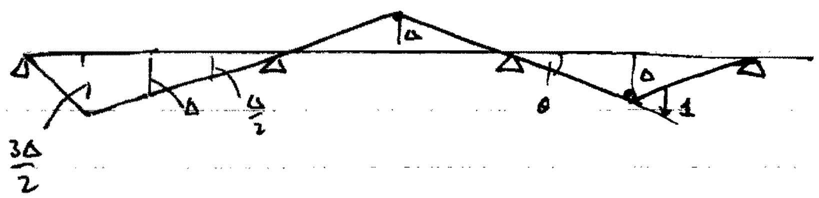
3D4. STRUCTURAL ANALYSIS + STABILITY 2013.

Q1. a) Put hinge at A with unit kink.



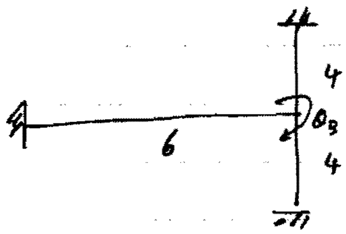
Influence line

Vertical scale: $\frac{\Delta}{4} = \theta$ and $2\theta = 1$. $\rightarrow \theta = 1/2$

$$\Delta = 4\theta = 4(\tfrac{1}{2}) = 2.$$

$$\text{BM at A} = F\left(\frac{3\Delta}{2} + \Delta + \frac{\Delta}{2}\right) = F.\ 3\Delta = \underline{\underline{6F}}$$

b).



by inspection.

$$M_B = K\,\theta_B$$

$$K = 2.4\,\frac{EI_c}{L_c} + 4\,\frac{EI_b}{L_b} = EI\left[\frac{2.4}{4} + \frac{4}{6}\right] = \underline{\underline{EI\left(\frac{19}{15}\right)}}$$

That is rotational stiffness if a moment M_B were to be applied at B.

Q1 b).

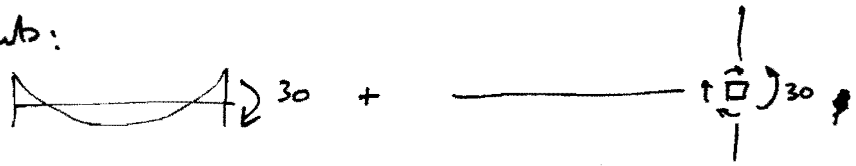
rotation at B = 0

rotation from $M_{applied} = K\theta_B$.

$$M_{FIXED} = M_{applied} = \frac{WL^2}{12} = \frac{10(36)}{12} = \underline{30\text{ kNm}}$$

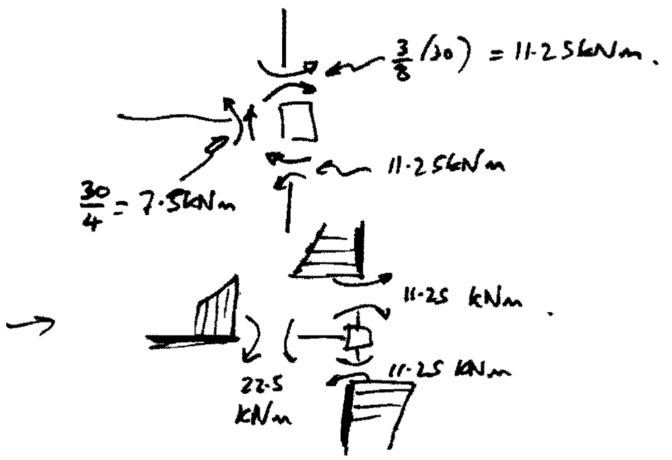
$$\therefore \text{Rotation} = \frac{M}{K} = \frac{30\text{ kNm}}{5\times10^4\text{ kNm}^2}\ \frac{3\text{m}}{8} = \frac{90}{40\times10^{-4}} = \underline{\underline{2.25\times10^{-4}\text{ radians}}}.$$

ii) Bending moments:

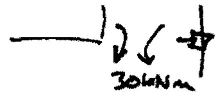


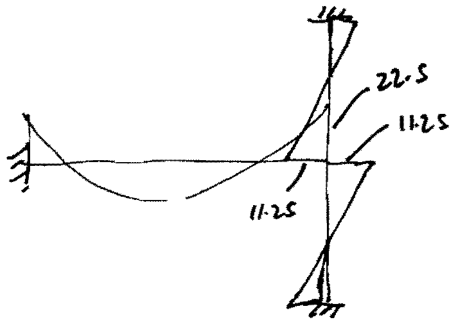
Proportion as stiffness. All EI same, all far end supports fixed.
$\therefore$ Stiffness $\propto 1/L$. so $1/6 : 1/4 : 1/4$
beam col col.

Total $= \frac{1}{6} + \frac{1}{4} + \frac{1}{4} = 2/3$. $\therefore$ Proportions $1/4 : 3/8 : 3/8$.



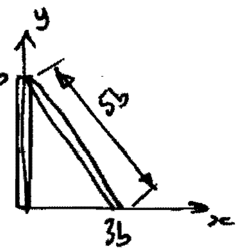
Now add on fixed end moment.





kNm

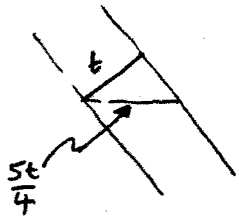
3D4 Q2.



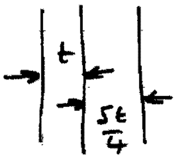
i) $J = \text{"}\sum \frac{bt^3}{3}\text{"} = \frac{9bt^3}{3} = \underline{\underline{3bt^3}}$

ii) By inspection $\bar{y} = 2b$

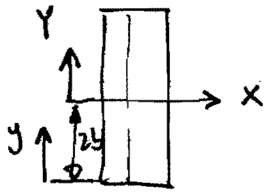
look at sloping part



$\therefore$ equiv. thickness of whole section



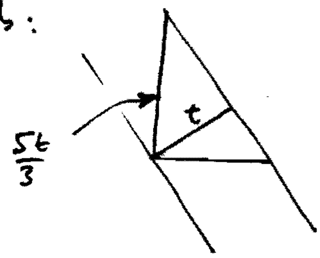
$t_{TOT} = \frac{9t}{4}.$



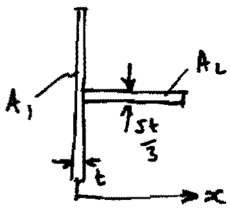
$$I_{xx} = \text{"}\frac{bd^3}{12}\text{"} = \left(\frac{9t}{4}\right)\frac{(4b)^3}{12}$$

$$= b^3t \quad \frac{3.\cancel{3}.\cancel{4}.\cancel{4}.4}{\cancel{4}.\cancel{4}.\cancel{3}} = \underline{\underline{12b^3t}}$$

Now look at x integrals:



$\therefore$ Equiv section



centroid $\bar{x} = \frac{\int x\,dA}{\int dA}$

$\int dA = A = 9bt$

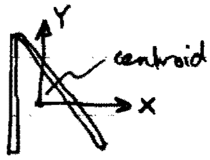
$$\int x\,dA = \int x\,dA_1 + \int x\,dA_2$$

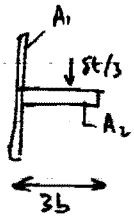
$$= 0 + \int_0^{3b} x\left(\frac{5t}{3}\right)dx = \left(\frac{5t}{3}\right)(3b)^2\frac{1}{2} = \frac{15b^2t}{2}$$

so $\bar{x} = \frac{15b^2t}{2(9bt)} = \frac{5.\cancel{3}.}{2.3.\cancel{3}}b = \underline{\underline{\frac{5}{6}b}}$

