ENGINEERING TRIPOS PART IIA

Friday 26 April 2013 9.30 to 11

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: Final

1 (a) Figure 1 shows a beam over three spans. The beam is continuous over the supports, and the left and centre spans each contain a pin. The left span is loaded by three concentrated forces.

(i) Derive the influence line for bending moment at location A. [30%]

(ii) Determine the bending moment at A due to the loading shown. [20%]

(b) Figure 2 shows a frame consisting of three members that are rigidly connected at B. The connections A, C and D are built-in. The uniform flexural stiffness of each member is $EI = 5 \times 10^4$ kNm^2 and axial stiffnesses can be assumed to be infinite. The beam AB carries a uniformly distributed load $w = 10$ kNm^{-1}. The stiffness matrix for beam bending is given in Fig. 3.

(i) Calculate the rotation of the connection B. [30%]

(ii) Calculate the bending moments in the three members at sections adjacent to connection B and sketch the bending moment diagram. [20%]

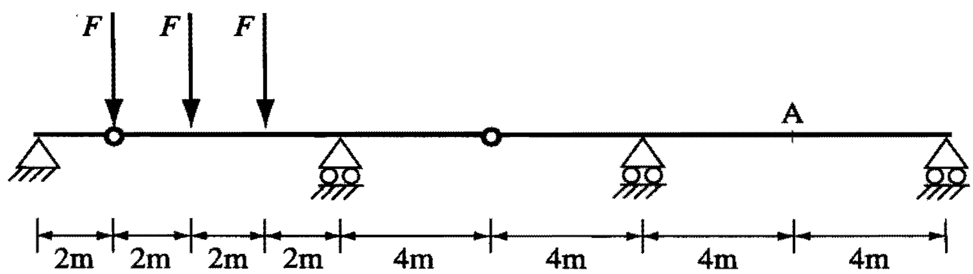


Fig. 1

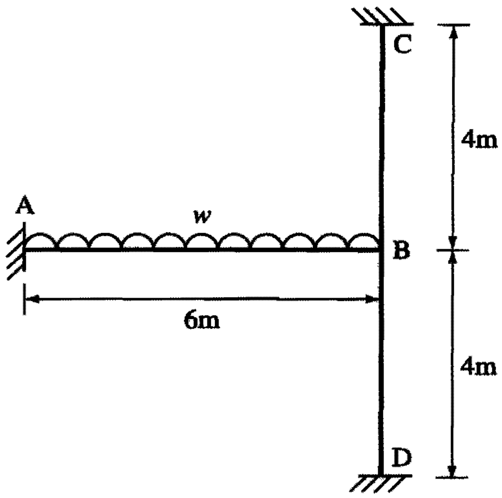
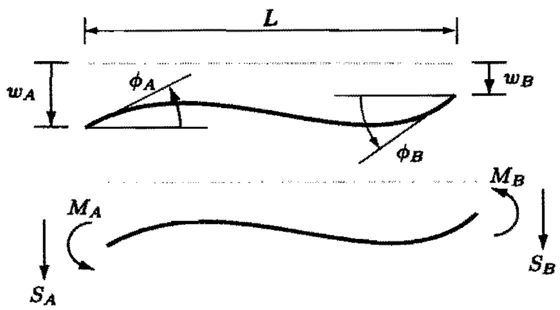


Fig. 2



$$
\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}
$$

Fig. 3: Beam stiffness matrix

2 (a) The thin-walled cross-section ABC shown in Fig. 4(a) has a uniform thickness t, Young's modulus E and shear modulus G.

(i) Calculate the torsion constant of the cross-section. [10%]

(ii) Calculate the principal second moments of area of the cross-section. [60%]

(b) Figure 4(b) shows a cantilever of length L with a cross-section as shown in Fig. 4(a). A stiff plate is welded to the tip of the cantilever, to which a vertical force F is applied, acting downwards at an offset of $2b$ from the vertical leg of the cantilever cross-section as shown.

