1. (a)

$$P_c = T_c \omega_c = -\left(\frac{T_0}{\omega_0}\right)\omega_c^2 + T_0 \omega_c$$

$$\frac{dP_c}{d\omega_c} = -2\left(\frac{T_0}{\omega_0}\right)\omega_c + T_0$$

equate to zero $\quad T_0 = \frac{2T_0}{\omega_0}\omega_c$

$$\underline{\underline{\omega_c = \frac{\omega_0}{2}}}$$

resistance to motion $\quad F = mg\alpha$

torque on wheel $\quad T_a = F.r = mg\alpha r$

power conserved $\quad T_c \omega_c = T_a \omega_a$

$$\therefore \quad T_c = \frac{\omega_a}{\omega_c} mg\alpha r$$

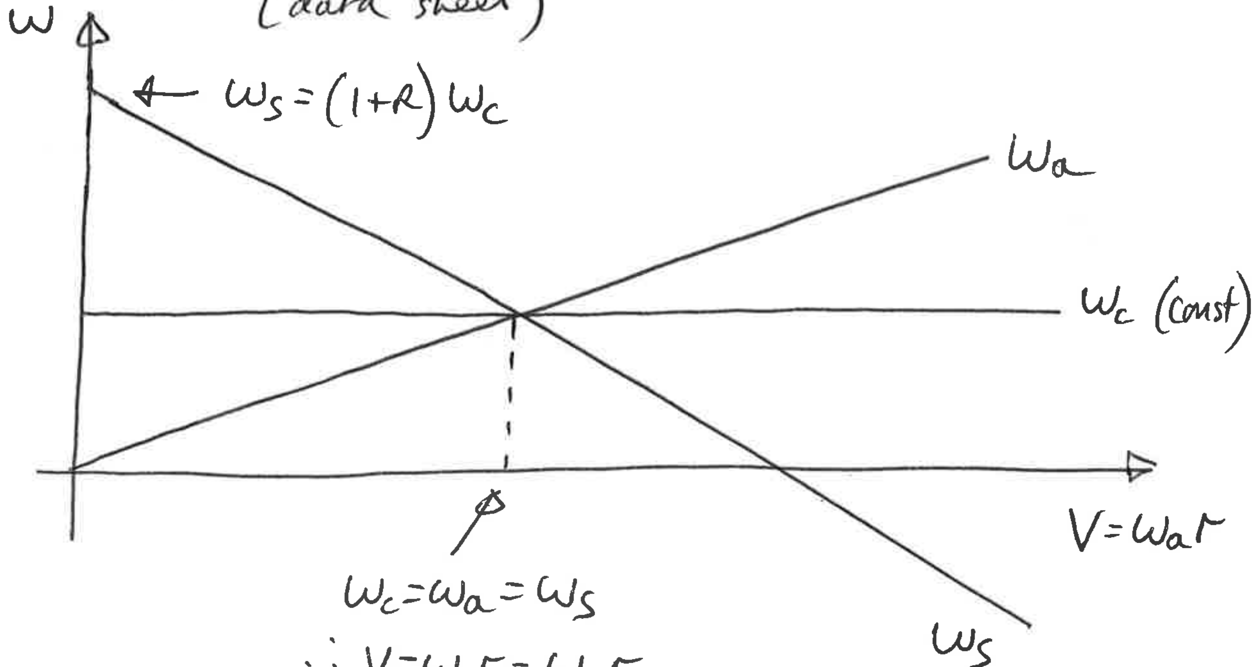
but $\frac{\omega_a}{\omega_c} = G \quad \therefore \quad T_c = \frac{mg\alpha r}{G}$.

max speed when max power, $\omega_c = \frac{\omega_0}{2}$ and $T_c = \frac{T_0}{2}$

thus $\quad T_c = \frac{mg\alpha r}{G} = \frac{T_0}{2}$.

