Q.1

(a) 80kPa $v = 2.858 - 0.161 \ln 80 = 2.152$ $\quad$ $\underline{e = 1.152}$ //

160kPa $v = 2.858 - 0.161 \ln 160 = 2.041$ $\quad$ $\underline{e = 1.041}$ //

320 kPa $v = 2.858 - 0.161 \ln 320 = 1.929$ $\quad$ $\underline{e = 0.929}$ //

(b) constant e in undrained failure

The critical state line from the Databook

$v = 2.759 - 0.161 \ln p'_{crit}$

$$p'_{crit} = \exp\left(\frac{2.759 - v}{0.161}\right) \qquad S_u = \frac{q_{ult}}{2} = \frac{\overset{0.89}{M} p'_{crit}}{2}$$

$p_0' = 80$ kPa $\quad v = 2.152$ $\quad$ $p'_{crit} = 43.3$ kPa $\quad$ $\underline{S_u = 19.2 \text{ kPa}}$ //

$p_0' = 160$ kPa $\quad v = 2.041$ $\quad$ $p'_{crit} = 86.5$ kPa $\quad$ $\underline{S_u = 38.5 \text{ kPa}}$ //

$p_0' = 320$ kPa $\quad v = 1.929$ $\quad$ $p'_{crit} = 173.0$ kPa $\quad$ $\underline{S_u = 77.0 \text{ kPa}}$ //

There is a linear relationship of $S_u = 0.24 p_0'$
undrained shear strength increases with depth for normally consolidated clay

(c) $v = 2.858 - 0.161 \ln 320 + 0.062 \ln (320/80)$

$= 2.015$

(i) $\dfrac{q}{p'} = M \ln\left(\dfrac{p_0}{p}\right)$ — cam-clay model

$q = M p' \ln\left(\dfrac{p_0}{p}\right)$ $\quad$ $p' = 80$ kPa until yielding
$p_0' = 320$ kPa

$q_y = 0.89 \cdot 80 \cdot \ln\left(\dfrac{320}{80}\right) = 98.7$ kPa

peak undrained shear strength $= \dfrac{98.7}{2} = \underline{49.4 \text{ kPa}}$ //

