## ENGINEERING TRIPOS PART IIA

Friday 3 May 2013 2.00 - 3.30

Module 3A5

## THERMODYNAMICS AND POWER GENERATION

Answer not more than three questions.

All questions carry the same number of marks.

The *approximate* percentage of marks allocated to each part of the question is indicated in the right margin.

There are no attachments.

STATIONERY REQUIREMENTS Single-sided script paper SPECIAL REQUIREMENTS Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final version

1 Figure 1 on the next page shows a schematic diagram of a chemical reformer that converts methane (CH<sub>4</sub>) to carbon monoxide and hydrogen within a power plant. The reformer is supplied with methane and (excess) steam in the ratio of 1 kmol of CH<sub>4</sub> to 6 kmol of H<sub>2</sub>O. At the reformer outlet, it may be assumed that there is no CH<sub>4</sub> present and the gas is a mixture of CO<sub>2</sub>, CO, H<sub>2</sub>O and H<sub>2</sub>. All streams enter and leave at a temperature of 1000 K and 1 bar pressure and the reaction proceeds isothermally with the heat required being supplied externally by a hot air stream also at 1000 K. It may be assumed that all species behave as ideal gases.

(a) Measurements show that for each kmol of  $CH_4$  supplied there are 0.366 kmol of CO in the outlet stream. Calculate the corresponding numbers of kmols of  $CO_2$ ,  $H_2$  and  $H_2O$  in the outlet stream. Hence, show that the gas mixture at outlet is in chemical equilibrium subject to the water-gas shift reaction,

$$-CO - H_2O + CO_2 + H_2 = 0$$
 [25%]

(b) With reference to the data given on the next page, show that  $\overline{Q}$  (the heat transferred from the air stream to the reformer per kmol of CH<sub>4</sub> supplied) is given by,

$$\overline{Q} = [\Delta \overline{H}_{1000}^{0}]_{ref} + 0.634 [\Delta \overline{H}_{1000}^{0}]_{shift}$$
[25%]

(c) With reference to the data given on the next page, derive an analogous expression for  $\Delta \overline{S}_{irrev}$ , the entropy created by irreversibility in the reformer per kmol of CH<sub>4</sub> supplied. In doing this, assume that the molar entropies of all gas species are to be evaluated at the mixture pressure of 1 bar rather than at their individual partial pressures (*i.e.*, neglect the entropy of mixing).

(d) With reference to the data given on the next page and assuming an environment temperature of 25 °C, calculate the maximum possible rational efficiency of a power plant supplied by methane at 25 °C and 1 bar which contains a reformer operating at the conditions considered above.

[20%]

[30%]

(cont.



Fig. 1

#### DATA FOR QUESTION 1

For the methane reforming reaction at 1000 K :  $CH_4 + H_2O \rightarrow CO + 3H_2$ 

 $[\Delta \overline{H}_{1000}^0]_{ref} = 225.70 \text{ MJ/(kmol CH_4)}, \quad [\Delta \overline{S}_{1000}^0]_{ref} = 0.2529 \text{ MJ/(kmol CH_4)/K}.$ 

For the water-gas shift reaction at 1000 K :  $CO + H_2O \rightarrow CO_2 + H_2$ 

 $[\Delta \overline{H}_{1000}^0]_{shift} = -34.74 \text{ MJ/(kmol CO)}, \quad [\Delta \overline{S}_{1000}^0]_{shift} = -0.0317 \text{ MJ/(kmol CO)/K}.$ 

For the oxidation of methane at 298.15 K :  $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$ 

$$\Delta \overline{G}_{298}^0 = -800.70 \text{ MJ/(kmol CH_4)}.$$

## **TURN OVER**

2 Note that parts (a) and (b) of this question are unrelated to each other.

(a) The Clausius-Clapeyron equation for the liquid-vapour equilibrium of a pure substance is,

$$\frac{dp_{sal}}{dT} = \frac{h_{fg}}{v_{fg}T} = \frac{h_g - h_f}{(v_g - v_f)T}$$

where subscripts f and g denote the saturated liquid and saturated vapour states, respectively. This equation may be used without proof.

