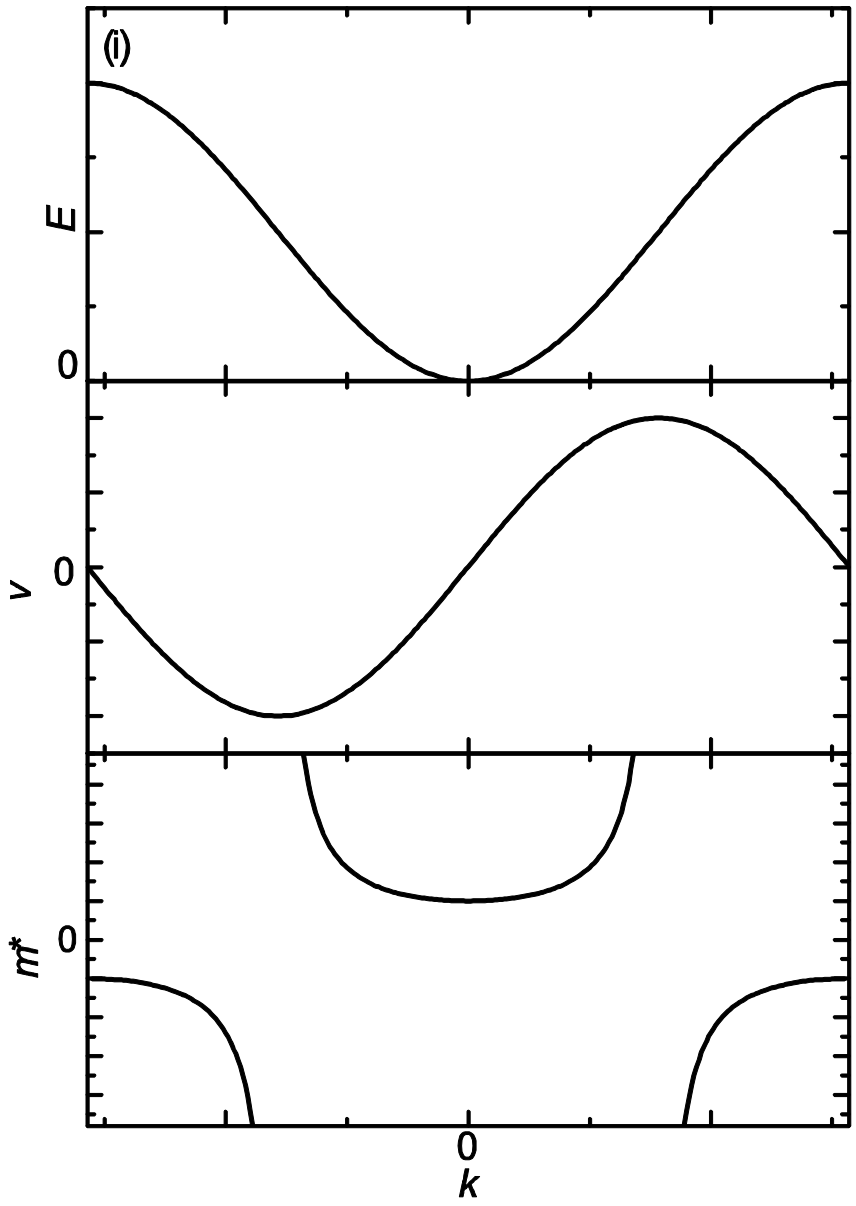

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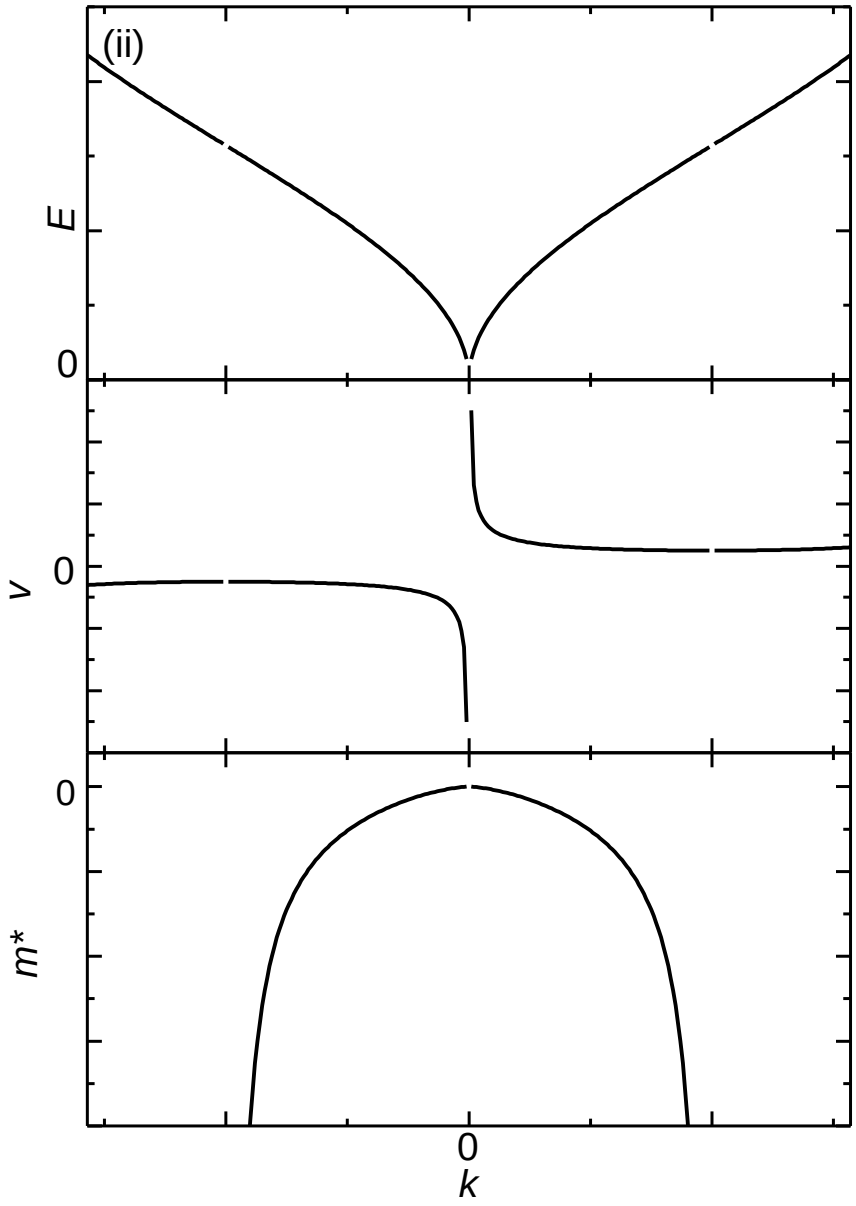