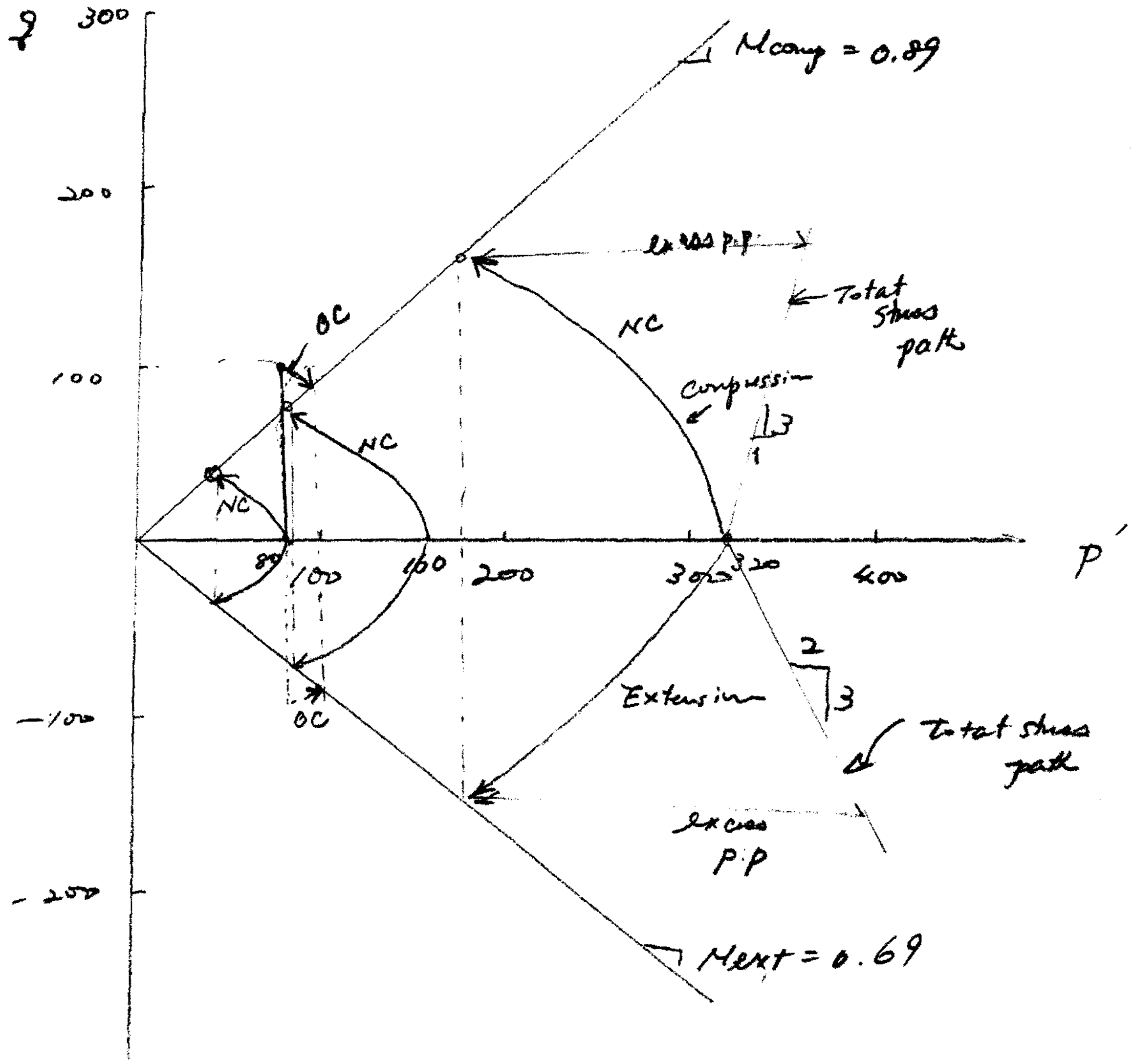
(ii) at critical state

$$p'_{crit} = \exp\left(\frac{2.759 - v}{0.161}\right) = \exp\left(\frac{2.759 - 2.015}{0.161}\right)$$

$$= 101.6 \text{ KPa}$$

$$S_u = \frac{M p'_{crit}}{2} = \frac{0.89 \times 101.6}{2} = \underline{45.2 \text{ KPa}}$$

(d)

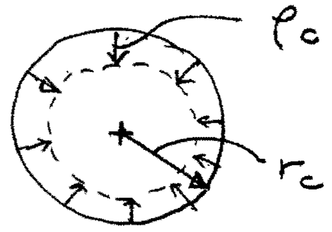


The mean effective stress at failure will be the same for the two cases. However, since Mcomp (=0.89) > Mext (0.69), the undrained shear strength in extension will be smaller than that in compression. For the excess pore pressure, the value in extension will be greater than that in compression, because the total stress path in extension has 2:3 slope rather that that in compression has 1:3 slope as shown in the figure above

Question 2 Solution

(a) Assuming axisymmetric conditions, tunnel construction can be viewed as a contraction of a cylindrical cavity of tunnel radius r_c

- uniform radial contraction $= \rho_c$
- tunnel lining radial stress $= \sigma_c$
- initial total stress in ground $= \sigma_o$



$$\delta\sigma_c = \sigma_o - \sigma_c$$

From Data Book:

$$\delta\sigma_c = c_u\left(1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right)$$

For small strains $\quad \dfrac{\delta A}{A} \simeq \dfrac{2\pi r_c \rho_c}{\pi r_c^2} = \dfrac{2\rho_c}{r_c}$

$c_u = 100\ kN/m^2$
$G = 15\ MN/m^2$ $\quad \therefore \dfrac{G}{c_u} = \dfrac{15\times10^3}{10^2} = 150$

$\rho_c = 20\ mm$
$r_c = 2.5m$ $\quad \therefore \dfrac{\delta A}{A} = \dfrac{2\rho_c}{r_c} = \dfrac{2\times20}{2500} = 0.016$

$$\therefore \delta\sigma_c = 100\,(1 + \ln 150 + \ln 0.016)$$
$$= 100\,(1 + 5.01 - 1.83) = 187\ kN/m^2$$

