ENGINEERING TRIPOS PART IIA

Monday 6 May 2013 9:30 to 11:00

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final Version

1 A stream of fluid with mass flow rate $\dot{m}$ and specific heat capacity c flows under steady state through a circular tube of diameter d and length L . The fluid enters the tube at a temperature T_h and leaves at a temperature T_c . The heat is transferred to ambient air at temperature T_∞ , which flows perpendicularly to the tube. Under all conditions, assume that the heat transfer coefficient for the inside of the tube is h_i, and for the outside, it is h_a. The heat exchanger has two modes of operation. Under normal conditions (A), the tube operates dry on the outside. Under high cooling demand (B), the tube is covered with a film of liquid water on the outer surface, which is maintained at a constant thickness by replacing the evaporated mass. The diffusivity of water vapour in air is given as $\mathscr{D}$, and the thermal conductivity of air is λ_a. Assume that the tube and the water film formed have negligible thermal resistance.

(a) Consider the operation under dry conditions (A), and starting from an energy balance for the fluid in the tube, sketch the bulk temperature distribution as a function of tube length, and write an expression for T_c, and the total heat transfer rate, $\dot{Q}_A$, as a function of given inlet and ambient temperatures, $\dot{m}$, heat transfer coefficients, geometry and fluid properties. [40%]

(b) Consider the operation under wet conditions (B). Evaporation takes place at the surface by transfer of heat and mass with the surrounding air. Assume that the mass concentration of vapour at the surface is $\rho_{v,s}$, and that the surrounding air has a moisture concentration $\rho_{v,\infty}$, which are related to the temperatures by $\rho_{v,s} - \rho_{v,\infty} = \beta\,(T_s - T_\infty)$, where β is a constant. The heat of vaporisation of water is h_{lv} . Assume perfect analogy between heat and mass transfer, so that the Nusselt number $\mathrm{Nu} = h_m d/\mathscr{D}$, with h_m as the mass transfer coefficient.

(i) Derive an expression for the vapour mass transfer flux, G_v, and the corresponding heat flux by evaporation, q_v, as a function of the heat transfer coefficients, fluid properties and the surface and ambient temperatures. [30%]

(ii) Show that the total rate of heat transfer from the tube to the surroundings is given as

$$\dot{Q}_B = \dot{m}c(T_h - T_\infty)\left[1 - \exp\left(\frac{-U_B \pi d L}{\dot{m}c}\right)\right] \qquad \text{where} \qquad U_B = \left[\frac{1}{h_a\left(1 + \frac{\mathscr{D}}{\lambda_a}\beta h_{lv}\right)} + \frac{1}{h_i}\right]^{-1}$$

Discuss how the heat transfer is enhanced or suppressed by the wet operation. [30%]

Final Version

2 A spherical liquid droplet of radius R and surface temperature T_s is heated via microwave radiation, leading to energy dissipation at a rate of $\dot{g}$ per unit volume. The liquid droplet has density ρ, heat capacity c, and thermal conductivity λ_l, and is surrounded by still ambient air at temperature T_∞, with thermal conductivity λ_a.

(a) Considering the heat conduction inside the droplet only, show that the temperature distribution within the droplet is governed by

$$\rho c \frac{\partial T}{\partial t} - \lambda_l \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \dot{g}$$

Sketch a diagram indicating the terms in your derivation. [30%]

(b) Assuming steady state, show that the temperature within the droplet is given by [15%]

$$T(r) = T_s + \frac{1}{6}\frac{\dot{g}}{\lambda_l}(R^2 - r^2)$$

(c) Assuming steady state, obtain an expression for the distribution of the air temperature surrounding the droplet. Express your answer as a function of the surface temperature of the liquid, ambient temperature and radius of the droplet. [15%]

(d) Consider the evaporation of liquid under steady state conditions.

