2013 IIA 3D5 - Water Engineering Dr D. Liang

1.

(a.i) The total volume of water generated by the rainfall at the catchment outlet is:

$$(4.2+7.6+5.2+3.6+2.2+1.2)\times 3\times 3600 = 259{,}200\ \text{m}^3$$

This should be equal to the volume of the rainfall.

$$3\times 10\times 10^{-3}\times A = 259{,}200$$

$$A = 8.64\times 10^{6}\ \text{m}^2 = 8.64\ \text{km}^2$$

(a.ii)

According to the unit hydrograph theory:

Duration (h)	0-3	3-6	6-9	9-12	12-15	15-18	18-24	24-27	27-30
Discharge due to 3am-6am rain	8.4	15.6	10.0	7.2	4.4	2.4	0.0	0.0	0.0
Discharge due to 6am-9am rain		8.4	15.6	10.0	7.2	4.4	2.4	0.0	0.0
Discharge due to 12pm-3pm rain				12.6	23.4	15.0	10.8	6.6	3.6
Total ($\text{m}^3\ \text{s}^{-1}$)	8.4	24.0	25.6	29.8	35.0	21.8	13.2	6.6	3.6

The peak flow rate is **35.0 $\text{m}^3\ \text{s}^{-1}$**, which occurs 12 hours after the rain starts, i.e. at 3pm-6pm.

(a.iii)

Convert the discharges by the three-hour rainfall into percentages.

Duration (h)	0-3	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Discharge (%)	17.5	32.5	20.8	15.0	9.2	5	0.0	0.0

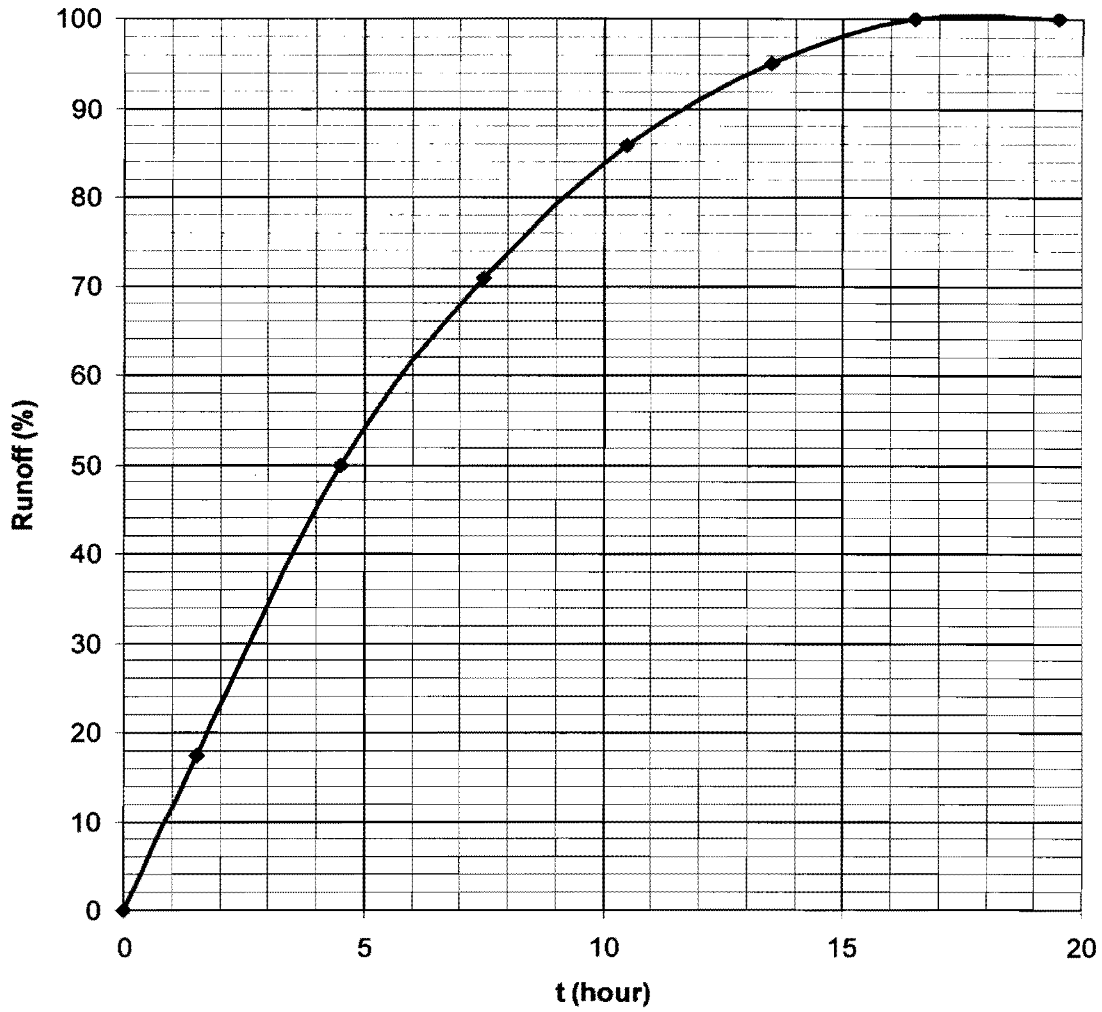
Construct the S curve (see the figure on the next page):