Tunnel axis at depth $25m$, $\gamma = 20\ kN/m^3$

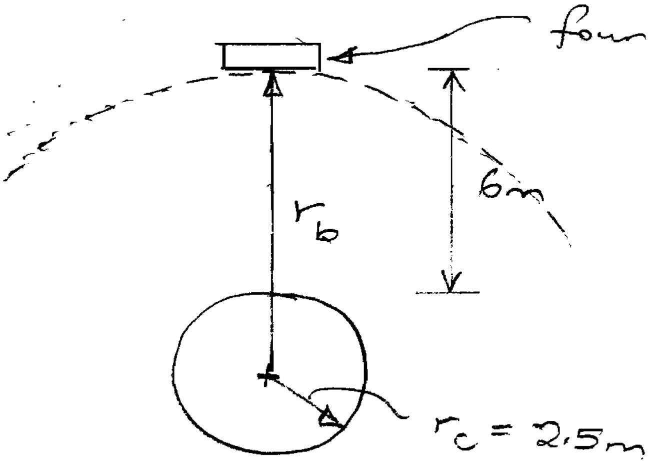
$$\therefore \sigma_o = 2 \times 20 = 500\ kN/m^2$$

$$\therefore \sigma_c = \sigma_o - \delta\sigma_c = 500 - 187 = \underline{\underline{313\ kN/m^2}}$$

[40%]

$$2\pi r \rho = \text{constant}$$

ρ = radial displacement at radius r



$$r_c \rho_c = r_b \rho_b$$

$$\therefore \quad \rho_b = \frac{r_c}{r_b} \cdot \rho_c$$

$$= \frac{2.5}{8.5} \cdot 20$$

$r_b = 3 + 6 = 9m$

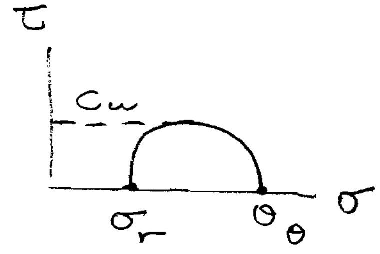
$$= \underline{\underline{5.9 \text{ mm}}}$$

[20%]

(c) In elastic zone

$$\left.\begin{aligned} \sigma_r &= \sigma_0 - \frac{G\,\delta A}{\pi r^2} \\ \sigma_\theta &= \sigma_0 + \frac{G\,\delta A}{\pi r^2} \end{aligned}\right\} \therefore \quad \sigma_\theta - \sigma_r = 2G\frac{\delta A}{\pi r^2}$$

In plastic zone, soil has failed at shear stress $\tau = c_u = \dfrac{\sigma_\theta - \sigma_r}{2}$



$\therefore$ at boundary of elastic and plastic zones $(r = r_p)$

$$2c_u = \frac{2G\,\delta A}{\pi r_p^2}$$

$$\therefore \quad r_p = \left(\frac{G\,\delta A}{\pi c_u}\right)^{1/2}$$

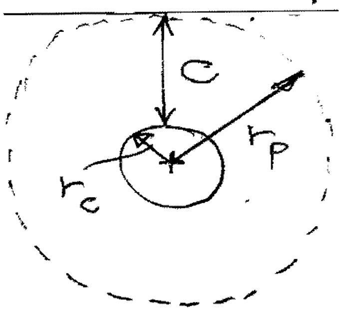
$$\frac{\delta A}{A} = 0.016 \quad (\text{from before})$$

for small strains $A \simeq \pi r_c^2$

$$\therefore \quad r_p = \left(\frac{15 \times 10^3 \times 0.016 \times \cancel{\pi} \times 2.5^2}{\cancel{\pi} \times 100} \right)^{1/2}$$

$$= 3.87 \text{ m}$$

level above plastic zone



$\therefore$ to be above plastic zone

$$C = r_p - r_c$$
$$= 3.87 - 2.5$$
$$= \underline{\underline{1.37 \text{ m}}}$$

[40%]

Q.3

(a) At depth of 7m, $\sigma_{ho} = 180 \text{ kN/m}^2$

Pore pressure $u_o = 7 \times 10 = 70 \text{ kN/m}^2$

$\therefore \sigma_{ho}' = 180 - 70 = 110 \text{ kN/m}^2$

$\sigma_{vo} = 20 \times 7 = 140 \text{ kN/m}^2$

$\sigma_{vo}' = \sigma_{vo} - u_o = 140 - 70 = 70 \text{ kN/m}^2$

$$K_o = \frac{\sigma_{ho}'}{\sigma_{vo}'} = \frac{110}{70} = \underline{\underline{1.57}}$$ [20%]

In the normally consolidated state

$K_o = 1 - \sin \phi' = 1 - \sin 25° = 1 - 0.42 = 0.58$

$\therefore$ If $K_o = 1.57$, clay is overconsolidated and substantial overburden would have been removed (by erosion or glaciation) in the past.

