ENGINEERING TRIPOS PART IIA

Thursday 25 April 2013 14 to 15.30

Module 3D2

GEOTECHNICAL ENGINEERING II

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Geotechnical Engineering Data Book (19 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A conventional triaxial compression test is performed on clay specimens that are isotropically consolidated prior to undrained shearing. The isotropic consolidation test gave a compression line of $v = 2.858 - 0.161 \ln p'$, where v is the specific volume and p' is the isotropic pressure in kNm^{-2}. The clay is found to have properties similar to those listed for London clay in the Geotechnical Engineering Data Book.

(a) Identify the void ratios when the clay is isotropically normally consolidated at $p_0' = 80$, 160 and 320 kNm^{-2}. [15%]

(b) Compute the values of undrained shear strength s_u of the three clay specimens consolidated at the pressures described in (a). Plot s_u versus p_0' and comment on the relationship. [25%]

(c) The clay is isotropically consolidated to $p_0' = 320$ kNm^{-2} and then swelled back to $p_1' = 80$ kNm^{-2}. The specimen is sheared in undrained conditions by increasing the axial strain, but keeping the confining stress constant.

(i) Using the Cam-clay model, estimate the peak undrained shear strength, $s_{u,peak}$. [20%]

(ii) Compute the undrained shear strength at the critical state, $s_{u,crit}$. [20%]

(d) If the above tests were performed in triaxial extension, in which the radial stress is increased while the axial stress is kept constant, do you expect the undrained shear strengths and the excess pore pressures at failure to be greater or smaller than those in triaxial compression? There is no need to compute the exact values, but explain the reason using the critical state soil mechanics concept. [20%]

2 A long tunnel of diameter 5 m is to be constructed in a uniform body of saturated clay with its axis at a depth of 25 m. The clay has a unit weight of 20 kNm^{-3}, an undrained shear strength s_u constant with depth of 100 kNm^{-2}, and a linear elastic shear modulus of 15 MNm^{-2}. The radial ground movement at the tunnel boundary is expected to be 20 mm.

(a) By assuming the tunnel construction to be an axisymmetric contracting cavity under undrained conditions, analogous to cylindrical cavity expansion, use the Geotechnical Engineering Data Book to estimate the average radial stress imposed on the tunnel lining, which should be assumed to be smooth. [40%]

(b) At one location the tunnel will pass beneath a deep foundation supporting a building. The base of the foundation is 6 m above the crown of the tunnel. Ignoring any effects of the loading of the foundation, and assuming the same average radial stress imposed on the lining as you have calculated for (a), estimate the ground settlement at the foundation. [20%]

(c) In the elastic zone of the soil, at any radius r, the following expressions apply:

$$\sigma_r = \sigma_0 - G\delta A/\pi r^2$$

$$\sigma_\theta = \sigma_0 + G\delta A/\pi r^2$$

where σ_r and σ_θ are the radial and circumferential stresses respectively, σ_0 is the original insitu total stress in the ground, G is the elastic shear modulus of the soil, and δA is the contraction of the cavity (expressed as a change in its cross-sectional area A).

Another more sensitive building is planned to be constructed prior to the tunnelling and it may be affected by the tunnel construction. By considering the radius of the elastic/plastic boundary, and making the same assumptions as for (a) and (b), calculate the distance above the tunnel crown which the foundation would have to be located to ensure that it remained above the plastic zone associated with the tunnel construction. [40%]

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3 A retaining wall is installed in clay, as shown in Fig. 1, and the soil is subsequently rapidly excavated on one side. During excavation the excavated soil is replaced by a support system, which generates a pressure distribution shown in the figure. Before excavation a self-boring pressuremeter test in the clay at a depth of 7 m showed the insitu total horizontal pressure to be 180 kNm^{-2}. The unit weight of the clay is 20 kNm^{-3}, the critical state angle of friction is 25 degrees, and the water table is at the ground surface. A piezometer is installed at X prior to excavation to monitor the pore pressure changes during excavation.