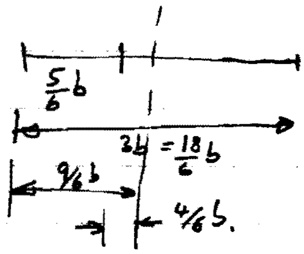
3D4

Q2 a) ii)





I_{YY}



$$I_{YY} = \int X^2 dA$$

$$= \int X^2 dA_1 + \int X^2 dA_2$$

$$= (4bt)\left(-\frac{5b}{6}\right)^2 + \left(\frac{5t}{3}\right)\frac{(3b)^3}{12} + \left[\frac{5t}{3} . 3b\right]\left[\frac{4b}{6}\right]^2$$

$$= b^3t\left[\frac{100}{36} + \frac{5.\cancel{3}.\cancel{3}.3}{\cancel{3}.\cancel{3}.4} + \frac{5}{\cancel{3}}\cancel{3}.\ \frac{2.2.}{3.3.}\right]$$

$$= b^3t\left[\frac{100}{36} + \frac{15}{4} + \frac{20}{9}\right]$$

$$= b^3t\left[\frac{100 + 15(9) + 20(4)}{36}\right] = \frac{315}{36} b^3t.$$

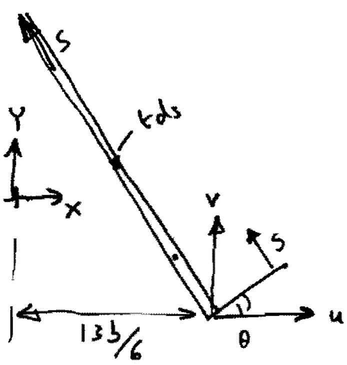
$$\underline{I_{YY} = 8.75\, b^3t = \frac{35}{4} b^3t.}$$

Need I_{XY}. Use line integral method.

$$I_{XY} = \int XY\, dA = \underbrace{\int_{A_1} XY\, dA}_{=0} + \int_{A_2} XY\, dA$$

because every Y has a −Y

so $I_{XY} = \int_{A_2} XY\, dA$ $\qquad = \int_0^{5b} XY\,t\,ds$



$u = -s \sin\theta$

$v = s \cos\theta$

$\sin\theta = 3/5 \qquad \cos\theta = 4/5$.

$X = u + \frac{13b}{6} \qquad Y = v - 2b.$

Q2 a) ii cont'd.

$$I_{xy} = \int_0^{5b} \left(-s\sin\theta + \frac{13b}{6}\right)\left(s\cos\theta - 2b\right) t \, ds$$

$$= \int_0^{5b} \left[-s^2 \sin\theta\cos\theta + \frac{13b}{6} s\cos\theta + 2b\, s\sin\theta - \frac{13}{3}b^2\right] t\, ds$$

$$= \left(-\frac{(5b)^3}{3}\cdot\frac{3}{5}\cdot\frac{4}{5} + \left[\frac{13}{6}\cdot\frac{4}{5} + 2\cdot\left(\frac{3}{5}\right)\right]\frac{(5.5)}{2}b^3 - \frac{13}{3}b^3(5)\right)t$$

$$= \left[-20 + \frac{13.5}{3} + 15 - \frac{13.5}{3}\right] b^3 t$$

$$= \underline{\underline{-5b^3t}}$$

$$\underline{M} = E \underline{\underline{I}}\, \underline{K}$$

$$\underline{I} = \begin{bmatrix} I_{xx} & -I_{xy} \\ -I_{xy} & I_{yy} \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 5 & 8.75 \end{bmatrix} b^3 t.$$