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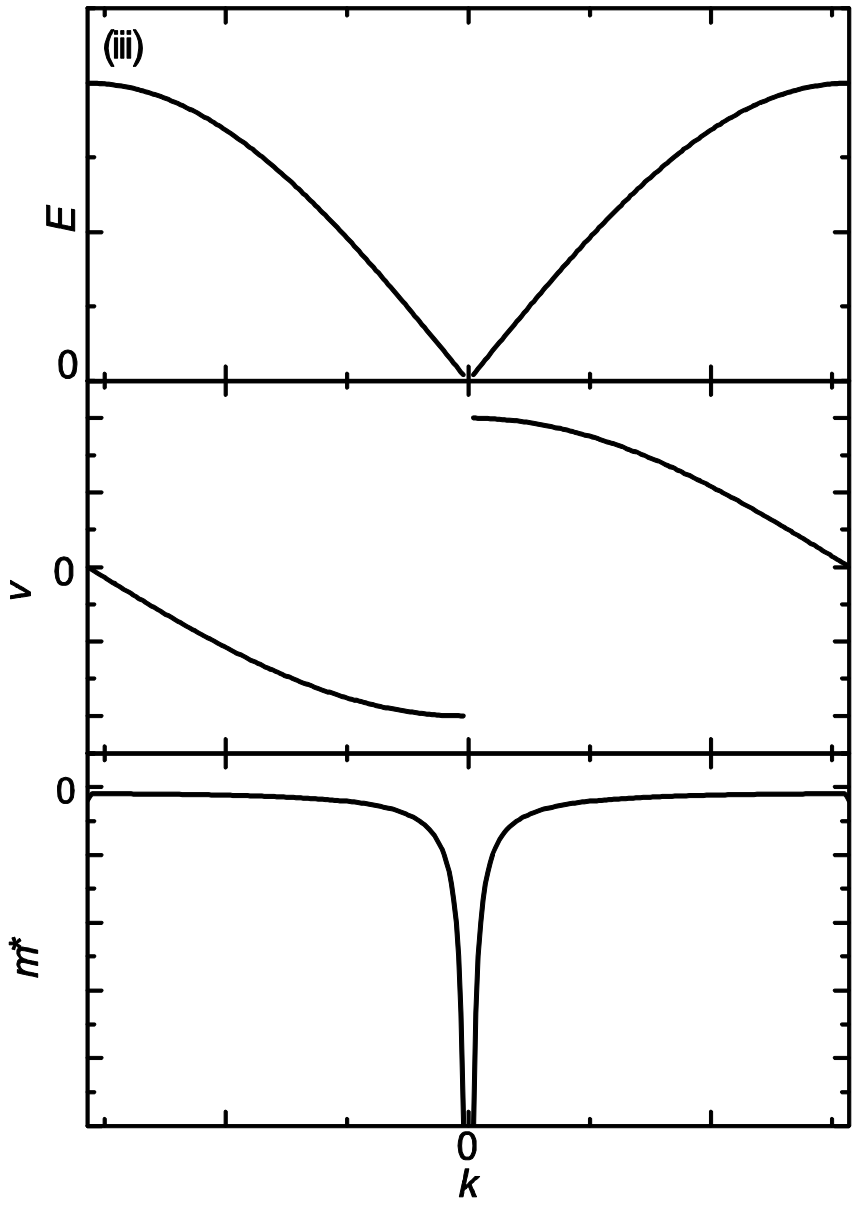