$$\underline{\underline{G = \frac{2mg\alpha r}{T_0}}}$$

(b) (i) epicyclic speed rule (data sheet) $\omega_s = (1+R)\,\omega_c - R\omega_a$



$\omega_c = \omega_a = \omega_s$

$\therefore V = \omega_a r = \omega_c r$

(ii) use virtual speeds to determine torque ratio

$\omega_s', \omega_c', \omega_a'$, let $\omega_s' = 0$

epicyclic speed rule $\quad 0 = (1+R)\,\omega_c' - R\omega_a'$

$$\frac{\omega_c'}{\omega_a'} = \frac{R}{1+R}$$

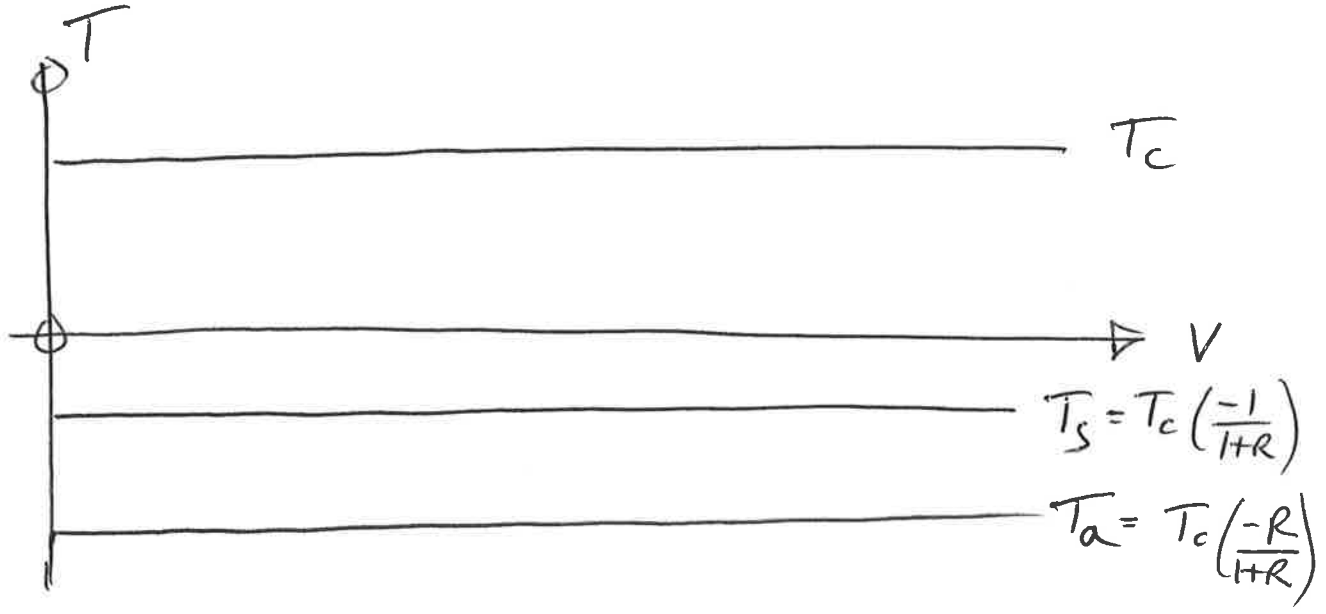
power $\quad T_a\omega_a' + T_c\omega_c' + T_s\omega_s' = 0$

$$\underline{\underline{\frac{T_a}{T_c} = -\left.\frac{\omega_c'}{\omega_a'}\right|_{\omega_s'=0} = -\frac{R}{1+R}}}$$