Time (hour)	0	1.5	4.5	7.5	10.5	13.5	16.5	19.5
Runoff (%)	0	17.5	50	70.8	85.8	95	100	100

Need to find the runoff proportions due to a one-hour uniform rainfall.

Read from S curve:

Time (hour)	0	1	3	5	7	9	11	13	15	17
S-curve value	0	12	35	54	68	79	88	94	98	100
Shift S-curve by 2 hours		0	12	35	54	68	79	88	94	98
2-hour hydrograph	0	12	23	19	14	11	9	6	4	2



(b)

It is a process to remove extremely small suspended solids in the water (colloids). Colloids have negligible settling velocities, and they often have negative charges. Coagulants (e.g. $AlSO_4$) are introduced to induce particle agglomeration into flocs, which can then be removed by sedimentation and filtration.

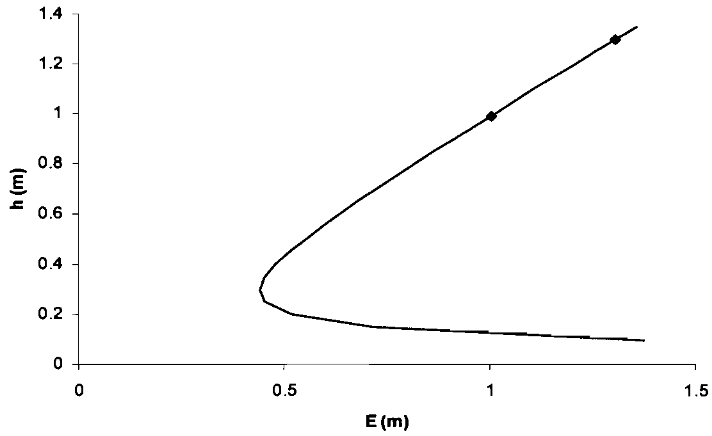
2 (a)

The flow varies rapidly close to the excavated area, so neglect the energy loss.

Specific energy in front of the excavated area: $h + \frac{(q/h)^2}{2g} = 0.99 + \frac{(0.5/0.99)^2}{2 \cdot 9.81} = 1.003$ m

Specific energy in the excavated area: 1.003+0.3 = 1.303 m

Plot specific energy vs. water depth:



Flow is subcritical, and water depth above the excavated area is 1.3 m.