Principal

$$0 = \begin{vmatrix} 12-\lambda & 5 \\ 5 & 8.75-\lambda \end{vmatrix} \rightarrow (12-\lambda)(8.75-\lambda) - 25 = 0$$

$$105 - 20.75\lambda + \lambda^2 - 25 = 0$$

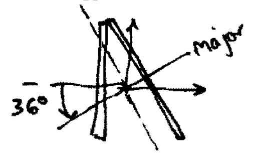
$$\lambda^2 - 20.75\lambda + 80 = 0$$

$$\lambda = \frac{20.75 \pm \sqrt{(20.75)^2 - 4(80)}}{2}$$

$$= \frac{20.75 \pm 10.51}{2} = \begin{matrix} 15.63\ b^3t & \text{major princ.} \\ 5.12\ b^3t & \text{minor princ.} \end{matrix}$$

Evec: $(12 - 15.63)x + 5y = 0$ $\qquad y = \frac{3.63}{5}x = 0.726x.$

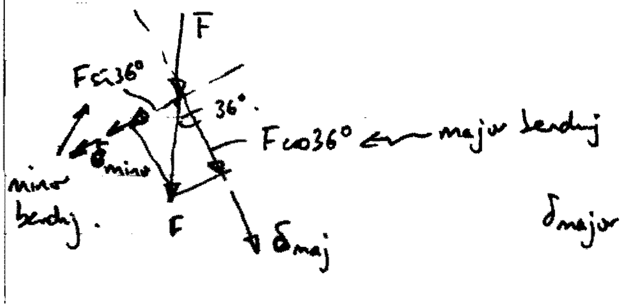
$\therefore \tan\theta = 0.726.$ $\quad \theta = \operatorname{atan}(0.726) = 0.628 = \underline{\underline{36^\circ}}$



3D4

Q2. b). 

$$\delta = \frac{FL^3}{3EI}$$ structures Data Book.



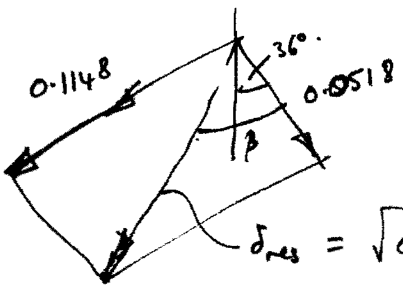
$$\delta_{major} = \frac{F\cos 36°.\,L^3}{E\,I_{major}}$$

$$= \frac{FL^3}{Eb^3t}\left(\frac{0.8092}{15.63}\right) = \underline{\underline{0.0518\,\frac{FL^3}{Eb^3t}}}$$

$$\delta_{min} = \frac{F\sin 36°\,L^3}{E\,I_{minor}}$$

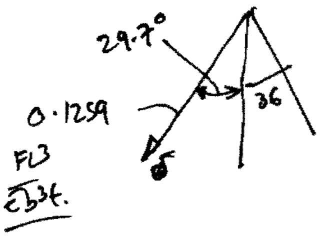
$$= \frac{FL^3}{Eb^3t}\left(\frac{0.5878}{5.12}\right) = \underline{\underline{0.1148\,\frac{FL^3}{Eb^3t}}}$$

$\times \frac{FL^3}{Eb^3t}$.



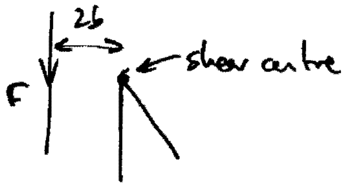
$$\delta_{res} = \sqrt{0.0518^2 + 0.1148^2} = 0.1259.\quad FL^3/Eb^3t.$$

Angle $\tan\beta = \frac{0.1148}{0.0518} \Rightarrow \beta = 1.1469 \text{ rads} = \underline{\underline{65.7°}}$



Q2 b)

Add on rotation.



Torque $= F.2b.$

Rotation is about shear centre.

$T = GJ \dfrac{d\theta}{dx}.$

$T =$ constant.

$\therefore \dfrac{d\theta}{dx} = \dfrac{\theta}{L}.$

$\therefore \quad F.2b = GJ.\dfrac{\theta}{L}$

$$\theta = \frac{F2b.L}{GJ.}$$

$$= \frac{F.2b\,L}{G.\,3bt^3} = \frac{2}{3}\frac{FL}{Gt^3}$$

3D4.