torque equilibrium $\quad T_a + T_c + T_s = 0.$

$$T_s = -(T_a + T_c)$$

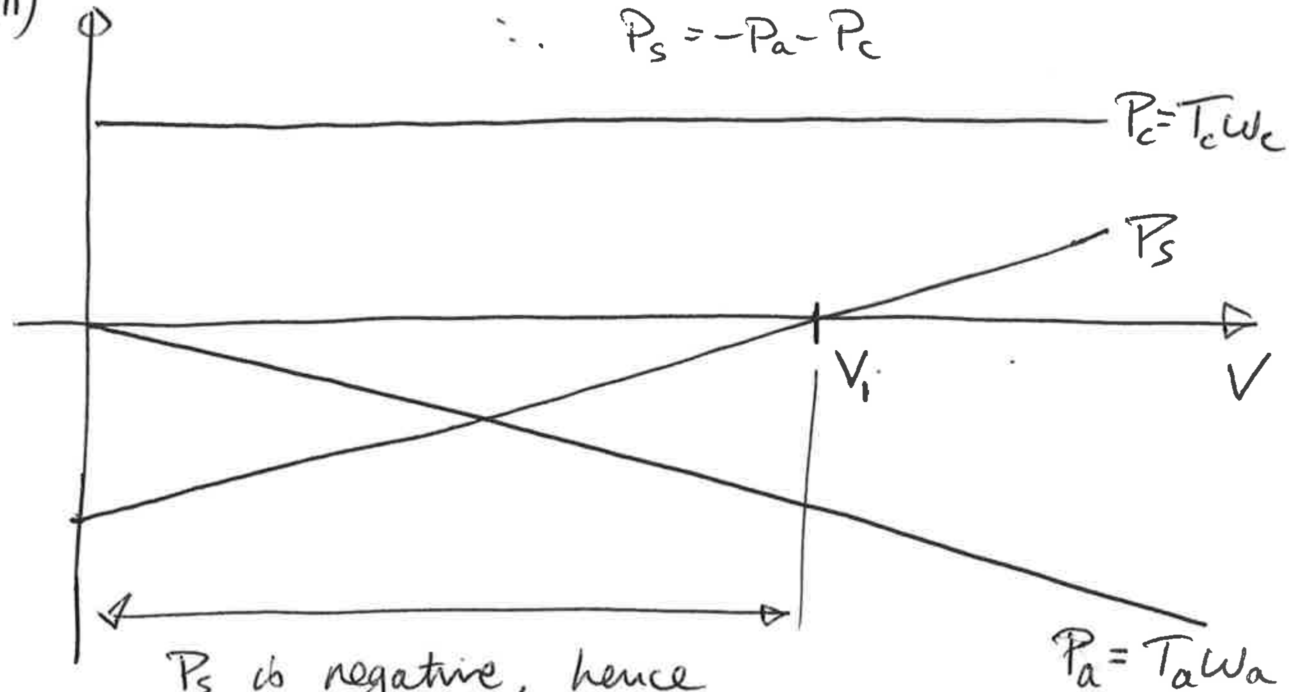
$$\underline{\underline{\therefore T_s = T_c\left(\frac{-1}{1+R}\right)}}$$



(iii) Power $= T\omega$

$P_a + P_c + P_s = 0$

$\therefore \quad P_s = -P_a - P_c$



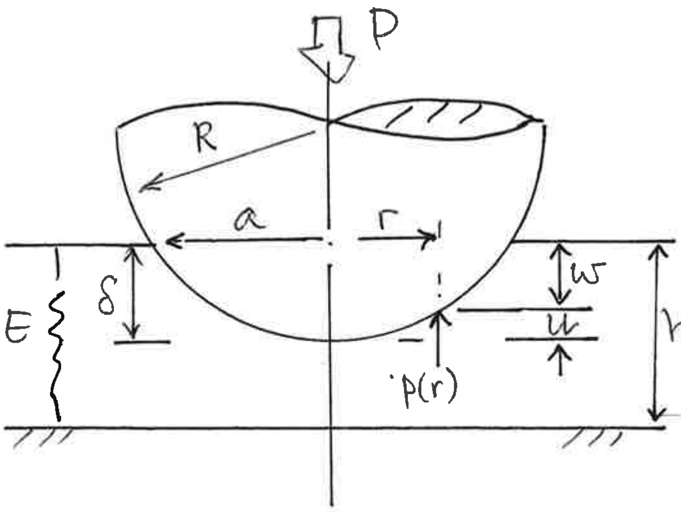
P_s is negative, hence power out of sun and into electric machine, generating electrical power.

