ENGINEERING TRIPOS PART IIA

Thursday 25 April 2013   9:30 to 12:30

Module 3A3

FLUID MECHANICS II

Answer not more than ***five*** *questions.*

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Compressible Flow Data Book (38 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A convergent-divergent nozzle has a cross-sectional area that increases linearly with distance downstream of the throat. The area of the nozzle exit is 20% greater than that of the throat. The upstream reservoir conditions are constant, and the flow through the nozzle is frictionless and adiabatic. The working fluid is air with $\gamma = 1.4$.

(i) Sketch the pressure distribution along the length of the nozzle as the back pressure is reduced. [20%]

(ii) Calculate the ratio of exit static pressure to reservoir stagnation pressure when the throat is first choked. [10%]

(iii) Calculate the ratio of exit static pressure to reservoir stagnation pressure when the flow downstream of the throat is fully supersonic and there are no shock waves. [10%]

(b) The nozzle is operated with a 1% loss of stagnation pressure between reservoir and exit. Calculate the location of the normal shock wave in the nozzle. [30%]

(c) What happens when the static pressure at nozzle exit falls between the value calculated in part (iii) and 67% of the reservoir stagnation pressure? [30%]

2 (a) The impulse function F is defined as

$$F = A\left(P + \rho V^2\right)$$

where A is the area, P is the pressure, ρ is the density and V is the velocity. Using this definition, together with the equations of mass conservation and energy conservation for a one-dimensional constant-area adiabatic flow with friction (Fanno flow), show that

$$dF = PA\left(M^2 - 1\right)\frac{dV}{V}$$

where M is the Mach number. [40%]

(b) Describe how the velocity varies with distance downstream in a Fanno flow that is

(i) subsonic and
(ii) supersonic at entry.

Illustrate both cases on a T-s diagram. [30%]

(c) A pipe of constant inside diameter 0.1 m is supplied with air ($\gamma = 1.4$) at a stagnation pressure of 5 bar. Just inside the entrance of the pipe the static pressure is 4 bar, and at a distance of 8 m downstream the static pressure is 2.5 bar. The flow is adiabatic. Calculate the skin friction coefficient. [30%]

3 (a) For a stationary hydraulic jump, show that the velocity V and depth h on either side of the jump are related according to

$$V_1 V_2 = \frac{1}{2} g\left(h_1 + h_2\right)$$

where g is the acceleration due to gravity. [35%]

(b) Water flows steadily along a horizontal channel of constant width. The depth is constant at 0.8 m. A sluice gate is lowered suddenly into the channel which immediately becomes completely blocked. The depth on the upstream side of the sluice gate is now measured as 1.3 m. Calculate the velocity of the original flow before the gate was lowered. [35%]

(c) Calculate the new depth immediately downstream of the sluice gate. [30%]

Note that: $V + 2c$ is constant for left-running infinitesimal waves
$V - 2c$ is constant for right-running infinitesimal waves

4 A two-dimensional aircraft jet engine ramp intake is sketched in Fig 1. The intake is designed to produce a focussed two-shock system for external compression at an inflow Mach number of 2.00 . Conditions may be assumed to be uniform in the direction into the page. Assume that the flow upstream of the intake is not affected by the aircraft fuselage and is parallel to the flight datum direction.

(a) (i) Draw a labelled sketch of the shock system for the design condition.

(ii) Calculate the static pressure recovery through the shock system.

(iii) Calculate the stagnation pressure loss through the shock system. [40%]

(b) (i) The aircraft now slows to a flight Mach number of 1.80 . A computer-controlled mechanism causes the intake ramp to translate parallel to the flight datum direction in order to preserve a focused shock system. Explain in which direction the ramp must translate and calculate the distance it must move, given that the cowl lip is 0.25 m from the flight datum direction.

(ii) Calculate the static pressure recovery through the new shock system.

(iii) Calculate the stagnation pressure loss through the new shock system.

(iv) Comment on these values compared with those calculated in section (a). [40%]

(c) If the intake ramp is locked in the original position that produces a focused shock system at $M = 2.00$, draw a carefully labelled sketch of the resulting shock system at $M = 1.80$ and briefly discuss its implications for both the engine and the airframe. [20%]

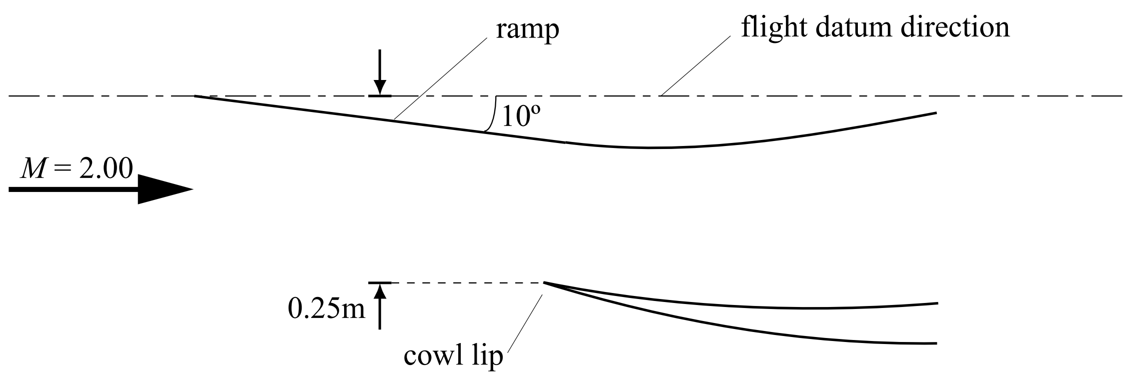

Fig. 1.

