ENGINEERING TRIPOS PART IIA

Tuesday 23 April 2013 9.30-12.30

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

- *3A1 Data Sheet for Applications to External Flows (2 pages);*
- *Boundary Layer Data Card (1 page);*
- *Incompressible Flow Data Card (2 pages).*

STATIONERY REQUIREMENTS
Single sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: Final

1 (a) An incompressible, irrotational flow has velocity $\underline{u} = (Ax, -Ay)$, with A a positive constant.

(i) Derive the streamfunction.

(ii) Sketch the streamlines in the entire $x-y$ plane. Include indications of the flow direction.

(iii) Explain carefully why your streamfunction could alternately represent an inviscid flow with a boundary along $y = 0$ ("stagnation point flow"). [40%]

(b) For a real fluid, with non-zero viscosity, the stagnation point flow has a rotational region near the boundary. Here it obeys the Navier-Stokes equation:

$$\underline{u}.\nabla\underline{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\,\underline{u}$$

Assuming that the streamfunction takes the form $\psi = xf(y)$

(i) show that $\partial p/\partial y$ is independent of x, and hence find the expression for $\partial p/\partial x$ that is consistent with the outer, irrotational flow;

(ii) find the ordinary differential equation that $f(y)$ satisfies. [40%]

(c) The thickness, δ , of the rotational region is defined as the value of y at which the x-component of velocity reaches 99% of its free-stream value. Find an expression which could be solved for δ. How does δ vary with x? [20%]

2 (a) State the Kutta condition for an aerofoil and explain briefly how it is applied in determining the lift on a flat plate in irrotational flow. [20%]

(b) Figure 1(a) shows a cylinder of radius R in the $z = x + iy$ plane, centered at $z = ih$ and passing through points $z = \pm a$. The transformation

$$\zeta = z + a^2/z$$

maps this cylinder into the curved plate shown in Fig.1(b).

(i) Show that

$$a - ih = Re^{-i\theta_o}$$

where θ_o is the angle defined in Fig.1(a).

(ii) The flow around the cylinder has free-stream velocity U at an angle α to the horizontal. Confirm that

$$F(z) = Uze^{-i\alpha} + \frac{UR^2}{z - ih}e^{i\alpha} - \frac{i\Gamma}{2\pi}\ln(z - ih)$$

is a valid complex potential for this flow (where Γ is a real constant). What does Γ represent?

(iii) Find an expression for the lift on the curved plate when it is placed in a flow of speed U at incidence α. [60%]

(c) Comment qualitatively on the flow velocity at the leading edge of the curved plate predicted by this analysis. Explain what would happen there in practice, and find the unique value of α for which this phenomenon is avoided. [20%]

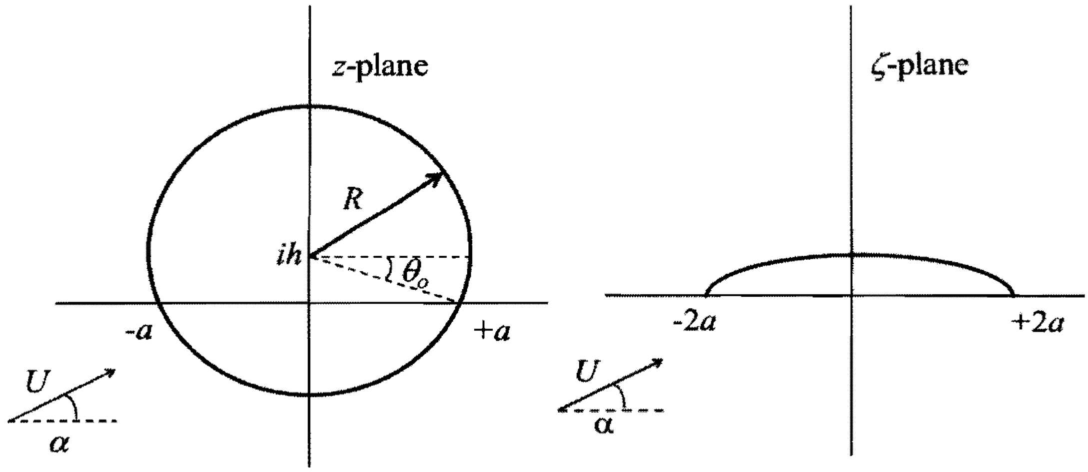

Fig. 1(a) Fig. 1(b)

TURN OVER

3 (a) State Kelvin's theorem and explain why it implies that an inviscid flow which is initially irrotational remains irrotational. [20%]

(b) Figure 2 shows a river entering a circular bend. The bulk of the upstream flow is irrotational and viscous effects in the bend can be neglected. After a short transition region the flow becomes independent of the azimuthal angle, θ.

(i) Show that the streamlines are circular.