(a) Calculate the value of K_0, the coefficient of horizontal earth pressure at rest at a depth of 7 m, and compare this with the value of K_0 for the same clay in a normally consolidated state. What does the value of K_0 indicate about the stress history of the clay? [20%]

(b) A clay sample is obtained from a depth of 7 m before construction of the wall and tested in a triaxial apparatus. A cell pressure of 125 kNm^{-2} is applied without allowing any drainage. If the sample was obtained with zero disturbance so that the insitu mean effective stress p' remains unchanged, what pore pressure would be measured? [20%]

(c) Jacks incorporated in the support system ensure that the support pressure p_s at 7 m depth is set at 100 kNm^{-2}. Predict the piezometer measurement at X, and show the total and effective stress paths on a plot of vertical stress against horizontal stress distinguishing clearly between the two lines. [30%]

(d) Assume that the undrained shear strength of the clay is sufficiently high that the clay does not fail in undrained conditions by the excavation. From your plot of the total and effective stress paths, estimate by how much the support pressure p_s at 7 m would have to be reduced for the clay at that depth to initiate fully active conditions in terms of effective stresses. [30%]

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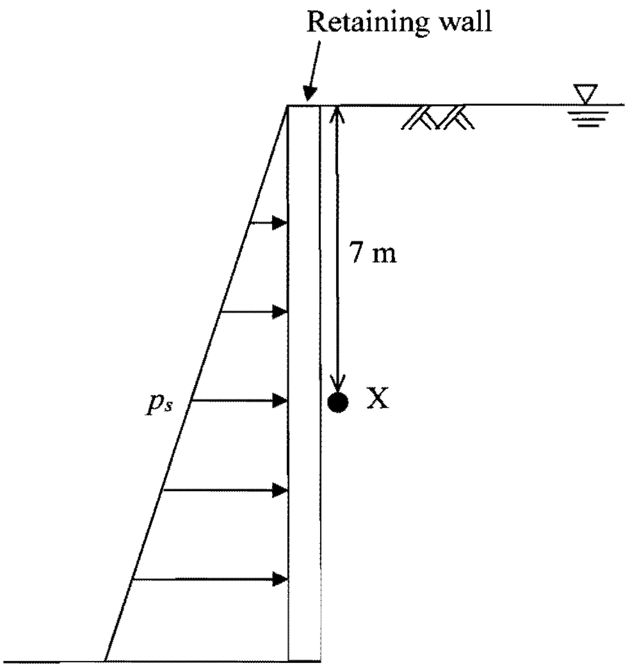

Fig. 1

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4 A natural sand has a maximum void ratio e_{max} of 0.85 and a minimum void ratio e_{min} of 0.5. The strength characteristics of the sand are evaluated using a plane strain testing apparatus, in which σ_1' is the axial effective stress and σ_3' is the lateral effective stress. The stress invariants are defined as $s' = (\sigma_1' + \sigma_3')/2$ and $t = (\sigma_1' - \sigma_3')/2$. The critical state friction angle of the sand is 32 degrees. The aggregate crushing strength of the sand σ_c is 20,000 kNm^{-2}. Using Bolton's relative dilatancy index, the $e - s'$ relationship at the critical state is estimated by the following equation.

$$e = e_{\max} - (e_{\max} - e_{\min}) / \ln(\sigma_c / s')$$

(a) The sand is placed in the plane strain testing apparatus and then isotropically consolidated to a void ratio of 0.80 at an effective stress of 100 kNm^{-2}. The specimen is then compressed in drained conditions by increasing the axial strain but keeping the lateral stress constant. The specimen is sheared until it reaches the critical state. Evaluate the strength and volumetric strain at the critical state. Plot the effective stress path in $s' - t$ space. [25%]

(b) Rather than failing in compression as described in (a), it is possible to fail the specimen by reducing the lateral stress but keeping the axial stress constant. Evaluate the strength and volumetric strain at the critical state. Plot the effective stress path in $s' - t$ space. [25%]

(c) The sand is compacted to a dense state and then isotropically consolidated to a void ratio of 0.65 at an effective stress of 100 kNm^{-2}.