V_1 speed is where $\omega_s = 0$, i.e. $\omega_a = \left(\frac{1+R}{R}\right)\omega_c$

but $V = \omega_a r \quad \therefore \quad V_1 = r\omega_c\left(\frac{1+R}{R}\right)$

Thus speed range for electrical power generation is $\underline{\underline{0 \text{ to } r\omega_c\left(\frac{1+R}{R}\right)}}$

2.



(a) Assuming parabolic profile $\delta = \frac{a^2}{2R}$, and $u = \frac{r^2}{2R}$

then at position r, deflection of foam $w = \delta - u$

$$\therefore \quad w = \frac{a^2 - r^2}{2R}$$

$\therefore$ compression strain in foam $= \frac{a^2 - r^2}{2Rh}$

$\therefore$ compressive stress $p(r) = \frac{E}{2Rh}(a^2 - r^2)$

(b) But for equilibrium $P = 2\pi \int_0^a r\, p(r)\, dr$

$$= \frac{2\pi E}{2Rh} \int_0^a a^2 r - r^3 \, dr$$

i.e. $P = \frac{\pi E}{Rh} \frac{a^4}{4}$

(c) Since $\delta = \frac{a^2}{2R}$ $\quad a^4 = 4(\delta R)^2$

$$\therefore \quad P = \frac{\pi E}{Rh} \cdot (\delta R)^2$$

or $\left(\frac{\delta}{R}\right)^2 = \frac{1}{\pi}\left(\frac{h}{R}\right)\left(\frac{P}{ER^2}\right)$

(a) • In the Hertzian solution, shear between adjacent elements of the material, i.e. the springs of the mattress model, is not neglected, so that deformation of the elastic material extends beyond $r=a$

• This, in turn, means that the displacement of any point on the surface depends on the way the pressure is distributed throughout the contact.

See for example Section 4.3 p 104 of K.L Johnson 'Contact Mechanics'.

3 (a) Advantages: shaft alignment accuracy

reduced noise (all rollers in contact)