(b) Insitu effective mean normal stress, p_o'

$p_o' = \frac{1}{3}(\sigma_{vo}' + 2\sigma_{ho}') = \frac{1}{3}(70 + 2 \times 110)$

$= 96.7 \text{ kN/m}^2$

Applied cell pressure $= 125 \text{ kN/m}^2$

$=$ total mean normal stress, p

Pore pressure $= p - p' = 125 - 96.7$

$= \underline{\underline{28.3 \text{ kN/m}^2}}$ [20%]

(c) Stress states at X prior to excavation:

total stresses [A] $\sigma_v = \sigma_{vo} = 140 \text{ kN/m}^2$

$\sigma_h = \sigma_{ho} = 180 \text{ kN/m}^2$

pore pressure $u_o = 70 \text{ kN/m}^2$

effective stresses [A'] $\sigma_v' = \sigma_{vo}' = 70 \text{ kN/m}^2$

$\sigma_h' = \sigma_{ho}' = 110 \text{ kN/m}^2$

Excavation under undrained (rapid) conditions:

total stresses $A \rightarrow B$ (σ_{vo}, p_s)

pore pressure $u_o \rightarrow u_1$

effective stresses $A' \rightarrow B'$

undrained, elastic $\Rightarrow$ $s' = \frac{1}{2}(\sigma_v' + \sigma_h') =$ constant

$\Delta s' = 0 \Rightarrow \Delta\sigma_h' = -\Delta\sigma_v'$

$$\Delta\sigma_v' = \sigma_{vB}' - \sigma_{vA}' = (\sigma_{vo} - u_1) - (\sigma_{vo} - u_o)$$
$$= u_o - u_1$$

(constant vertical total stress, σ_{vo})

$$\Delta\sigma_h' = \sigma_{hB}' - \sigma_{hA}' = (p_s - u_1) - (\sigma_{ho} - u_o)$$

$$\therefore \; p_s - u_1 - \sigma_{ho} + u_o = -(u_o - u_1)$$

$$\therefore \; u_1 = u_o - \tfrac{1}{2}(\sigma_{ho} - p_s)$$
$$= 70 - \tfrac{1}{2}(180 - 100) = \underline{\underline{30 \text{ kPa}}}$$

Stress states B : $\sigma_v = \sigma_{vo} = 140 \text{ kN/m}^2$

$\sigma_h = p_s = 100 \text{ kN/m}^2$

B' : $\sigma_v' = \sigma_{vo} - u_1 = 140 - 30 = 110 \text{ kN/m}^2$

$\sigma_h' = p_s - u_1 = 100 - 30 = 70 \text{ kN/m}^2$

Total stress path (TSP) $A \rightarrow B$ } see plot
Effective stress path (ESP) $A' \rightarrow B'$ }

[30%]

(d) 'Active' condition in terms of effective stress:

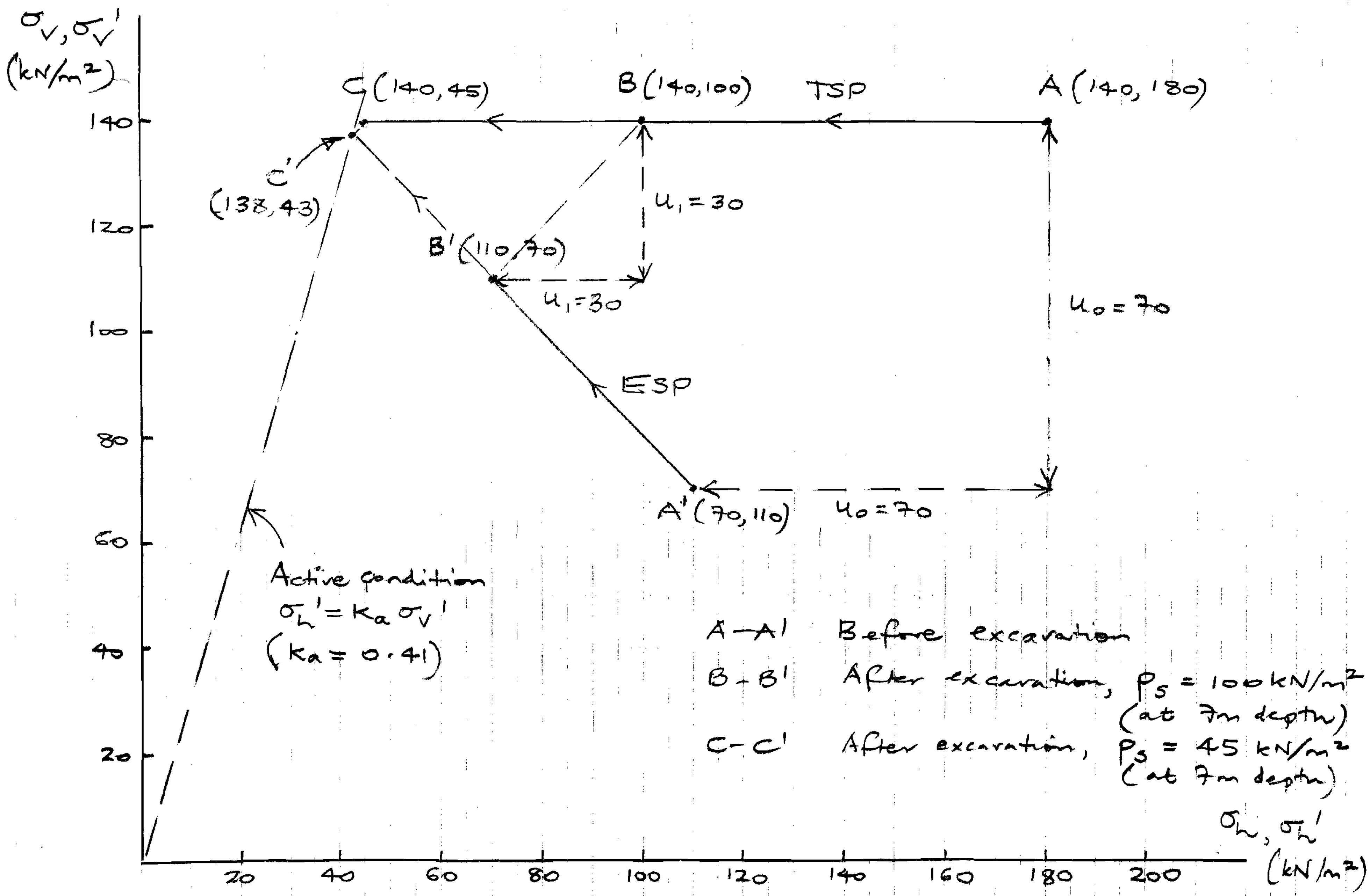
$$\sigma_h' = K_a \sigma_v' = \frac{1 - \sin\phi'}{1 + \sin\phi'} \sigma_v'$$

for $\phi' = 25°$, $K_a = 0.41$

Extrapolating A'B' to B'C' intersects the active condition at C' (138, 43)

Pore pressure $u_2 = 2$ kPa

$\therefore \; \sigma_h = p_s = \sigma_h' + 2 = 43 + 2 = \underline{\underline{45 \text{ kPa}}}$ [30%]



σv, σv' (kN/m²)
140
120
100
80
60
40
20
C (140, 45)
C' (138, 43)
B (140, 100)
B' (110, 70)
TSP
A (140, 180)
u1 = 30
u1 = 30
u0 = 70
ESP
A' (70, 110)
u0 = 70
Active condition
σh' = Ka σv'
(Ka = 0.41)
A – A' Before excavation
B – B' After excavation, ps = 100 kN/m² (at 7m depth)
C – C' After excavation, ps = 45 kN/m² (at 7m depth)
σh, σh' (kN/m²)
20
40
60
80
100
120
140
160
180
200