The velocity is $\frac{0.5}{1.3} = 0.38$ m/s

(b.i)

Manning formula: $U = \frac{1}{n} \cdot h_0^{2/3} \cdot S_b^{1/2}$

$$q = U \cdot h_0 = \frac{1}{n} \cdot h_0^{5/3} \cdot S_b^{1/2}$$

$$4.646 = \frac{1}{0.02} \cdot h_0^{5/3} \cdot 0.0003^{1/2}$$

$$h_0 = 2.74 \text{ m}$$

$$U = \frac{4.646}{2.74} = 1.70 \text{ m/s}$$

$$Fr = \frac{U}{\sqrt{gh_0}} = \frac{1.7}{\sqrt{9.81 \cdot 2.74}} = 0.328 < 1.0$$

So, the flow is subcritical.

(b.ii)

Gradually varied flow: $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$

When h = 0.457 m: $U = \frac{4.646}{0.457} \approx 10.17$ m/s

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.02^2 \cdot 10.17^2}{0.457^{4/3}} \approx 0.1175$$

$$Fr = \frac{U}{\sqrt{gh}} = \frac{10.17}{\sqrt{9.81 \times 0.457}} \approx 4.803$$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.0003 - 0.1175}{1 - 4.803^2} \approx 0.0053$$

When h = 0.5 m: $U = \frac{4.646}{0.5} = 9.292 \text{ m/s}$

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.02^2 \cdot 9.292^2}{0.5^{4/3}} \approx 0.0870$$

$$Fr = \frac{U}{\sqrt{gh}} = \frac{9.292}{\sqrt{9.81 \times 0.5}} \approx 4.196$$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.0003 - 0.0870}{1 - 4.196^2} \approx 0.0052$$

So $\frac{0.5 - 0.457}{\Delta x} = \frac{0.0053 + 0.0052}{2}$

$\Delta x = 8.19 \text{ m}$

Or $$\frac{0.5 - 0.457}{\Delta x} = \frac{0.0003 - \frac{0.1175 + 0.0870}{2}}{1 - \left(\frac{4.803 + 4.196}{2}\right)^2} = 0.00530$$

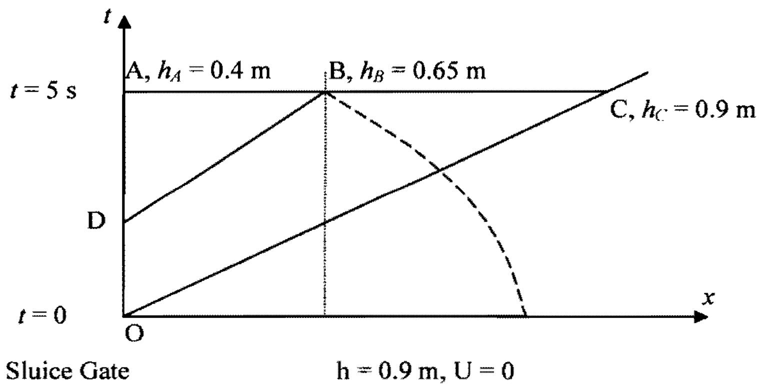
$\Delta x = 8.11 \text{ m}$

Or using $$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

$$\frac{\left(0.5 + \frac{9.292^2}{2 \times 9.81}\right) - \left(0.457 + \frac{10.17^2}{2 \times 9.81}\right)}{\Delta x} = 0.0003 - \frac{0.1175 + 0.0870}{2}$$

$\Delta x = 8.12 \text{ m}$

(c.i)



The positive characteristic OC divides the moving water and still water.

Positive line OC is straight: $\frac{dx}{dt} = U_O + \sqrt{gh_O}$

$$\frac{x_C - 0}{5 - 0} = 0 + \sqrt{9.81 \times 0.9} \quad \Rightarrow \quad \mathbf{x_C = 14.86\ m}$$

(c.ii) Along the negative characteristic through B: $\left(U - 2\sqrt{gh}\right) = const$

$$U_B - 2\sqrt{9.81 \cdot 0.65} = 0 - 2\sqrt{9.81 \times 0.9} \quad \Rightarrow \quad U_B = -0.89 \text{ m/s}$$

Along the positive characteristic through B, depth and velocity remain constants, so

$h_D = 0.65$ s => $t_D = 2.5$ s (The water depth drops linearly at sluice gate.)