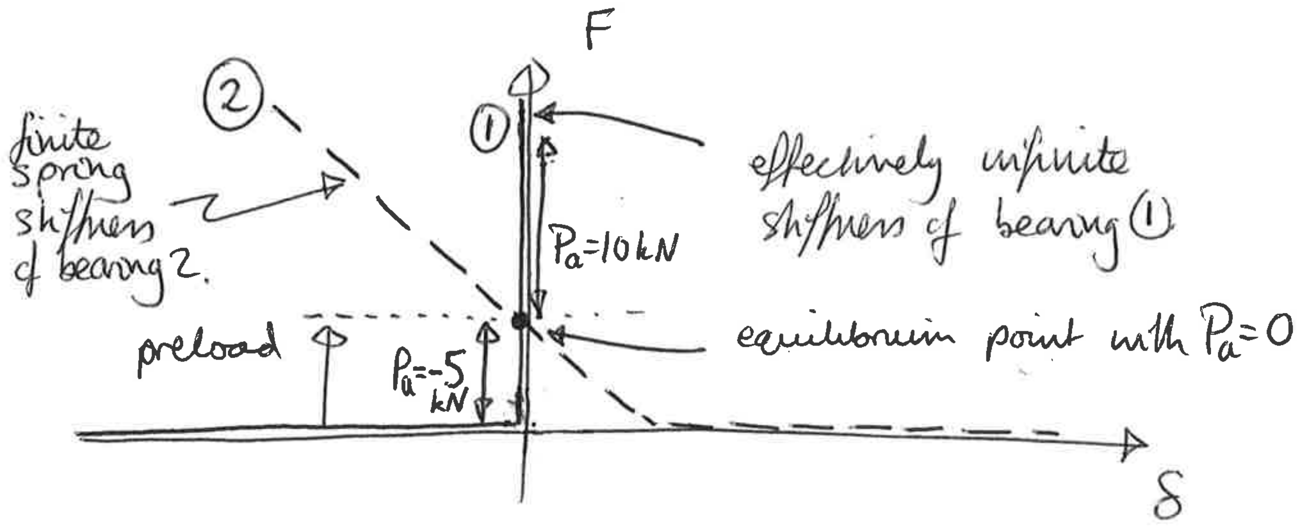
longer life

compensation for wear

Disadvantages: preload needs to be set accurately

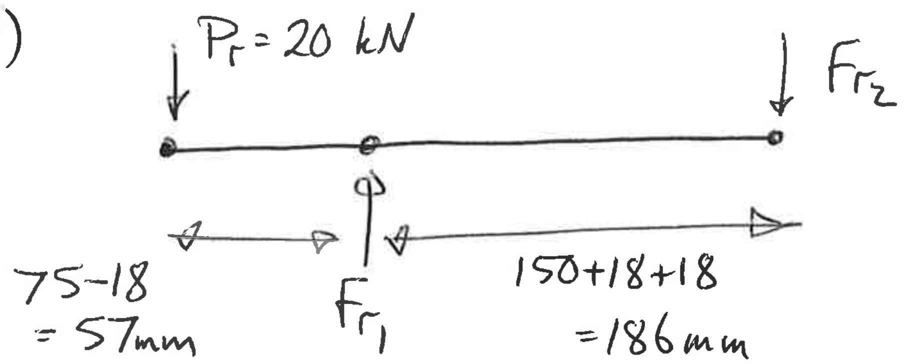
careful design is required

(b) Use graphical solution method.
Superimpose force-deflection characteristics of each bearing so as to embody the axial force equilibrium and compatibility of axial displacements



$P_a = +10 kN$ acts on the very high stiffness of bearing 1.
$P_a = -5 kN$ acts on the low stiffness spring of bearing 2.
A <u>preload of 5 kN</u> is needed to prevent axial clearance arising in bearing 1.

(c) (i)