Q3 a) .i) $$M_{cr} = \frac{\pi}{L}\sqrt{GJEI_y}$$

$$J = \sum \frac{bt^3}{3} = \frac{(0.2)(0.01)^3}{3} = 6.667 \times 10^{-8}\ m^4$$

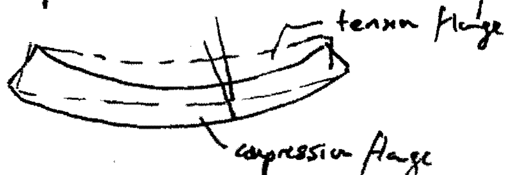
$$I_y = \frac{bt^3}{12} = \frac{1}{4} J \qquad (\therefore J = 4I)$$

$$M_{cr} = \frac{\pi}{L}\, 2EI \sqrt{\frac{G}{E}}$$

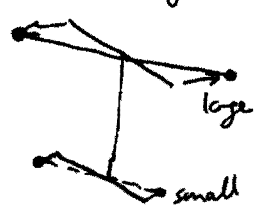
$$= \frac{\pi}{2.}\, 2.\, (210 \times 10^9)\left(\frac{6.667 \times 10^{-8}}{4}\right)\sqrt{\frac{81}{210}}$$

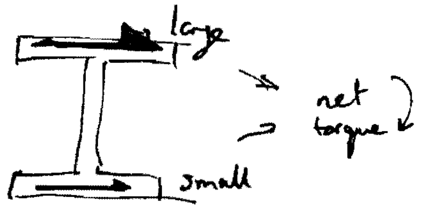
$$= 6828\ Nm = \underline{\underline{6.83\ kNm}}$$

a) ~~b~~.ii) Warping is deviation from the "plane sections remain plane" assumption. An I-beam undergoing lateral torsional buckling is liable to warp, as can be seen in plan.



Since the compression flange deflects further it has greater curvature than the tension flange, therefore original section is no longer planar.

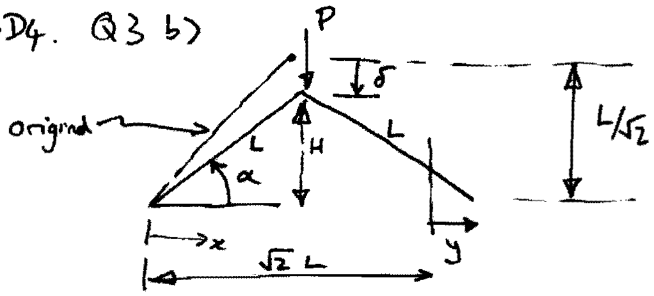




The differing moments in the two flanges lead to differing shear stresses, which have an unbalanced component causing a torque around the longitudinal axis. Consideration of this resistance to warping leads to an extra factor in LTB critical moment

$$M_{LTB} = \frac{\pi}{L}\sqrt{GJEI_y}\left[1 + \text{warping term}\right]^{1/2}$$

3D4. Q3 b)



$H = L\sin\alpha$

$\delta = \frac{L}{\sqrt{2}} - L\sin\alpha$.

$x = 2L\cos\alpha$

$y = 2L\cos\alpha - \sqrt{2}L$

Total Potential Energy

$$\Pi(\alpha) = \tfrac{1}{2}kL^2(2\cos\alpha - \sqrt{2})^2 - P\left(\frac{L}{\sqrt{2}} - L\sin\alpha\right)$$