Positive line DB is straight: $\frac{dx}{dt} = U_B + \sqrt{gh_B}$

$$\frac{x_B - 0}{5 - 2.5} = -0.89 + \sqrt{9.81 \times 0.65} \quad \Rightarrow \quad \mathbf{x_B = 4.09\ m}$$

3 (i)

Total shear: $\tau_b = \rho g R_h S_b = 1000\times 9.81\times 2\times 4\times 10^{-4} = 7.848$ **Pa**

Grain-related roughness: $k_s{}' = 2d = 2\times 0.3\times 10^{-3} = 6\times 10^{-4}$

Grain-related Chezy: $C' = 7.8\ln\left(\dfrac{12h}{k_s{}'}\right) = 7.8\ln\left(\dfrac{12\times 2}{6\times 10^{-4}}\right) = 82.65\ \text{m}^{1/2}\ \text{s}^{-1}$

Grain-related shear: $\tau_b{}' = \dfrac{g}{C'^2}\rho\cdot U^2 = \dfrac{9.81}{82.65^2}\cdot 1000\cdot 1^2 = 1.436$ **Pa**

Form-related shear: $\tau_b{}'' = \tau_b - \tau_b{}' = 7.848 - 1.436 = 6.412$ **Pa**

(ii)

For uniform flow: $U = C\sqrt{R_h S_b}$

$$C = \frac{U}{\sqrt{R_h S_b}} = \frac{1}{\sqrt{2\times 4\times 10^{-4}}} = 35.355\ \text{m}^{1/2}\ \text{s}^{-1}$$

$$C = 7.8\ln\left(\frac{12h}{k_s}\right) \Rightarrow \mathbf{k_s = 0.258\ m}$$

$$k_s{}'' = k_s - k_s{}' \approx k_s$$

(iii)

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{gR_h S_f} = \sqrt{9.81\times 2\times 4\times 10^{-4}} = 0.0886\ \text{m/s}$$

$$u_*{}' = \sqrt{\frac{\tau_b{}'}{\rho}} = \sqrt{\frac{1.436}{1000}} = 0.0379\ \text{m/s}\ \text{(This value is not needed.)}$$

$$d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.3\times 10^{-3}\times\left(\frac{9.81\times(2.65-1)}{10^{-12}}\right)^{1/3} = 7.589$$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049\cdot d_*^3} - 10.36\right]$$

$$w_s = \frac{10^{-6}}{0.3\times 10^{-3}}\left[\sqrt{10.36^2 + 1.049\times 7.589^3} - 10.36\right] = 0.0448\ \text{m/s}$$

$$\theta' = \frac{\tau_b{}'}{g(\rho_s - \rho)d} = \frac{1.436}{9.81\times(2650-1000)\times 0.3\times 10^{-3}} = 0.296$$

$$\bar{c}(2d) = \frac{0.331\cdot(\theta'-0.045)^{1.75}}{1+0.72\cdot(\theta'-0.045)^{1.75}} = \frac{0.331\cdot(0.296-0.045)^{1.75}}{1+0.72\cdot(0.296-0.045)^{1.75}} = 0.0277$$

This is the volume concentration.

According to $$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

Take $a = 2d = 0.6\times 10^{-3}$ m and $z = 1$ m.

$$\frac{\bar{c}(1)}{\bar{c}(0.6\times 10^{-3})} = \left(\frac{2-1}{1}\cdot\frac{0.6\times 10^{-3}}{2-0.6\times 10^{-3}}\right)^{\frac{0.0448}{0.4\times 0.0886}} = 3.55\times 10^{-5}$$

So, the concentration at mid-depth is:

$$\bar{c}(1) = 3.55 \times 10^{-5} \times 0.0277 \times 2650 = \mathbf{0.0026\ kg/m^3}$$

(iv)

This is a two-dimensional continuous-release problem.