Moments about F_{r_2} $\quad P_r(57+186) = F_{r_1} 186$

$$F_{r_1} = 20 \cdot \frac{57+186}{186}$$

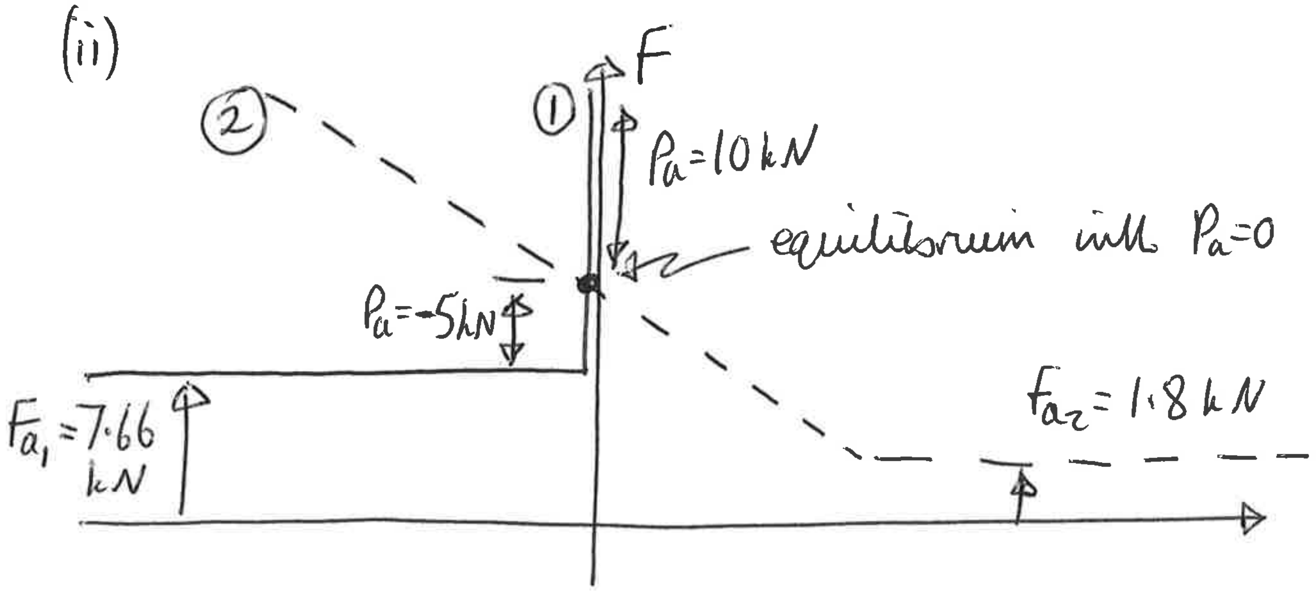
$$F_{r_1} = 26.13 \text{ kN}$$

$$\therefore F_{r_2} = 6.13 \text{ kN}$$

$$F_{a_1} = \frac{0.5}{1.7} \cdot F_{r_1} = \frac{0.5}{1.7} \cdot 26.13 = 7.66 \text{ kN}$$

$$F_{a_2} = \frac{0.5}{1.7} F_{r_2} = \frac{0.5}{1.7} \cdot 6.13 = 1.80 \text{ kN}$$

(ii)



so total preload required is 5 kN + 7.66 kN

$= \underline{12.66 \text{ kN}}$

(d) Spring loading is not suitable when high stiffness is required, when the loading direction changes, and when undefined shock loading is present.