TURN OVER

5 A supersonic aircraft is flying at high altitude at a flight Mach Number, $M = 2.00$. Its wing consists of a section with wedge-shaped leading and trailing edges of $4°$ included angle and a flat bottom surface, as sketched in Fig 2. The wing is at $6°$ incidence to the oncoming flow. The flow can be considered to be two-dimensional.

(a) Draw a carefully labelled sketch of the shock / expansion system around the airfoil including a short length of its wake. [30%]

(b) If the ambient static air pressure is $2.0\ \text{Nm}^{-2}$, each wedge section of the airfoil is $0.5\ \text{m}$ long and the parallel portion is $1\ \text{m}$ in length, calculate:

(i) the section lift coefficient

(ii) the section pressure drag coefficient

(iii) the section pitching moment coefficient, evaluated about a point on the section centreline, a quarter of the distance from the leading to the trailing edge. [60%]

(c) As the aircraft slows to a subsonic flight speed, and assuming that the flow remains attached, do you expect the centre of lift to move fowards or aft? Briefly explain your answer and comment on the implications for flight safety. [10%]

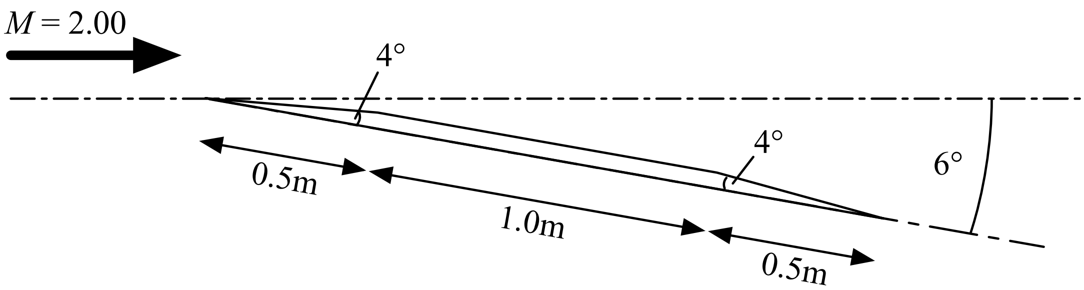

Fig. 2.

6 The unsteady flow of an incompressible viscous fluid between two long parallel plates is subject to the equation

$$\frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial y^2}$$

where V is the velocity, ν is the kinematic viscosity, t is the time and y is the distance perpendicular to the plates. A finite-difference approximation to this equation using the uniform discretisation $t^n = n\,\Delta t$ and $y_i = i\Delta y$ is given by

$$V_i^{n+1} = V_i^n + \frac{\nu \Delta t}{\Delta y^2}\left(V_{i+1}^n - 2V_i^n + V_{i-1}^n\right)$$

(a) Show that this approximation is first-order accurate in time and second-order accurate in space. [50%]

(b) Using discrete perturbation stability analysis, or otherwise, find the maximum stable time step for the method. [40%]

(c) Indicate how the stability limit might be improved. [10%]

7 *Note that part (a) of this question is not related to part (b).*

(a) Partial differential equations can be classed as hyperbolic, elliptic or parabolic. Give a simple example of an equation from each class, describe the relevant initial and boundary conditions, and explain how its signal propagation behaviour can be exploited to devise a numerical solution method. [50%]

(b) The flow through a single-stage axial-flow turbine has a constant axial velocity and the blade speed is U. The stator exit absolute flow angle is α_2 and the rotor exit relative flow angle is α_3^{rel}.

(i) Show that the stagnation enthalpy drop across the turbine, Δh_0, is related to the flow coefficient, ϕ, by:

$$\frac{\Delta h_0}{U^2} = \phi\left(\tan\alpha_2 - \tan\alpha_3^{rel}\right) - 1$$

and sketch this relationship. [25%]

(ii) At design operation, the axial velocity through the turbine is $250\ \mathrm{ms^{-1}}$ and the blade speed is $300\ \mathrm{ms^{-1}}$. If the stator exit absolute flow angle is $60°$ and the *absolute* flow angle downstream of the rotor is $-30°$, calculate the stage loading coefficient. Comment on this value. [10%]

(iii) Assuming that the axial velocity remains fixed at the design value, estimate the turbine blade speed at the aerodynamic limiting, no-load, over-speed condition. Comment on the practical implications of the value. [15%]

8 The flow through the tip section of the first rotor blade in an axial-flow compressor is at a constant radius of $0.5\ \mathrm{m}$. The compressor rotates at $4800\ \mathrm{rpm}$, the inlet stagnation pressure is $1.0\ \mathrm{bar}$, the inlet stagnation temperature is $280\ \mathrm{K}$ and there is no inlet swirl.

(a) The compressor inlet Mach number is 0.4 at the design operating conditions. Calculate the relative flow angle, the relative Mach number, the relative stagnation temperature and the relative stagnation pressure at the inlet to the rotor tip section. [20%]

(b) The rotor stagnation pressure loss coefficient for the tip section is 0.05 and the rotor relative exit flow angle is $-48°$ at the design operation. Assuming that the exit cross-sectional area of the annulus is 89.6% of the inlet area, calculate the relative stagnation pressure and the relative Mach number at the exit from the rotor tip section. [30%]

(c) Calculate the inlet stagnation to exit static pressure ratio for the tip section of the rotor blade. [10%]

(d) By calculating the absolute stagnation temperature and absolute stagnation pressure downstream of the rotor tip section determine the corresponding total-to-total isentropic efficiency. Comment on this value. [25%]

(e) Comment on the likely size of the inlet total to exit static pressure ratio across the rotor hub section compared with the calculated value for the tip section. Explain your reasoning. [15%]

You may assume that the working fluid has $R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$, $\gamma = 1.4$ and $c_p = 1005\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

END OF PAPER