(ii) How does the flow speed depend on the radial position, r. [40%]

(c) On the bed of the river there is a rotational, boundary layer region. Indicate on a sketch how the vortex lines associated with this region evolve as the river traverses the bend. Describe the resulting secondary flow. [40%]

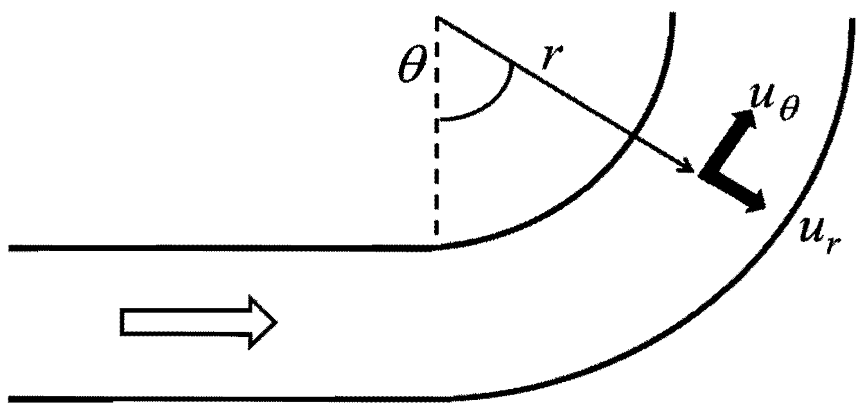

Fig. 2

4 The shear stress in a turbulent boundary layer is defined by

$$\frac{\tau}{\rho} = -\overline{u'v'} + \nu\frac{d\bar{u}}{dy}$$

where $\bar{u}$ is the local time-averaged mean velocity at a wall-normal distance y and ν is the kinetic viscosity.

(a) Explain the physical significance of the turbulent shear stress term. [Hint: consider the sign.] [40%]

(b) Consider a turbulent boundary layer in a zero pressure gradient flow. In a region close to the wall the velocity profiles collapse according to the "law of the wall" scaling:

$$\frac{\bar{u}}{U_\tau} = f\left(\frac{yU_\tau}{\nu}\right)$$

where $U_\tau = \sqrt{\tau/\rho}$ is the wall shear stress velocity scale and f is an unknown function.

(i) Use the turbulent boundary layer equations to evaluate the function f as $y \to 0$. What name is given to this region?

(ii) Further away from the wall the velocity profile is dominated by $-\overline{u'v'}$. This region of the flow can be characterised by the following dimensionless group

$$A = \frac{y}{U_\tau}\frac{d\bar{u}}{dy}$$

Using this, and the law of the wall, evaluate the function f to show

$$\frac{\bar{u}}{U_\tau} = A\ln\left(\frac{yU_\tau}{\nu}\right) + C$$

Note the wall coordinate is defined by $y^+ = yU_\tau/\nu$. [60%]

TURN OVER

5 (a) Consider classical, finite wing theory and derive the corresponding *lifting line equation* relating circulation to the effective angle of attack. [40%]

(b) For an *elliptic* loading distribution use the result from Part (a) to describe how:

(i) the chord varies at constant wing twist;
(ii) the twist varies at constant chord;
(iii) the desired loading distribution is achieved if both chord and twist are constant. [20%]

(c) If the elliptically loaded wing has linear taper with the tip chord half that at the wing root, derive an expression for the spanwise variation of twist and sketch it. [20%]

(d) If the elliptically loaded wing has constant twist, derive an expression showing how the lift slope vs. angle of attack is modified. What is the effect of the wing aspect ratio? [20%]

6 (a) Consider two-dimensional thin airfoil theory. For a symmetric, chordwise thickness distribution, $t(x)$, write down boundary conditions in terms of the y-wise velocity perturbation, v', and free stream velocity U. [20%]

(b) The thickness is represented by a chordwise distribution of source and sinks with strength $\sigma(x)$ per unit length. How is $\sigma(x)$ related to the thickness distribution? [20%]

(c) Write down an expression for the x-wise velocity perturbation, u', in terms of $\sigma(x)$. [20%]

(d) Given that the thickness distribution can be represented by the following

$$t = c\sum_{n=1}^{\infty} \tau_n \sin(n\theta)$$

derive a corresponding relationship for u'/U. [20%]

(e) Given the following values

$\tau_1 = 0.0780$; $\tau_2 = -0.0256$; $\tau_3 = -0.0017$; $\tau_4 = -0.0013$

compute the pressure coefficients at the airfoil leading and trailing edges and sketch the variation in-between. Comment on the result. [20%]

7 (a) Consider a two-dimensional laminar boundary layer on a flat plate and derive the momentum integral equation either using an integral approach or by integrating the boundary layer equations away from the wall, $y = 0$, to $y = h$ (where $u = U$ at $y = h$). [40%]

(b) The velocity distribution in a laminar boundary layer can be approximated by

$$\frac{u}{U} = \eta(2 - \eta)$$

where $\eta = y/\delta^*$ and δ^* is the displacement thickness. Find the momentum thickness θ in terms of δ^* and an expression for the local skin friction coefficient c_f'. [40%]

(c) Find an expression for the variation of δ^* with respect to x assuming a zero pressure gradient. [20%]

8 (a) Sketch the basic flow (in side view) you expect for the three fundamental car shapes: hatchback, saloon and fastback. Assume that the car in each category has been designed for good aerodynamic resistance. [30%]

(b) For each of the shapes of Part (a) briefly list the pros and cons. Why might hatchback cars be so popular? [20%]

(c) Which geometric feature determines whether the flow over the rear of a car is typical for a fastback or hatchback shape? What would you expect the geometry to be in an efficient fastback shape? [30%]

(d) What aerodynamic detailing can be applied to the rear of a hatchback car to improve the drag? [20%]

END OF PAPER