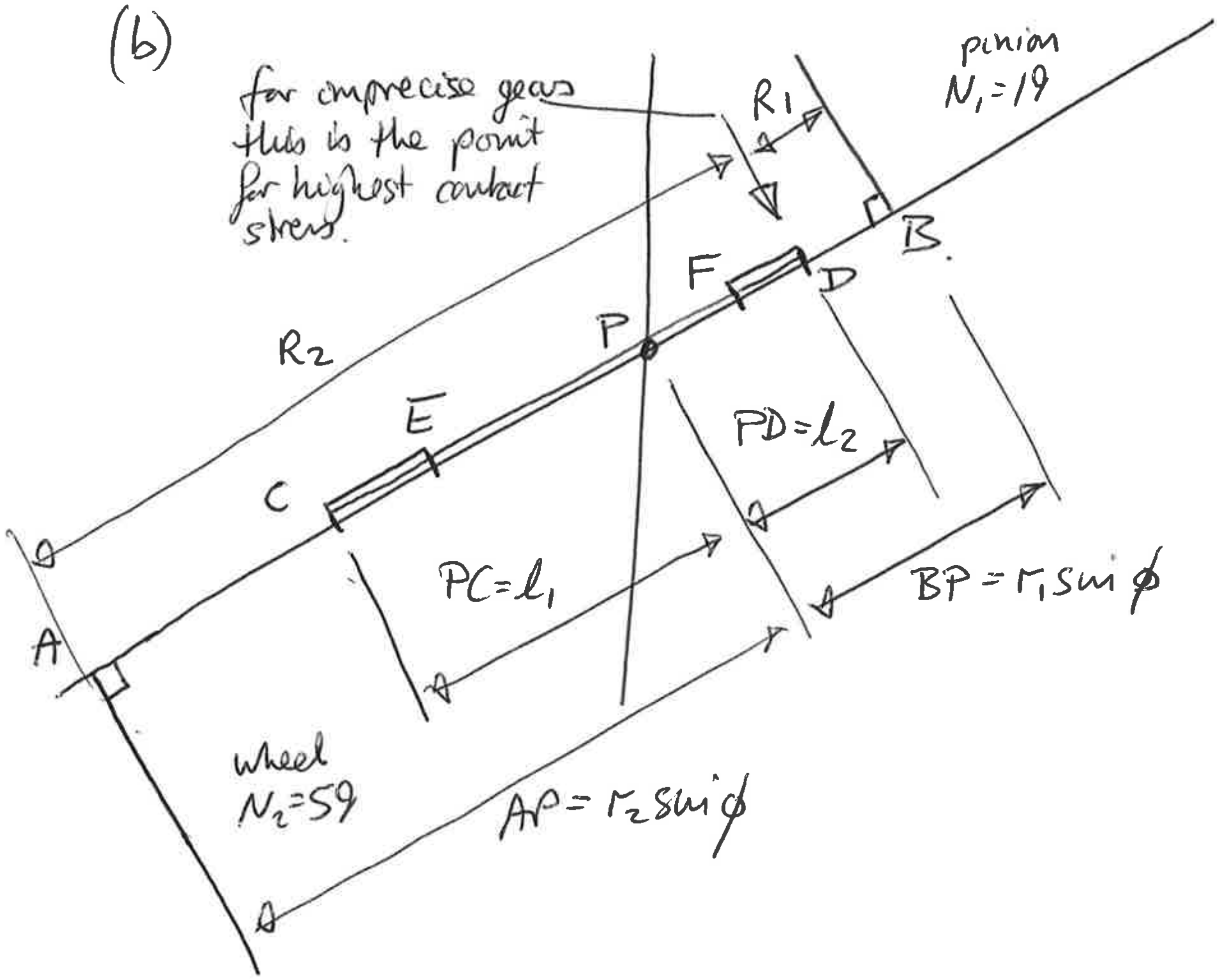
4 (a) Failure by contact stress, leading to pitting.

Failure by bending stress, leading to tooth breakage.

Failure by wear or scoring due to poor lubrication.

Imprecisely made gears might not share load between contacts, so assume load is carried by a single contact.

(b)



Imprecise gears, so assume single contact everywhere between C and D. Highest contact stress is at D, since this point is closest to a base circle tangent point (B).

effective contact radius at D is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{AD} + \frac{1}{DB}$$

where $AD = AP + PD = r_2 \sin\phi + l_2$

$$= \frac{mN_2}{2}\sin\phi + m\left[\sqrt{0.02924N_2^2 + N_2 + 1} - 0.171\,N_2\right]$$

$$= \frac{3.59}{2}\sin 20 + 3\left[\sqrt{0.02924.59^2 + 59 + 1} - 0.171.59\right]$$

$$= 30.269 + 7.891$$

$$\underline{AD = 38.160\,mm}$$

and $DB = BP - PD = r_1 \sin\phi - l_2$

$$= \frac{mN_1}{2}\sin\phi - l_2$$

$$= \frac{3.19}{2}\sin 20 - 7.891$$

$$DB = 1.857\,mm.$$

$$\therefore\ R = \frac{38.16 \times 1.857}{38.16 + 1.857} = 1.771\ mm$$

contact stress $p_0 = \sqrt{\frac{P'E^*}{\pi R}}$

where $P' = \frac{p_0^2\,\pi R}{E^*} = \frac{(1200.10^6)^2\,\pi.\,1.771.10^{-3}}{115.10^9}$