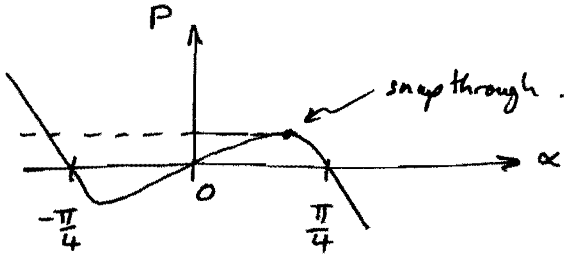
Equlib: $\frac{d\Pi}{d\alpha} = 0 = -\frac{1}{2}kL^2(2)(2\sin\alpha)(2\cos\alpha - \sqrt{2}) + PL\cos\alpha$

$$\therefore \quad P = 2kL\frac{\sin\alpha}{\cos\alpha}(2\cos\alpha - \sqrt{2})$$

$$= 2kL(2\sin\alpha - \sqrt{2}\tan\alpha)$$

Zero? $P = 0$ when i) $\tan\alpha = 0 \rightarrow \alpha = 0$

ii) $\cos\alpha = 1/\sqrt{2} \rightarrow \alpha = \pm\pi/4 = \pm 45°$ (initial condition).



Snap through: P_{cr} when $\frac{dP}{d\alpha} = 0$

$$P = 2kL(2\sin\alpha - \sqrt{2}\tan\alpha)$$

$$\frac{dP}{d\alpha} = 2kL\left(2\cos\alpha - \sqrt{2}\frac{1}{\cos^2\alpha}\right)$$

$\Rightarrow \quad \cos^3\alpha = \frac{1}{\sqrt{2}} \quad$ so $\quad \cos\alpha = \frac{1}{2^{1/6}} \quad \alpha = \cos^{-1}\left(\frac{1}{2^{1/6}}\right)$

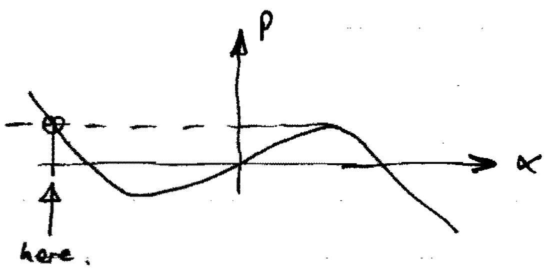
$\alpha = \cos^{-1}(0.8901) = 0.4715$ rads $= \underline{27.01°}$

$$P_{cr} = 2kL(2\sin(0.4715) - \sqrt{2}\tan(0.4715)) = \underline{\underline{0.3748\,kL}}$$

3D4

Q 3 b) cont'd

Other α

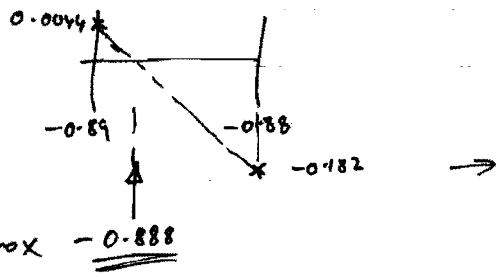


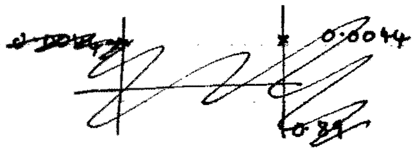
Need solutions to $P_{cr} = 2kL\left(2\sin\alpha - \sqrt{2}\tan\alpha\right)$

$$0.1874 = 2\sin\alpha - \sqrt{2}\tan\alpha$$

Solve numerically.

