ENGINEERING TRIPOS PART IIA

Tuesday 30 April 2013 2.00 to 3.30

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than three questions.

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final version

1 An electron exists in a one-dimensional potential well of the form

$$V(x) = \frac{cx^2}{2}$$

where c is a constant and x is position in the one dimension. Such a potential well is commonly called a *harmonic oscillator well.*

(a) Under what circumstances would the electron's behavior be determined by quantum mechanics rather than classical mechanics? [15%]

(b) The wavefunction of the quantum mechanical ground (lowest energy) state of the electron in the harmonic oscillator well is given by

$$\psi(x) = A_0 \exp\left(\frac{-\alpha^2 x^2}{2}\right)$$

where A_0 is a constant.

(i) For this wavefunction to be a valid solution of the Time-Independent Schrödinger Equation, show that $\alpha^2 = \sqrt{cm}/\hbar$. [35%]

(ii) Hence, derive an expression for the total energy of the electron in this ground state, E_0. [10%]

(iii) Calculate the value of x at which the probability of finding the electron will have decreased to 0.1% of the probability of finding the particle at $x = 0$ if it is in this ground state and $c = 3.054$ J m^{-2}. [25%]

(c) The full solution of the Schrödinger equation for the harmonic oscillator well shows that the electron can only take discrete values of energy given by $E_n = (2n+1)E_0$ where $n = 0,1,2,3,\ldots$. Explain qualitatively how and why this situation would change if there were many evenly spaced and overlapping harmonic oscillator wells along the one dimension. [15%]

NOTE: The Time-Independent Schrödinger Equation is

$$\frac{-\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V\psi = E\psi$$

2 (a) Draw the energy-wavenumber (E-k) diagram predicted by the *nearly-free electron theory* for a periodic series of Coulombic potential wells with a spacing of a. Clearly show where the E-k diagram deviates from that predicted by the free electron theory, and explain the physical origin of this deviation. [30%]

(b) Starting from the Einstein postulate $E = \hbar\omega$ and by considering the total energy of a particle to be both kinetic and potential, show that the velocity of a particle in a particular state is given by

$$v = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$ [20%]

(c) Fig. 1 shows the first Brillouin zone of the E-k diagram for electrons in three different materials. In each case, sketch how the electron velocity and effective mass vary with k. [30%]

(d) Why is the concept of effective mass helpful when considering the behavior of electrons in a material? [20%]

NOTE: The de Broglie postulate states that $\lambda = h/p$. Also, the effective mass, m^*, of an electron is given by

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

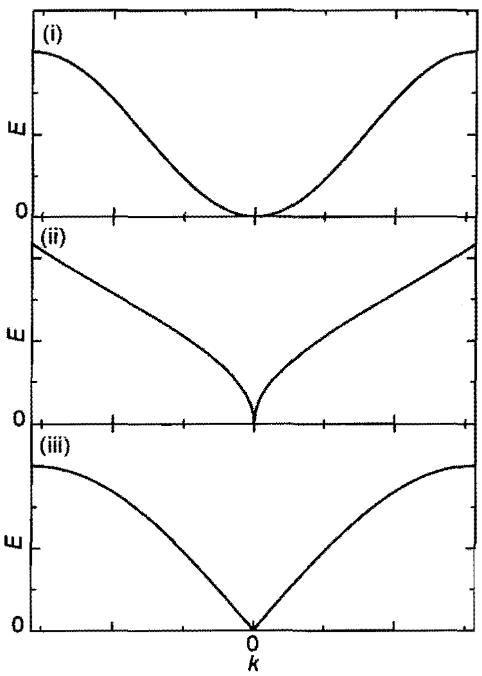

Fig. 1

3 (a) Sketch an equilibrium band diagram for each of the following metal-semiconductor junctions and determine in each case if it is a Schottky or ohmic contact. Assume Ge has an electron affinity of 4V and effective densities of states of $N_C = 1\times10^{25}$ m^{-3} in the conduction band and of $N_V = 6\times10^{24}$ m^{-3} in the valence band, respectively. Assume no interface traps and room temperature conditions. State all other assumptions.

(i) A metal with a work function of 4.6 eV deposited onto Ge doped with a density of 10^{22} m^{-3} of phosphorus atoms. [15%]

(ii) A metal with a work function of 4.2 eV deposited onto Ge doped with a density of 10^{22} m^{-3} of boron atoms. [15%]

(b) Starting from the Poisson equation, show that the width w of the depletion region of a Schottky barrier diode with an n-type semiconductor may be derived to be

$$w = \left(\frac{2\varepsilon_0\varepsilon_r V_0}{eN_D}\right)^{1/2}$$

where ε_r is the relative permittivity, V_0 the built-in potential and N_D the donor doping density of the semiconductor. State all assumptions made. [30%]

(c) A Schottky barrier diode is formed on n-type Si. The donor density is 10^{22} m^{-3} and the difference between the work functions of the metal and n-type Si is 0.5 eV. A bias of -1V is applied to the metal. Assume $\varepsilon_r = 11.8$ for the n-type Si.

(i) Draw a band diagram of the biased junction and indicate the Fermi levels. [10%]

(ii) Calculate the capacitance per unit area of the biased junction. [20%]

(d) The current flow I through the Schottky barrier diode as a function of applied bias V can be derived based on the thermionic emission theory as

$$I = I_s \left[\exp\left(\frac{eV}{\eta kT} \right) - 1 \right]$$

where I_s is the reverse saturation current and η the ideality factor. Explain how and why η will change when the diode is cooled with liquid nitrogen. [10%]

4 (a) The Continuity equation for excess holes in a semiconductor may be written as

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_h} - \mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}$$

Explain briefly what the various terms represent. [15%]

(b) A bar made of n-type Si with a minority carrier life time of 2μs is illuminated with light of wavelength 600 nm. Discuss why this illumination will produce excess holes in the sample and estimate how long after the illumination is switched off it will take for this excess hole density to fall to 50% of its initial value. [25%]

(c) With the aid of energy band diagrams, explain why the direct recombination of an electron-hole pair accompanied by the release of a photon is much more likely in semiconductors with direct band gap compared to semiconductors with an indirect band gap. Give an example of a direct band gap semiconductor. [15%]

(d) In a Haynes-Shockley experiment a pulse of excess holes is locally created by a laser focussed on a bar of n-type Ge that has an electric field applied along its length.

(i) Sketch the change in shape of the hole pulse with time as it drifts across the Ge bar of length L. [10%]

(ii) The length L of the Ge bar is 1 cm and the electric field across it is 2 Vcm^{-1}. The peak of the hole pulse propagated across the whole length of the bar in 0.26 ms. Calculate the hole mobility. [10%]

(iii) The experiment is carried out at room temperature. Estimate the value of the hole diffusion coefficient that can be expected to be measured. [10%]

(e) Sketch the I-V characteristics of a p-n junction made out of Si that is illuminated with light of wavelength 600 nm. [15%]

END OF PAPER

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