(i) The specimen is sheared in drained conditions by increasing the axial strain, but keeping the lateral stress constant. Assuming that the change in void ratio is small until the specimen reaches the peak strength, show that the peak friction angle is approximately 39.3 degrees. [25%]

(ii) After the peak, the specimen reaches the critical state by further shearing. Evaluate the strength and volumetric strain at the critical state. Plot the effective stress path in $s' - t$ space. [10%]

(d) For the three cases (a), (b) and (c), sketch the deviator stress – shear strain relationship and the volumetric strain – shear strain relationship on the same graph, giving salient values. [15%]

END OF PAPER

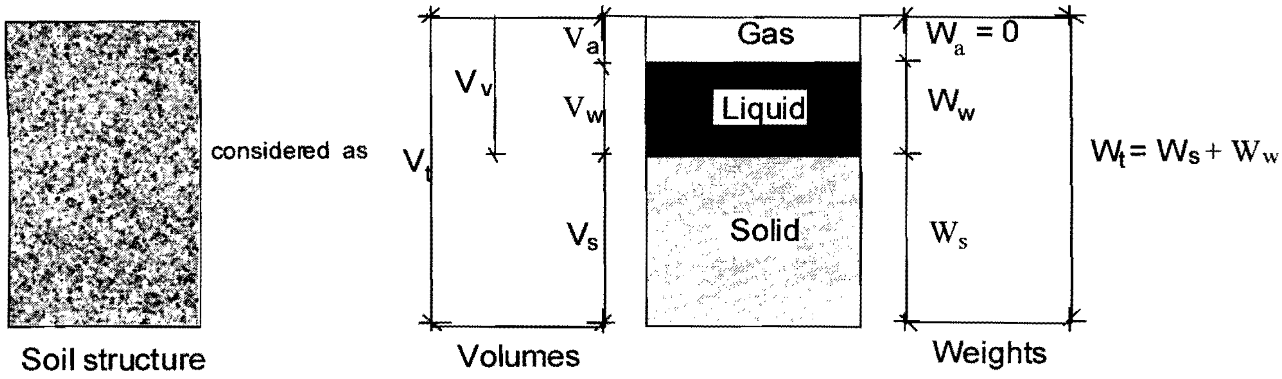
Engineering Tripos Part IIA

3D1 & 3D2 Geotechnical Engineering

Data Book 2012-2013

Contents	**Page**

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\, G_s/e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

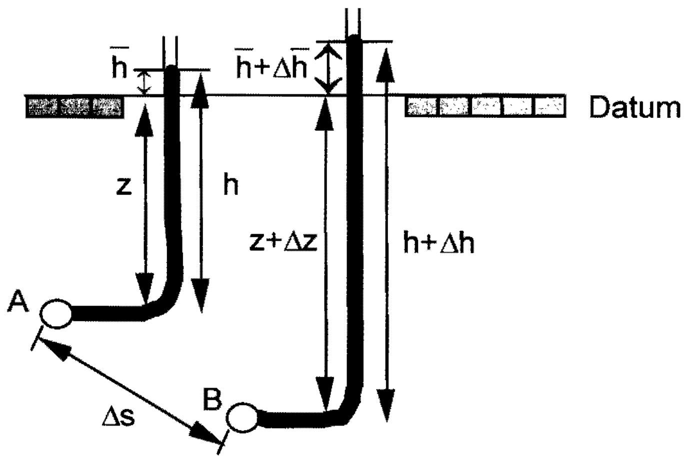
D equivalent diameter of soil particle

D_{10}, D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} > 1\mu$m	:	$k \cong 0.01$ (D_{10} in mm)^2 m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

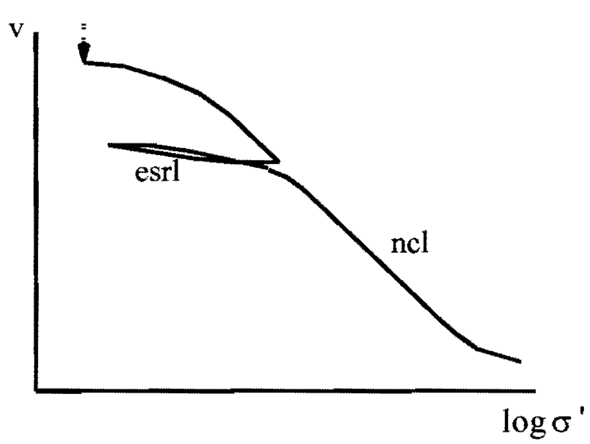
$$h_c = \frac{4T}{\gamma_w d}$$: capillary rise in tube diameter d, for surface tension T

$$h_c \approx \frac{3\times10^{-5}}{D_{10}} \text{ m}$$: for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