$$= 69667\ N/m$$

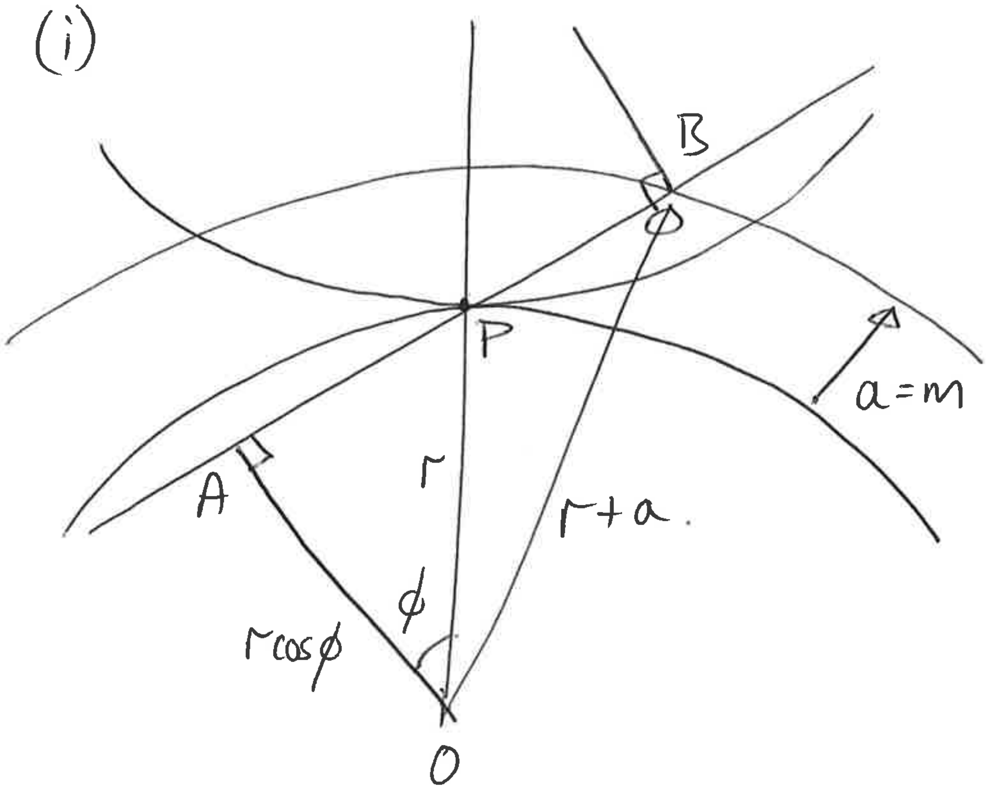
force $P = P'w \doteq 69667 . 0.03 = 2090\ N$

pinion torque $T_1 = P.\,r_{b1} = P r_1 \cos\phi$

$$= 2090 . \frac{\pi.19}{2} . \cos 20 . 10^{-3}$$

$$\underline{\underline{T_1 = 58.6\ Nm}}$$

(c) (i)



Interference occurs when the addendum circle reaches the tangent point of the pressure line on the base circle (point B).

triangle OAB $\quad (r\cos\phi)^2 + (2r\sin\phi)^2 = (r+a)^2$

$$r^2\cos^2\phi + 4r^2\sin^2\phi = r^2 + 2ra + a^2$$

$$r^2(\cos^2\phi + \sin^2\phi) + 3r^2\sin^2\phi = r^2 + 2ra + a^2$$

$$3r^2\sin^2\phi = 2ra + a^2$$

$$0 = \frac{2a}{r} + \frac{a^2}{r^2} - 3\sin^2\phi$$

$$\therefore \frac{a}{r} = \frac{-2 \pm \sqrt{4 + 4.3\sin^2\phi}}{2}$$

+ve solution $\quad \frac{a}{r} = \sqrt{1 + 3\sin^2\phi} - 1$

if $\phi = 20°$, $a = m$ then $\quad r = 6.16m = \frac{Nm}{2}$

$\therefore N = 12.32$

$\underline{\underline{N_{min} = 13}}$

(ii) a larger module means a larger addendum, moving the contact point closer to the base circle tangent point. Thus the radius of curvature becomes smaller and the contact stress increases.