Q4

(a) plane strain compression $\Delta t/\Delta s' = 1$

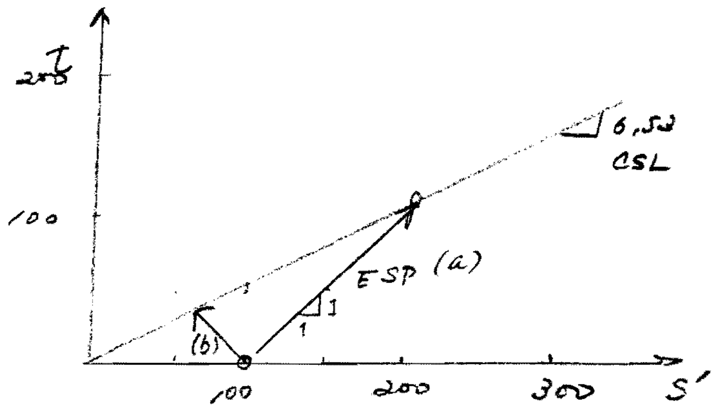
$\Delta\sigma_1 > 0$ $\Delta\sigma_3 = 0$

critical state

$t/s' = \sin\phi'$

$= \sin 32^\circ$

$= 0.53$



$$\frac{\sigma_1' - 100}{\sigma_1' + 100} = \sin 32^\circ = 0.53$$

$\sigma_1' = 325$ kPa

$s' = \frac{325+100}{2} = 212.5$ kPa

$t = \frac{325-100}{2} = \underline{112.5 \text{ kPa}}$

$e = 0.85 - (0.85 - 0.5)/\ln(20{,}000/212.5)$

$= 0.773$

$\varepsilon_v = \frac{0.80 - 0.773}{1.80} = 0.015 = 1.5\%$ contraction

(b) $\Delta\sigma_1' = 0$ $\Delta\sigma_3' < 0$ $\Delta t/\Delta s' = -1$

$$\frac{100 - \sigma_3'}{100 + \sigma_3'} = \sin 32^\circ = 0.53$$

$\sigma_3' = 30.7$ kPa

$s' = \frac{100 + 30.7}{2} = 65.4$ kPa

$t = \frac{100 - 30.7}{2} = 34.65$ kPa

$e = 0.85 - (0.85 - 0.5)/\ln(20{,}000/65.4)$

$= 0.789$

$\varepsilon_v = \frac{0.80 - 0.789}{1.80} = 0.006 = 0.6\%$ contraction

(c) $e = 0.65$ $\sigma_0' = 100$ kPa

(i)

The sample failes when $\phi' = 39.3°$

$$\frac{\sigma_1' - 100}{\sigma_1' + 100} = \sin 39.3° =$$

$$\sigma_1' = \frac{100 + 100 \cdot \sin 39.3°}{1 - \sin 39.3°} = 446 \text{ kPa}$$

$$s' = \frac{446 + 100}{2} = 272.8 \text{ kPa}$$

$$t = \frac{446 - 100}{2} = 172.7 \text{ kPa}$$

$$I_R = \ln\left(\frac{20.000}{272.76}\right)\left(\frac{0.85 - 0.65}{0.85 - 0.5}\right) - 1$$

$$= 1.45$$

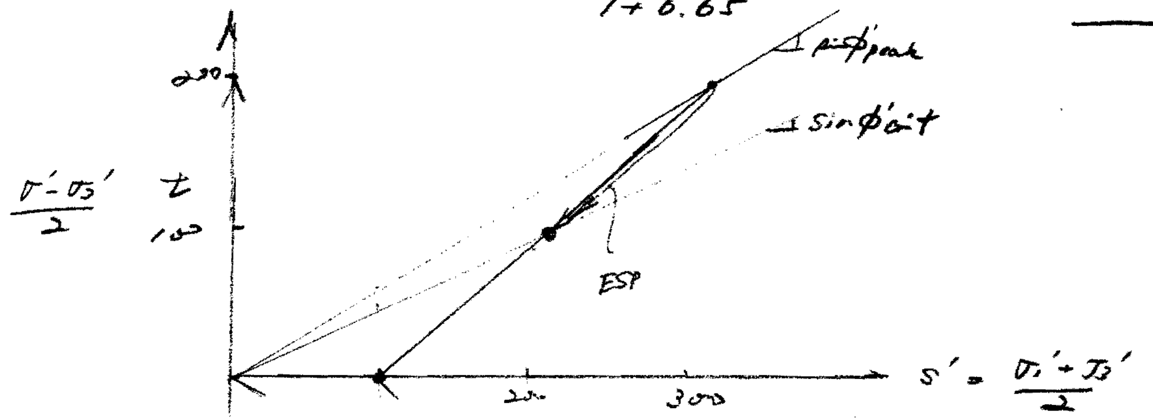
$$\phi_{peak} = 32' + 5 I_R \approx 39.3° \quad \underline{OK}$$