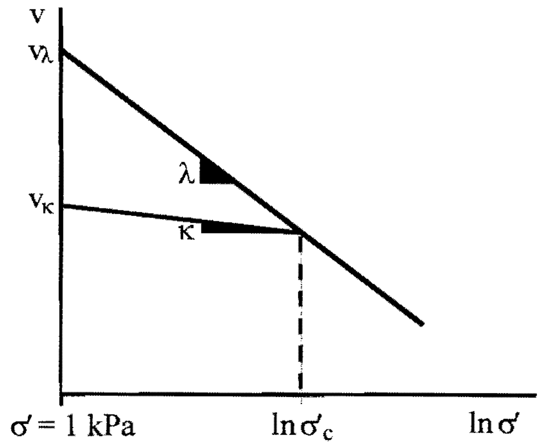
One-Dimensional Compression

- **Fitting data**

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

- **Deriving confined soil stiffnesses**

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

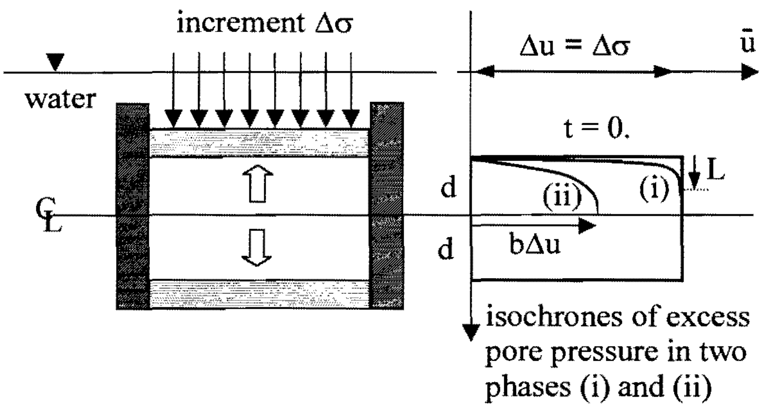
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

- **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma_1', \sigma_2', \sigma_3'$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

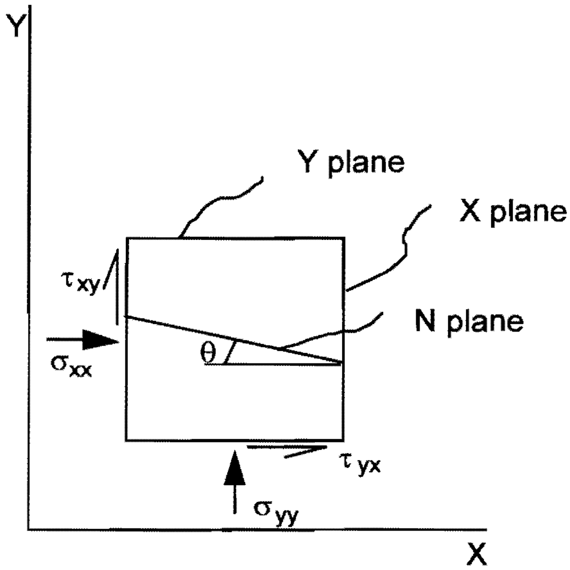
isotropic compression in which p' increases at zero q