At temperatures far below its critical temperature, the saturated vapour of a certain pure substance behaves as a perfect gas with specific gas constant R and isobaric specific heat capacity  $c_{pg}$ . Also,  $v_f$  may be neglected in comparison with  $v_g$  and  $h_{fg}$  is independent of temperature.

- (i) Derive the equation for the saturated vapour pressure  $p_{sat}$  as a function of T, subject to  $p_{sat} = p_{sat,0}$  at  $T = T_0$ . [15%]
- (ii) Derive the equation for the specific entropy of saturated vapour  $s_g$  as a function of T, subject to  $s_g = s_{g,0}$  at  $T = T_0$ . [25%]

(b) Oxygen gas  $O_2$  dissociates into atomic oxygen O according to the reaction,

$$O_2 \rightarrow (1-\alpha)O_2 + 2\alpha O$$

where  $\alpha$  is the degree of dissociation. It may be assumed that both O<sub>2</sub> and O behave as ideal gases.

A sample of oxygen is heated to a temperature where it is half dissociated. If the pressure is then reduced isothermally by a factor of 3, show that the volume increases by a factor of  $(2 + \sqrt{2})$ .

[60%]

3 A recuperated gas turbine cycle comprises a compressor, combustor, turbine and recuperator. The compressor and turbine have *polytropic* efficiency  $\eta$ , there are no losses of stagnation pressure in the combustor or recuperator, and the gas is assumed perfect with constant  $c_p$  and  $\gamma$  throughout. The *isentropic* temperature ratio of the compressor and turbine is  $r_t$  and the ratio of turbine inlet temperature  $T_3$  to compressor inlet temperature  $T_1$  is denoted by  $\theta = T_3/T_1$ .

(a) Explain the potential benefit of a recuperated gas turbine and sketch the cycle on a T-s diagram.

(b) Show that recuperation is only possible if,

$$\theta > r_t^{(\eta^2 + 1)/\eta} \tag{20\%}$$

[10%]

[10%]

(c) For a particular recuperated gas turbine with  $\eta = 0.9$ ,  $r_t = 2.0$ ,  $\theta = 5.5$  and a recuperator heat exchanger effectiveness  $\varepsilon = 0.8$ , evaluate the cycle efficiency. [40%]

(d) Explain, qualitatively, how the finite pressure drop in a real recuperator will affect the cycle efficiency.

(e) Suggest, with reasons, how the cycle efficiency of a recuperated gas turbine cycle could be increased further. [20%]

# **TURN OVER**

4 The steam cycle of a power plant comprises a feed pump, boiler, turbine and condenser. The boiler pressure is 150 bar and the temperature of the steam at exit from the boiler is 550 °C. The condenser pressure is 0.04 bar and the fluid is saturated liquid at condenser exit. The turbine has an isentropic efficiency of 85 %. Pressure losses in the boiler and condenser, and the work needed to drive the feed pump, may be neglected.

- (a) Calculate the cycle efficiency.
- (b) The cycle is modified by the addition of a single stage direct contact feedheater. A fraction *m* of the mass flow rate of steam at boiler exit is bled from the turbine at a pressure of 10 bar and is mixed with feed water at the same pressure. At exit from the feed heater, the water is saturated liquid. Assuming that the turbine expansion is a straight line on an enthalpy-entropy chart, evaluate:
  - (i) the mass fraction of bled steam m,

(ii)	the cycle efficiency.	[30%]
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(c) Stating your assumptions, evaluate the maximum possible cycle efficiency that could be achieved with a steady-flow device operating between the turbine entry and feed heater exit states used in (b). [25%]
(d) (i) Show that the overall efficiency of the power plant is given by the product of the boiler efficiency and the cycle efficiency. [5%]
(ii) Discuss briefly a strategy that is commonly employed for raising the

(ii) Discuss, briefly, a strategy that is commonly employed for raising the boiler efficiency.

## **END OF PAPER**

[25%]

[15%]