(i) Write the energy flux balance at the interface between droplet and air as a function of the local temperatures, their gradients and the fluid properties, assuming a heat of vaporisation of the liquid, h_{lv}, and an evaporative mass flux G_v. [20%]

(ii) Sketch the steady temperature around the surface of the droplet, indicating clearly the difference between cases with zero or non-zero evaporation. Assume $\lambda_l = 10\,\lambda_a$. [20%]

3 A solar collector is shown in Fig. ??. It is positioned at an angle of 60° to the horizontal. It receives collimated solar radiation flux, $G_S = 900\,\mathrm{Wm}^{-2}$, at an angle of 60° to the horizontal. The collector is constructed using a cover plate above an absorber surface. The absorber plate has a coating with absorptivity, $\alpha_{\lambda,a}$ of 0.9 below 2 μm and 0.3 at longer wavelengths. The cover plate is completely transparent ($\tau = 1$) to radiation below 2 μm and fully opaque ($\tau = 0$) with absorptivity $\alpha_{\lambda,c} = 0.75$ at longer wavelengths. These spectral characteristics are also shown in Fig. ??. Both surfaces are diffuse and grey at wavelengths longer than $\lambda = 2$ μm. The solar radiation can be approximated as a black body at $T_s = 5800\,\mathrm{K}$. The system is at steady state, supplying useful power, with the surface temperatures of the absorber plate and the cover plate equal to $T_a = 343$ K, and $T_c = 300$ K, respectively. The Stefan-Boltzmann constant is $\sigma = 5.670 \times 10^{-8}$ W m^{-2} K^{-4}.

(a) In the questions below, indicate approximate magnitudes in relation to an appropriate black body emission curve as a function of wavelength. You may use Wien's law, $\lambda_{max}T = 2897.8\,\mu\mathrm{m\,K}$.

(i) Sketch the black body emission, $E_{b,\lambda}$, for an emitter at the solar temperatures, and the incident solar irradiation $G_{i,\lambda}$. [20%]

(ii) Sketch the spectral emission for the absorber plate alone, $E_{a,\lambda}$, the collector plate alone, $E_{c,\lambda}$ and for the total radiation emitted by of the collector assembly, $E_{a+c,\lambda}$. [20%]

(iii) Briefly explain the purpose of the absorber coating and the cover plate with reference to the sketched curves. [10%]

(b) Calculate the gross solar energy flux absorbed by the absorber plate alone, $G_{a,\lambda}$. You may use black body radiation cumulative fractions the tabulated in Table ??. [20%]

(c) Assume that the net flux absorbed is entirely transformed into useful power, and the only losses in the system are due to radiation. Calculate the efficiency of the collector and comment on your answer. You may neglect both end effects and radiative losses below the collector. State any additional assumptions clearly. [30%]