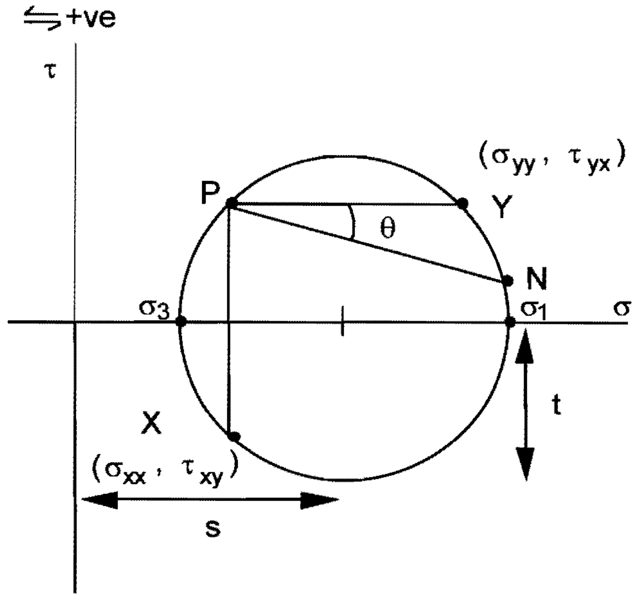
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

- **Interchangeable parameters for stress combinations at yield, and plastic strain increments**

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

- **General equations of plastic work**

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

- **General yield surface**

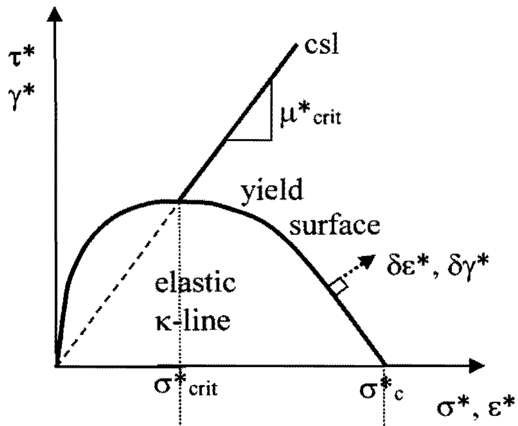
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

- **Parameter values which fit soil data**

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

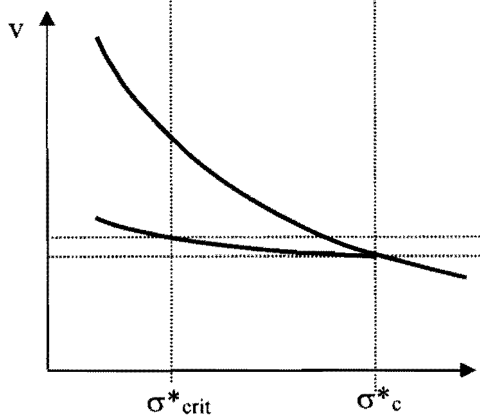
- **The yield surface in (σ*, τ*, v) space**

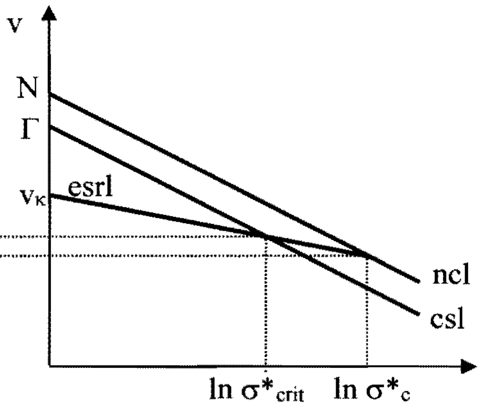


ncl: normal compression line
$v = N - \lambda \ln \sigma^*$

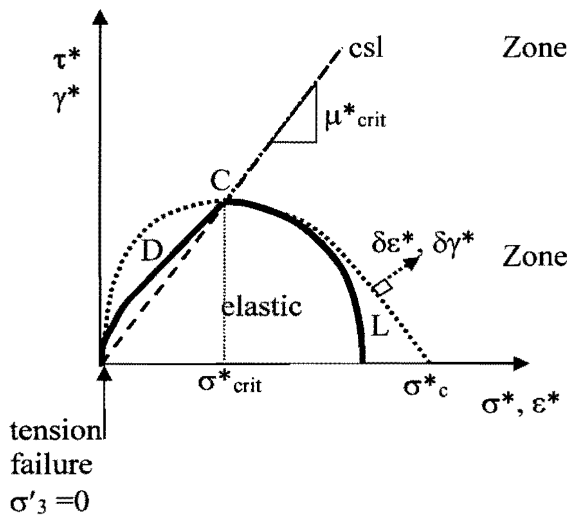
csl: critical state line
$v = \Gamma - \lambda \ln \sigma^*$