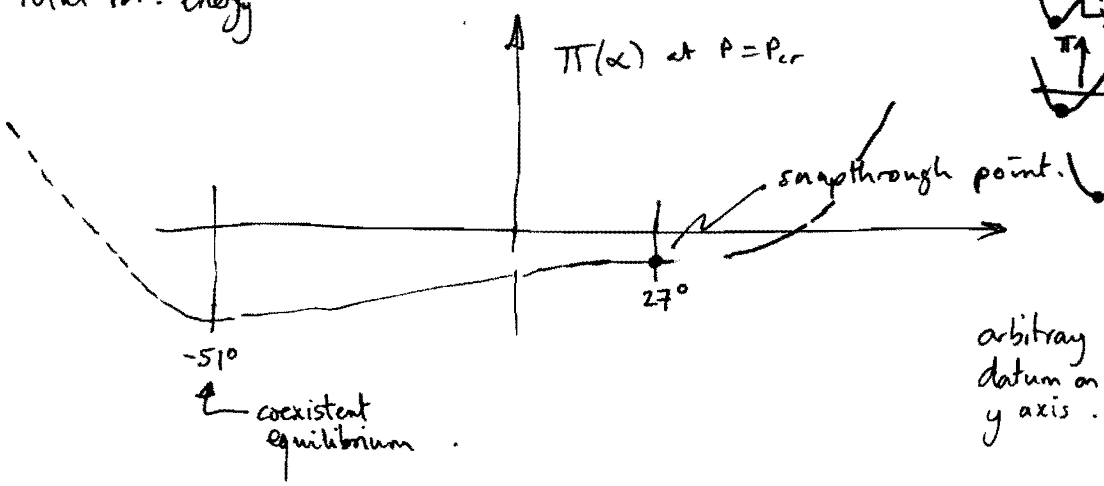
α	$2\sin\alpha$	$-\sqrt{2}\tan\alpha$	RHS	error
-1.0	-1.6829	2.2025	0.5196	
-0.9	-1.5667	1.7821	0.2155	
-0.89	-1.5541	1.7460	0.1918	0.0044
-0.88	-1.5415	1.7107	0.1692	-0.0182

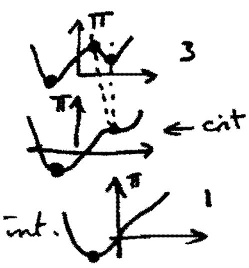




$\therefore$ Other $\alpha = -0.888$ rads $= -\underline{50.9^\circ}$

ii) Total Pot. Energy





Q3 b) iii).

Rayleigh-Ritz does not work for snapthrough problems such as this, as ~~it does~~ the problem does not satisfy the assumptions req'd by Rayleigh-Ritz

eg. – a demarcation between axial and lateral effects

– ~~later~~ axial displacements go as (lateral displacements)^2

so cannot partition ~~as~~ the total potential energy into

$$\Pi = \underbrace{\text{internal strain energy}}_{\sim\ \delta_{lateral}^2} \ \cancel{+} - \ \underbrace{\text{ext W.D}}_{\sim\ P_{axial} \times \delta_{axial}\ \sim\ P_{axial} \times \delta_{lateral}^2}$$

not true in this case.

3D4. 2013

Q4(a)

	I_{maj}	L	EI/L $\quad Nm^{-2}m^4/m = Nm$
356 x 127 x 33 UB	$8249 \times 10^{-8}\,m^4$	6	$210\times10^9 \times I/L = 2.887\times10^6$ Nm.
203 x 203 x 86 UC	$9449 \times 10^{-8}\,m^4$	3	$= 6.614\times10^6$ Nm.

$$\frac{k_{beam}}{k_{col}} = \frac{2.887}{6.614} = 0.4365$$

stiffness matrix.

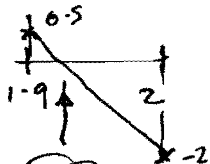
$$\begin{pmatrix} M_B \\ M_E \end{pmatrix} = \begin{bmatrix} k_c s + k_{top} & k_c sc \\ k_c sc & k_c s + k_{bot} \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$$

$$= K_{col.} \begin{bmatrix} s + 2(3)(0.4365) & sc \\ sc & s + 2(4)(0.4365) \end{bmatrix}$$

$$= \begin{bmatrix} s + 2.619 & sc \\ sc & s + 3.492 \end{bmatrix}$$

Buckles when det = 0.