$D_{ty} = 0.15 h u_* = 0.15 \times 2 \times 0.0886 = 0.02658\ \text{m}^2/\text{s}$

Considering the image of one channel bank:

$$\bar{c}(x, y) = 2 \times \frac{\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{y^2}{4D_y\, x/U}\right)$$

Knowing $\bar{c}(x = 50, y = 1) = 10\ \text{mg/l} = 0.01\ \text{kg/m}^3$:

$$0.01 = 2 \times \frac{\dot{M}/2}{1 \times \sqrt{4 \times 3.14 \times \frac{50}{1} \times 0.02658}} \exp\left(-\frac{1^2}{4 \times 0.02658 \times 50/1}\right)$$

$$\dot{M} = 0.049\ \ \text{kg/s}$$

The total mass discharged in a day is: 0.049×24×3600 = **4233.6 kg = 4.2 tonne**

4. (a.i)

$$H_f = \lambda \frac{L}{D}\frac{U^2}{2g} = \lambda \times \frac{20000}{0.4} \times \frac{\left(\frac{Q}{3.14\times 0.4^2/4}\right)^2}{2\times 9.81} = \lambda \cdot 161544 \cdot Q^2$$

$$H_l = \sum \zeta \frac{U^2}{2g} = 10 \times \frac{\left(\frac{Q}{3.14\times 0.4^2/4}\right)^2}{2\times 9.81} = 32.31 \cdot Q^2$$

k_s/D = 0.16/400 = 0.0004

Assume the flow is hydraulically rough, $\lambda = 0.016$ from Moody diagram.

So, $H_f = 0.016 \cdot 161544 \cdot Q^2 = 2584.70 \cdot Q^2$

$160 = H_f + H_l = 2617.01 \cdot Q^2$ => Q = 0.24726 m^3/s

$$U = \frac{Q}{3.14 \times 0.4^2/4} = 1.97 \text{ m/s}$$

$$\text{Re} = \frac{UD}{\nu} = \frac{1.97 \cdot 0.4}{10^{-6}} = 7.88 \times 10^5$$

From Moody diagram, $\lambda = 0.0165$

So, $H_f = 0.0165 \cdot 161544 \cdot Q^2 = 2665.476 \cdot Q^2$

$160 = H_f + H_l = 2697.8 \cdot Q^2$ => Q = 0.2435 m^3/s

$$U = \frac{Q}{3.14 \times 0.4^2/4} = 1.94 \text{ m/s}$$

$$\text{Re} = \frac{UD}{\nu} = \frac{1.94 \cdot 0.4}{10^{-6}} = 7.76 \times 10^5$$

From Moody diagram, λ is still around 0.0165. So, **Q is about 243.5 litre/s.**

If λ is taken to be 0.017, then Q can be calculated to be **240 liter/s**.

(a.ii)

$$U = \frac{0.10}{3.14 \times 0.4^2/4} = 0.796 \text{ m/s}$$

$$\text{Re} = \frac{UD}{\nu} = \frac{0.796 \cdot 0.4}{10^{-6}} = 3.2 \times 10^5$$

From Moody diagram, $\lambda = 0.0176$

$$H_f = \lambda \frac{L}{D}\frac{U^2}{2g} = 0.0176 \times \frac{20000}{0.4} \times \frac{0.796^2}{2\times 9.81} = 28.4 \text{ m}$$

$H_l = 160 - H_f = 131.6$ m

$$H_{valve} = H_l - 10\frac{U^2}{2g} = 131.6 - 10\frac{0.796^2}{2\times 9.81} = 131.3 \text{ m}$$

$$P_{valve} = \rho g Q H_{valve} = 1000 \times 9.81 \times 0.1 \times 131.3 = 128805 \text{ W} = \mathbf{128.8\ kW}$$

(b.i)

This problem needs to solve simultaneously the head-discharge relationships for the pump and pipeline.