where $N = \Gamma + \lambda - \kappa$





- **Regions of limiting soil behaviour**



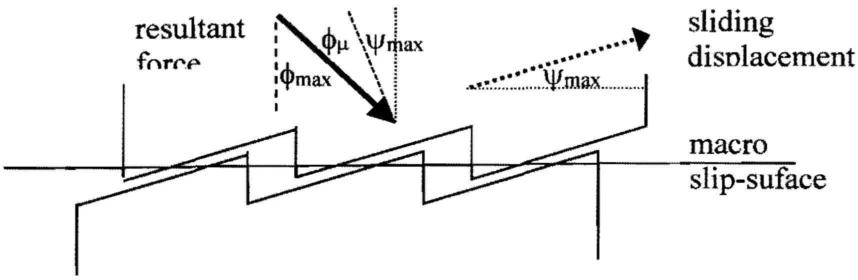
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

- **Friction and dilatancy: the saw-blade model of direct shear**

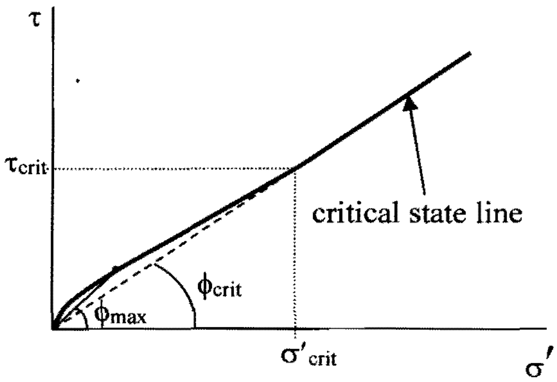


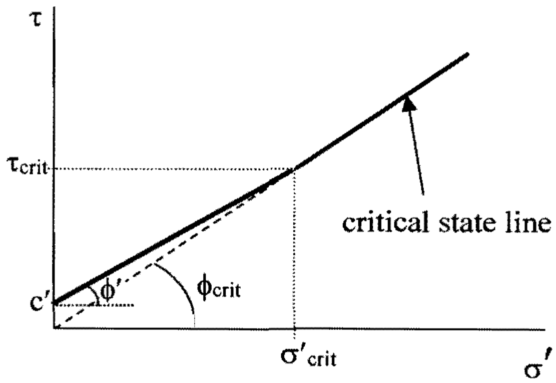
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

- **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\ \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})} \quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p') \quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

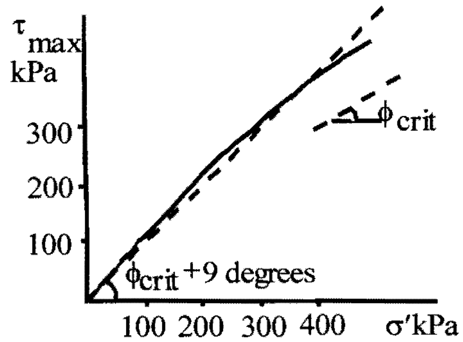
Relative dilatancy index $I_R = I_D I_C - 1 \quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

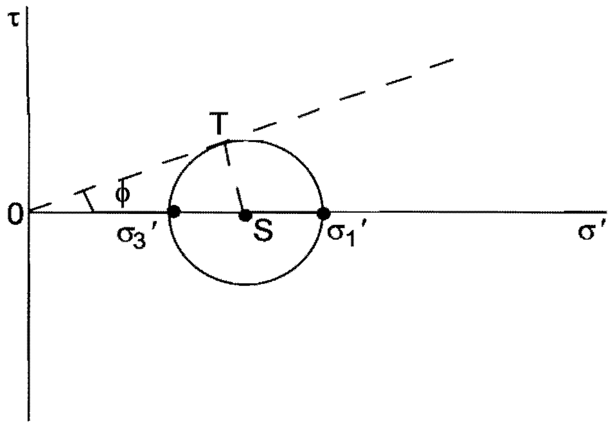
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' = 400$ kPa