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(d) The effective mass allows the interaction of the electron with the lattice (scattering) to be taken into account whilst still allowing simple Newtoniam mechanics to be applied (i.e. the lattice appears to modulate the mass).

3 (a) (i) for n-type Ge:

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

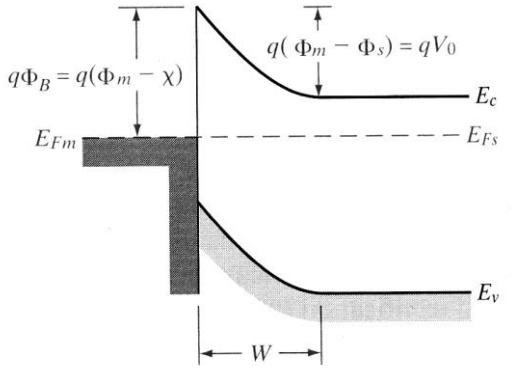
assuming all donors are ionised, ie n=N_D

$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right) = 0.862\times10^{-4} \times 298 \ln\left(\frac{10^{22}}{1\times10^{25}}\right)$$

$$E_F - E_C = -0.18\,\text{eV}$$

hence the work function of the n-type Ge is

$$e\Phi_{SC} = e\chi + (E_C - E_F)$$
$$= 4eV + 0.18eV = 4.18eV$$



hence $\Phi_m > \Phi_{sc}$ which for n-type semiconductor means it is a Schottky contact

(ii) for p-type Ge:

$$p = N_V \exp\left(\frac{-(E_F - E_V)}{kT}\right)$$

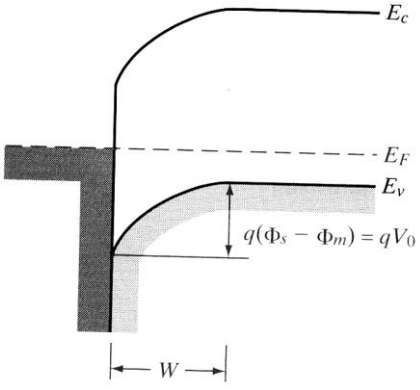
assuming all acceptors are ionised, ie p=N_A

$$E_F - E_V = -kT \ln\left(\frac{N_A}{N_V}\right) = -0.862\times10^{-4} \times 298 \ln\left(\frac{10^{22}}{6\times10^{24}}\right) = 0.16eV$$

hence the work function of the p-type Ge is

$$e\Phi_{SC} = e\chi + [E_{gap} - (E_F - E_V)]$$
$$= 4eV + (0.67eV - 0.16eV) = 4.51eV$$

hence $\Phi_m < \Phi_{sc}$ which for p-type semiconductor means it is a Schottky contact



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(b) For the Schottky barrier junction the depletion region may be considered to be in the semiconductor alone. The Poisson equation states

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0 \varepsilon_r} = \frac{-eN_D}{\varepsilon_0 \varepsilon_r}$$

where N_D is the donor density. This assumes abrupt junction model, and donors to be fully ionised.