For the pipeline, the head required to produce a discharge Q is:

$H = H_{st} + H_f + H_l$

$H_{st} = 85\text{-}52 = 33$ m

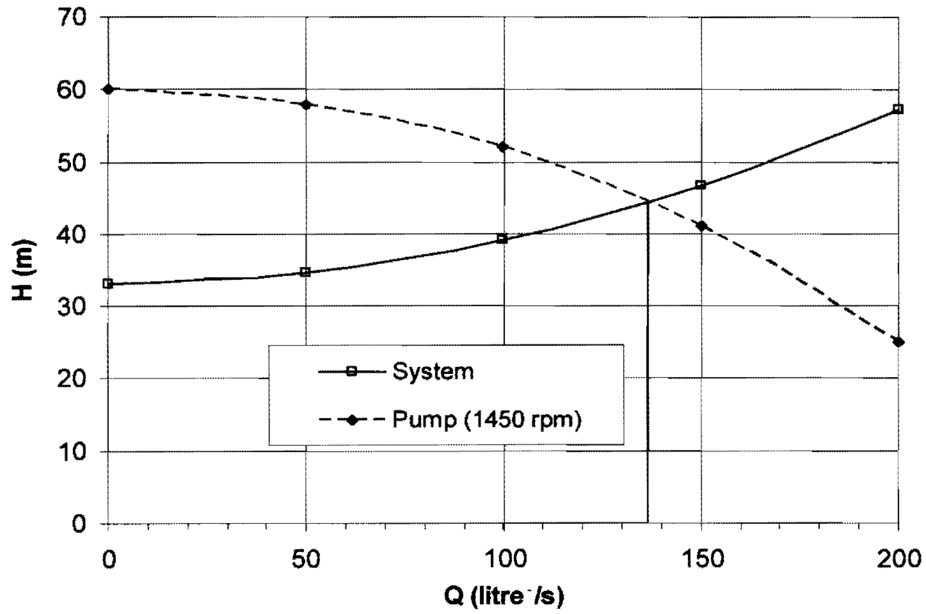
$$H_f = \lambda \frac{L}{D}\frac{U^2}{2g} = 0.0175 \times \frac{2000}{0.35} \times \frac{\left(\frac{Q}{3.14 \times 0.35^2/4}\right)^2}{2 \times 9.81} = 551.2Q^2$$

$$H_l = \sum \zeta \frac{U^2}{2g} = 10 \times \frac{\left(\frac{Q}{3.14 \times 0.35^2/4}\right)^2}{2 \times 9.81} = 55.1Q^2$$

System curve: $H = 33 + 551.2Q^2 + 55.1Q^2 = 33 + 606.3Q^2$

Q (litre/s)	0	50	100	150	200
System	33	34.51575	39.063	46.64175	57.252

Plot the system curve and the pump curve (see figure): **Q = 137 litre/s**



(b.ii)

Transform the working conditions of the pump at 1450 rpm to the equivalent conditions at 1200 rpm. For the same pump, D_p is the same. At the equivalent running conditions, the following two parameters should be the same.

$$\frac{Q_p}{N_p}, \frac{H_p}{N_p^2}$$

At 1450 rpm (given in the question):

Discharge (litre s^{-1})	0	50	100	150	200
Total head (m)	60	58	52	41	25
Efficiency (%)	0	44	65	64	48

At 1200 rpm ($\frac{1200}{1450}$ times the discharge given, $\left(\frac{1200}{1450}\right)^2$ times the head given):

Discharge (litre s^{-1})	0	41.4	82.8	124.1	165.5
Total head (m)	41.1	39.7	35.6	28.1	17.1
Efficiency (%)	0	44	65	64	48

Plot the system curve and the pump curve (see figure):

Q = 76 litre/s, H = 36.5 m, η = 63%

$P_p = \rho g Q_p H_p / \eta = 1000 \times 9.81 \times 0.076 \times 36.5 / 0.63 = 43195$ W = **43 kW**