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

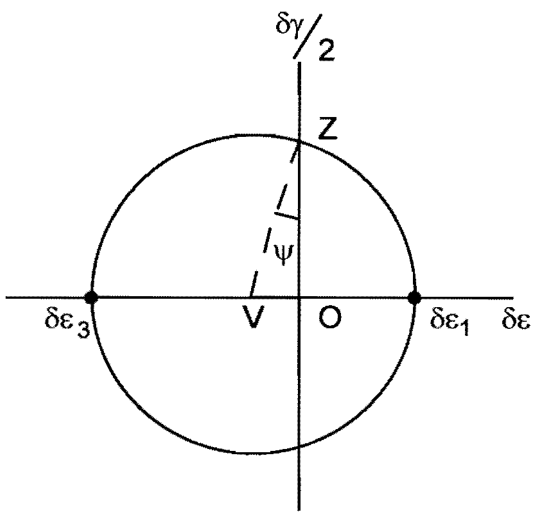
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ = -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2} = -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

• **Limiting stresses**

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

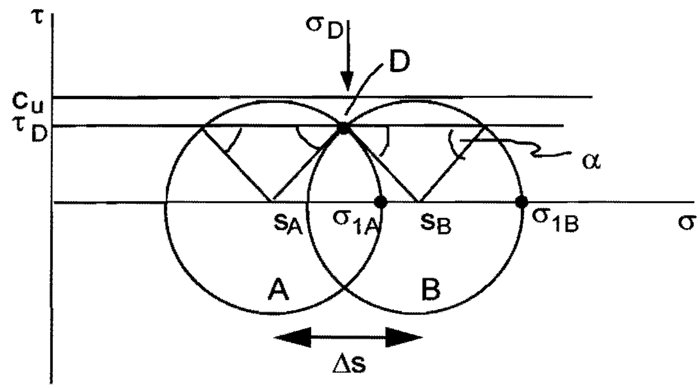
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = Ac_u x$$

• **Stress conditions across a discontinuity**

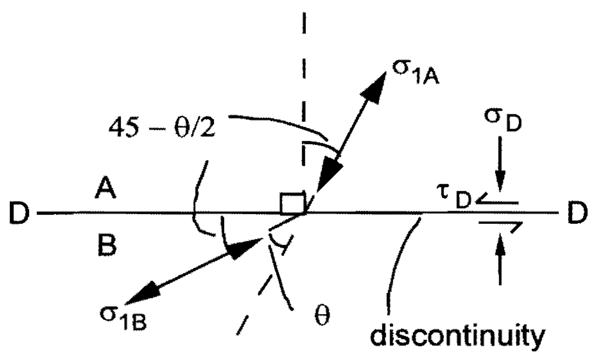


Rotation of major principal stress θ

$s_B - s_A = \Delta s = 2c_u \sin\theta$
$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$

In limit with $\theta \to 0$

$ds = 2c_u d\theta$



Useful example:

$\theta = 30°$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

- **Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

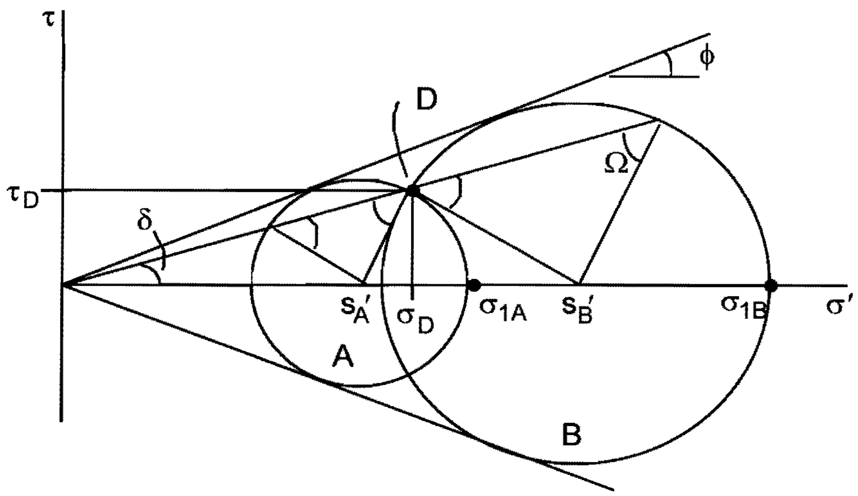
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