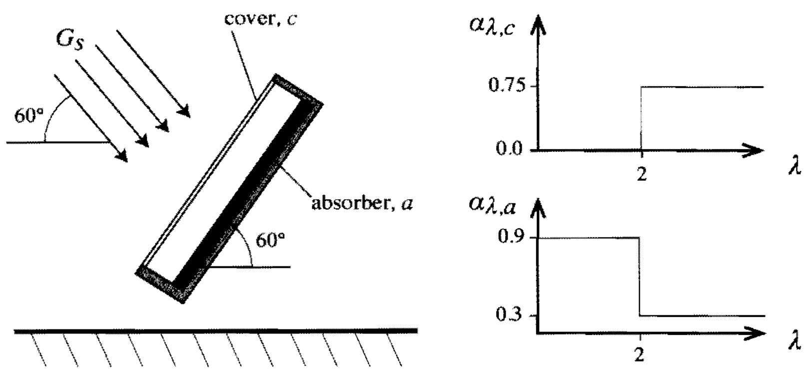

Fig. 1

Table 1

λT[μm K]	$F_{(0-\lambda)}$	λT[μm K]	$F_{(0-\lambda)}$	λT[μm K]	$F_{(0-\lambda)}$
400	0.000000	4,200	0.516014	8,500	0.874608
600	0.000000	4,400	0.548796	9,000	0.890029
800	0.000016	4,600	0.579280	9,500	0.903085
1,000	0.000321	4,800	0.607559	10,000	0.914199
1,200	0.002134	5,000	0.633747	10,500	0.923710
1,400	0.007790	5,200	0.658970	11,000	0.931890
1,600	0.019718	5,400	0.680360	11,500	0.939959
1,800	0.039341	5,600	0.701046	12,000	0.945098
2,000	0.066728	5,800	0.720158	13,000	0.955139
2,200	0.100888	6,000	0.737818	14,000	0.962898
2,400	0.140256	6,200	0.754140	15,000	0.969981
2,600	0.183120	6,400	0.769234	16,000	0.973814
2,800	0.227897	6,600	0.783199	18,000	0.980860
2,898	0.250108	6,800	0.796129	20,000	0.985602
3,000	0.273232	7,000	0.808109	25,000	0.992215
3,200	0.318102	7,200	0.819217	30,000	0.995340
3,400	0.361735	7,400	0.829527	40,000	0.997967
3,600	0.403607	7,600	0.839102	50,000	0.998953
3,800	0.443382	7,800	0.848005	75,000	0.999713
4,000	0.480877	8,000	0.856288	100,000	0.999905

4 Consider a two-dimensional steady flow of a constant density fluid at temperature T_∞ and free stream velocity U_∞ parallel to a block with surface temperature T_s, as shown in Fig. ??. The fluid has a density ρ, specific heat capacity at constant pressure c_p, and thermal diffusivity α. The block has thermal conductivity λ, and is insulated on the vertical sides, and has a temperature T_0 at the bottom surface.

(a) We define the displacement thickness δ_θ as:

$$\delta_\theta = \int_0^\infty \frac{u(T - T_s)}{U_\infty (T_\infty - T_s)} dy$$

(i) Starting from the boundary layer equations in Table 2, show that:

$$\frac{d\delta_\theta}{dx} = \frac{h}{\rho c_p U_\infty}$$

where h is the convective heat transfer coefficient.

(ii) Sketch δ_θ for a plate at constant temperature and one with constant heat transfer rate to the wall. Provide a physical interpretation for δ_θ. [50%]

(b) Now consider the interior of the block. Sketch a diagram and derive the energy balance for appropriate differential solid elements as a function of the element temperature, coordinates, fluid properties and the convective heat transfer coefficient when the element is located in the following positions:

(i) internal to the block (A);

(ii) at the surface of the element (B);

(iii) at the corner of the block (C). [30%]

(c) Discretise each of the equations in part (b), and create an appropriate matrix for solution of the steady state problem. Describe how the system of equations can be solved numerically. Comment on how the system could be solved if the top surface were also subject to radiative heat transfer. [20%]

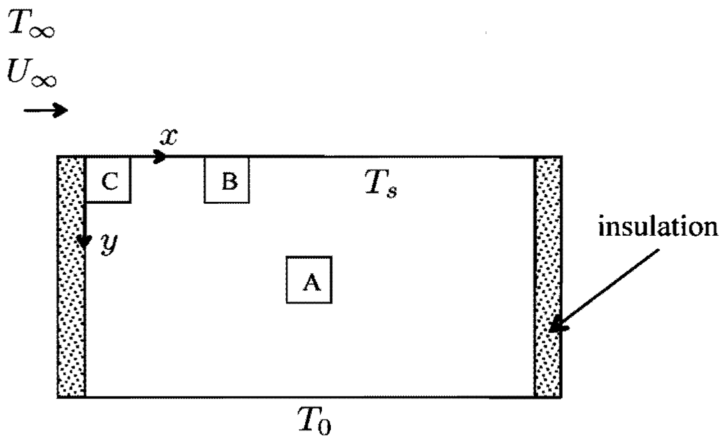

Fig. 2

Table 2

Item	Equation
Mass	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
Energy	$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$

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