(ii) the critical failure is the same as (a)

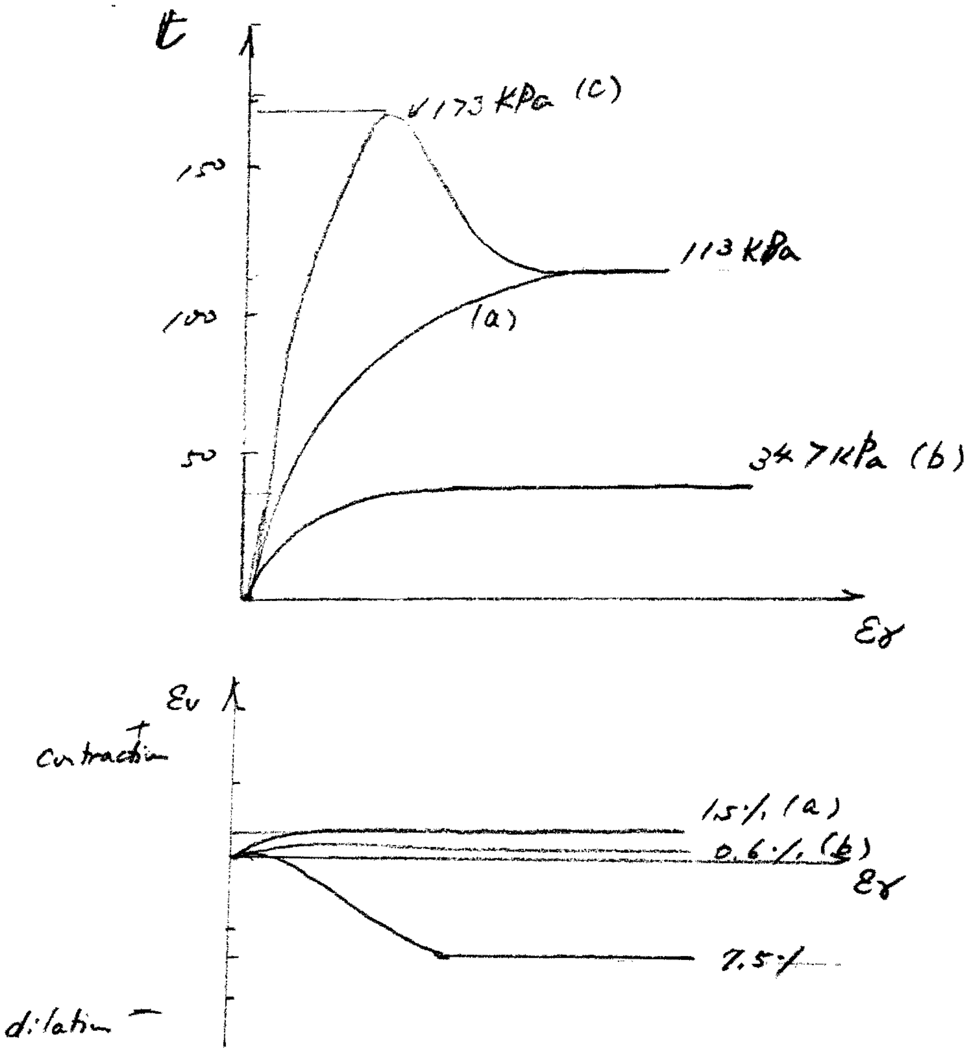
$t = 112.5$ kPa

$e_{crit} = 0.773$

$$\varepsilon_v = \frac{0.65 - 0.773}{1 + 0.65} = -0.075 = \underline{-7.5\,\%}$$ dilation



(d)


t
173 KPa (C)
150
113 KPa
100
(a)
50
347 KPa (b)
εγ
εv
contraction +
1.5% (a)
0.6% (b)
εγ
7.5%
dilation −