- **Stress conditions across a discontinuity**



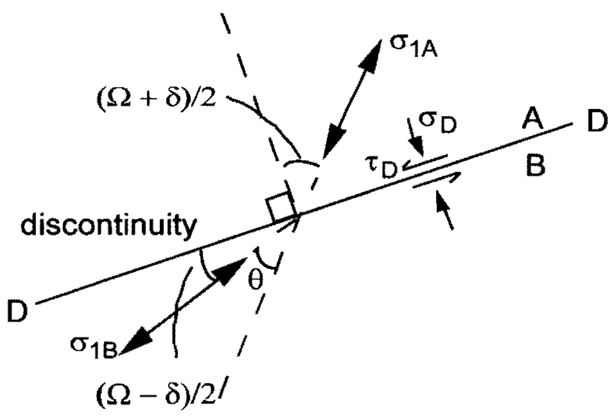
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \frac{\delta A}{2\pi r}$

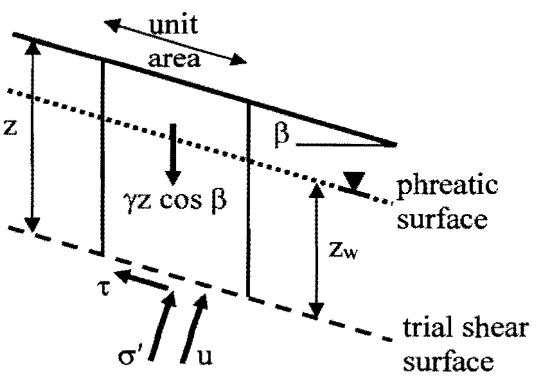
small shear strain $\gamma = \frac{2\rho}{r}$

Radial equilibrium: $r\frac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\frac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$

Infinite slope analysis



$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w) \cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\, B / L$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1} (h/B)$ (or h/D for a circular foundation)

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$$ (Taiebet & Carter 2000)

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

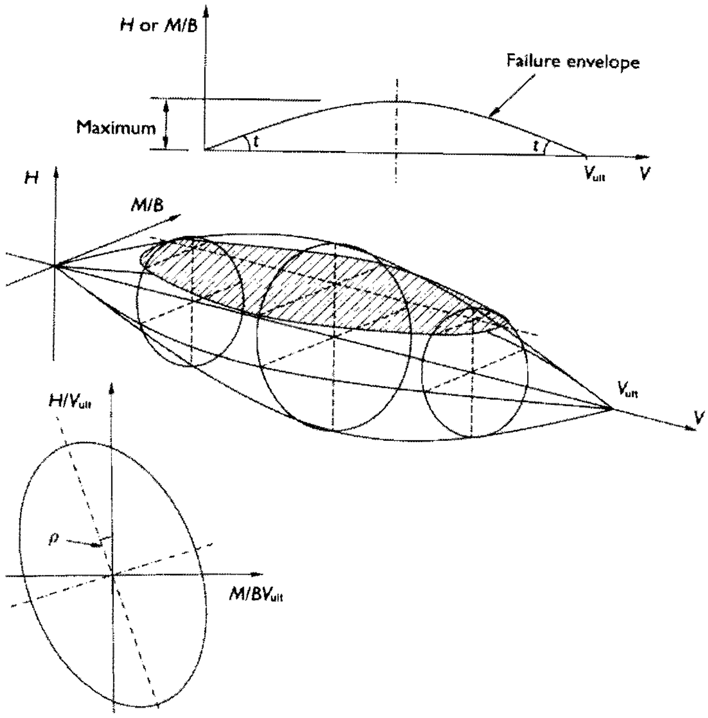
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$ (Prandtl 1921)

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$N_\gamma = 2\,(N_q - 1) \tan \phi$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054 e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663 e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_q = 1 + (B \sin \phi) / L$
$s_\gamma = 1 - 0.3\, B / L$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, H/V= $\tan\phi$, during sliding.