Given that V only varies in the x direction across the junction this becomes

$$\varepsilon_0 \varepsilon_r \frac{\mathrm{d}^2 V}{\mathrm{d}x^2} = -eN_D$$

Assuming that there are no electric fields outside the depletion region, i.e. the E-field is 0 at x=w (whereby w is the width of the depletion region), integration gives

$$-\mathrm{E} = \frac{dV}{dx} = \frac{eN_D}{\varepsilon_0 \varepsilon_r}(w - x)$$

Integration again with the boundary condition that V=0 at x=0 gives

$$V = \frac{eN_D}{\varepsilon_0 \varepsilon_r}\left(wx - \frac{x^2}{2}\right)$$

which may be evaluated at x=w to give the contact potential

$$V_0 = \frac{eN_D w^2}{2\varepsilon_0 \varepsilon_r}$$

Rearranging gives

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r V_0}{eN_D}\right)^{1/2}$$

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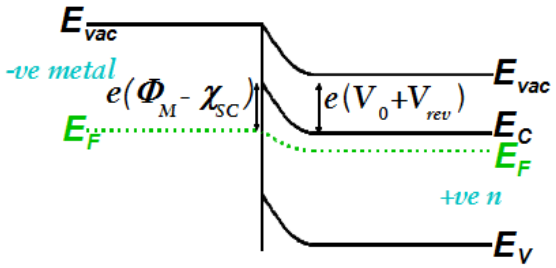
(c) (i)



(ii) $$V_0 = \frac{\Phi_M - \Phi_{SC}}{e} = 0.5V$$

for external bias V

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r (V_0 - V)}{eN_D} \right)^{1/2} = \left(\frac{2\varepsilon_0 \times 11.8 \times \ (1.5V)}{e \times 10^{22}\, m^{-3}} \right)^{1/2} = 0.4\mu m$$

Capacitance per unit area:

$$C = \frac{\varepsilon_0 \varepsilon_r}{w} = 240 \mu F m^{-2}$$

(d) Upon cooling the ideality factor will increase. This is due to the relative increase in tunneling current, which is not considered in thermoionic emission theory.

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4 (a) $\frac{\partial(\Delta p)}{\partial t} =$ total rate of change in excess hole concentration

$-\frac{\Delta p}{\tau_h} =$ net rate of change of excess holes due to difference in recombination and generation

$-\mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} =$ rate of change due to drift of excess holes

. $D_h \frac{\partial^2(\Delta p)}{\partial x^2} =$ rate of change due to diffusion of excess holes

(b) λ=600nm, hence $E = \frac{hc}{\lambda} \cong 2.1eV$

This is larger than Si bandgap (1.1 eV), so e-h pairs will be produced

$$\Delta p(t) = \Delta p(0) \exp(-\frac{t}{\tau})$$

$$t = -\tau \ln(\frac{\Delta p(t)}{\Delta p(0)}) = -2 \times 10^{-6} s \times \ln(\frac{1}{2}) = 1.4 \mu s$$

(c) Enery and momemtum must be conserved. So, unlike to direct semiconductors, in indirect semiconductors electron must loose energy and momentum. Indirect transisitions hence are based on three particle interactions, involving eg phonons or defect states. GaAs is example of direct band gap semiconductor.

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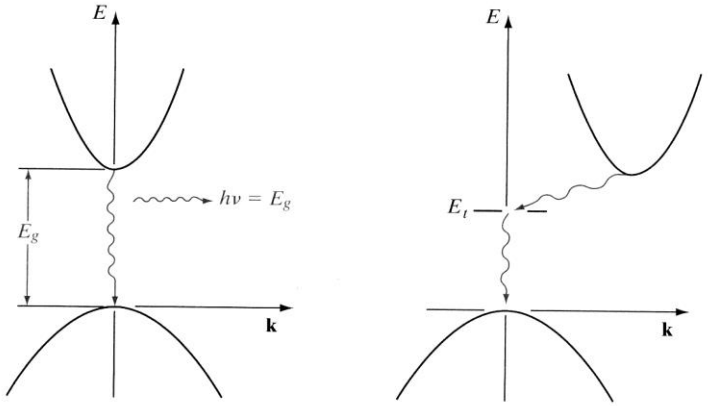

(d) (i) see lecture notes:

(ii) $\mu_h = \dfrac{v_d}{\varepsilon}$ with $v_d = \dfrac{L}{t_d}$

so

$$\mu_h = \frac{v_d}{\varepsilon} = \frac{1/(0.26\times10^{-3})}{2}\frac{cm^2}{Vs} \cong 1900\frac{cm^2}{Vs}$$

(iii) Einstein relation:

$$D_h = \frac{kT}{e}\times\mu_h = 0.026V\times1900\frac{cm^2}{Vs} = 49.4\frac{cm^2}{s}$$

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(e) I-V for illuminated pn junction (photon energy is larger than Si bandgap):