P/P_E	s	c	sc	① (s + 2.619)	② (s+3.492)	①×②−(sc)²
0	4	1/2	2	6.619	7.492	45.6.
1	2.5	1	2.5	5.119	5.992	24.42.
1.5	1.4	2	2.8	4.019	4.892	11.82
2	0.1	[illegible]	3.5	2.719	3.592	−2.483
1.9.	0.4		3.35	3.019	3.892	+0.5.



Conclude $P/P_E \sim 1.92$

~1.92.

(1.923 by Matlab).

$$\therefore \frac{\pi^2 EI / L_{eff}^2}{\pi^2 EI / L^2} = 1.92 \Rightarrow L_{eff} = \frac{L}{\sqrt{1.92}}$$

$$= \frac{3}{\sqrt{1.92}} = \underline{\underline{2.17\,m}}$$

Effective length.

Q4 a) cont'd.

$$P_{crit} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 (210\times10^9)(4449\times10^{-8})}{(2.17)^2}$$

$$= \underline{41.6\ MN}$$

b) Need relative magnitudes of rotations at B and E

At $P = P_{crit}$

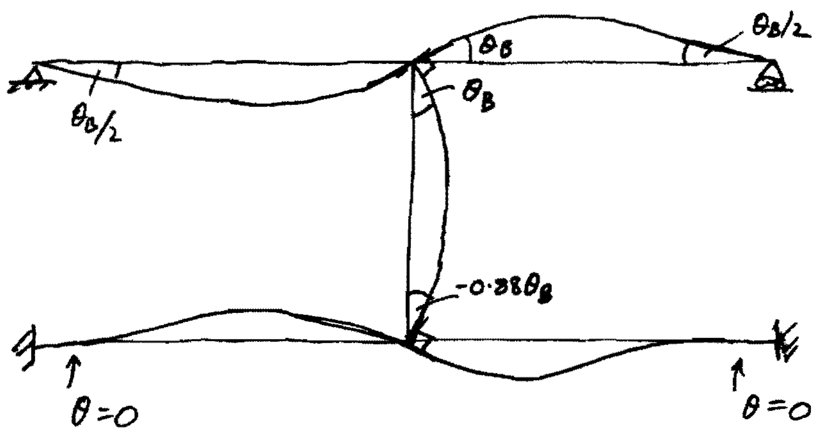
$s \approx 0.35$, $sc \approx 3.36$

$$\begin{bmatrix} 2.969 & 3.36 \\ 3.36 & 3.842 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_E \end{bmatrix} = \begin{bmatrix} M_B \\ M_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

no applied external couples at B, E joints

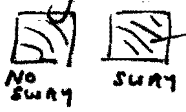
1st equation $\quad 2.969\,\theta_B + 3.36\,\theta_E = 0$

$$\therefore \quad \theta_E = -\frac{2.969}{3.36}\,\theta_B = -0.88\,\theta_B$$



3D4. Q4 (c).

If A, C on rollers, get sway. Elastic buckling load would be greatly reduced. Would need to use s_1, c_1 sway stability factors (or the sway effective length graphs.



— much greater effective lengths.)

Sway restraint can be provided by diagonal bracing, or by the floors/beams connecting to a stiff concrete services core (such as containing lift shaft and emergency stairs) — or the floor plates may connect to shear walls which provide lateral restraint in the correct direction.

d) $$P_{squash} = \underline{3905 \text{ kN}}. \quad \left(= 355 \times 10^6 \text{ N/m}^2 \times 110 \times 10^{-4} \text{ m}^2\right)$$

This is much less than $P_{Elastic,maj} = 41{,}600$ kN.

The two would typically be combined in practice using Perry-Robertson's formula (based on first yield) to produce a design capacity less than both $P_{plastic}$ and $P_{elastic}$.

Since $P_{elastic} > P_{plastic}$ for this column, it ~~fig~~ follows that $P_{perry\text{-}rob}$ will be marginally less than $P_{plastic}$

so rough estimate would be say $P_{Design} = \underline{3800 \text{ kN}}$.