Calculate the elastic deflection at point B at the cantilever tip and the torsional rotation due to the applied force. [30%]

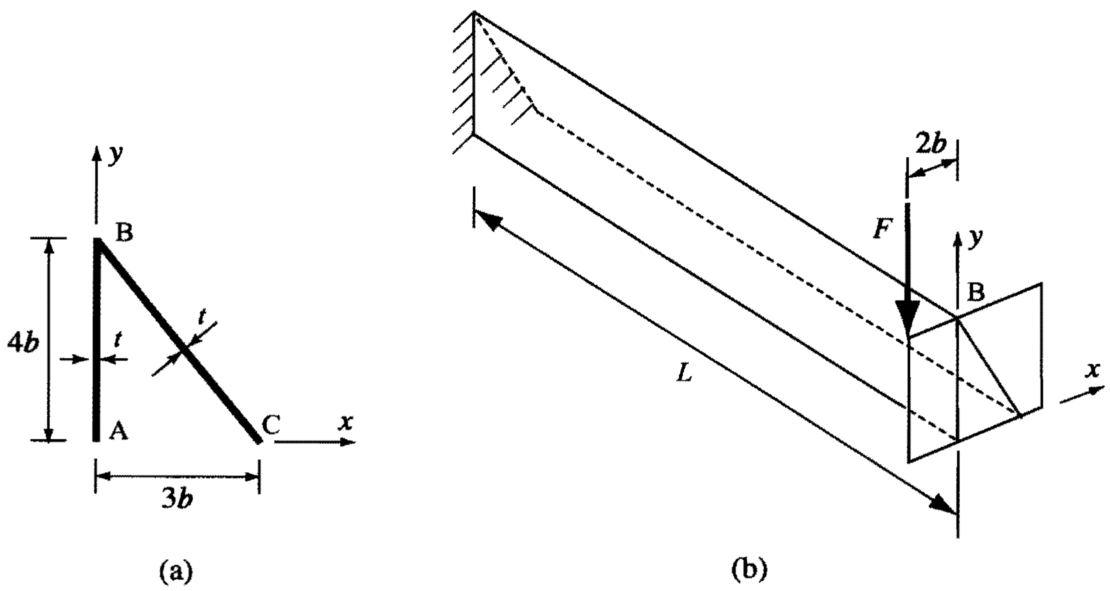


Fig. 4

3 (a) A steel beam has a thin rectangular cross-section with the dimensions illustrated in Fig. 5a. The supports are simply supported with respect to both major and minor axis flexure, and prevent any twist rotation about the longitudinal axis at the ends. Self-weight may be ignored.

(i) Determine the magnitude of the equal-and-opposite major axis end moments that will cause lateral-torsional buckling. [25%]

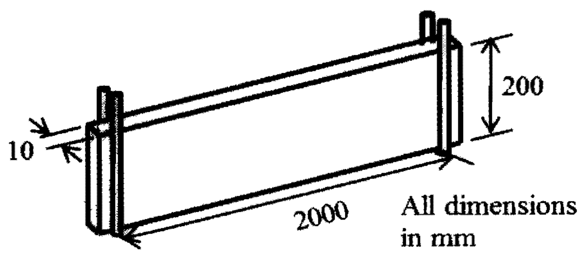
(ii) Briefly explain what is meant by warping and explain the role played by resistance to warping in determining the critical moment for lateral-torsional buckling of I-beams. [25%]

(b) The mechanism illustrated in Fig. 5b consists of two rigid rods of length L connected by a linear spring which has stiffness k and unstressed length $\sqrt{2}L$. It is loaded by a vertical load P at the central hinge.

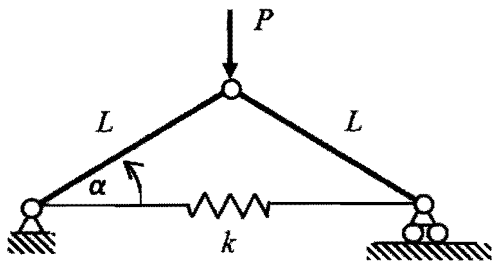
(i) By considering the total potential energy, determine the positive downward load $P = P_{cr}$ at snapthrough and the two corresponding equilibrium values of the angle α. [30%]

(ii) Sketch the total potential energy function as a function of α at $P = P_{cr}$. [10%]

(iii) Explain why a Rayleigh-Ritz approach to this problem would be unproductive. [10%]



(a)



(b)

Fig. 5

4 The subframe shown in elevation in Fig. 6 consists of four horizontal beams with $356 \times 127 \times 33$ UB section connected to a vertical $203 \times 203 \times 86$ Universal Column, with support conditions as shown. The joints between beams and columns at B and E are fully rigid. Each beam is 6 m long and the column is 3 m high. All webs are in the plane of the diagram and all deformation out of the plane of the diagram is prevented. A vertical load P is applied at B. Ignore self-weight. A graph of s and c stability functions is provided in Fig. 7.

(a) Determine the effective length of the column for major axis elastic buckling, and the corresponding critical axial load in the column. [40%]

(b) Determine the relative magnitudes of the rotations at B and E as the subframe undergoes elastic buckling, and sketch the buckling mode shape. [20%]

(c) Explain why the lack of rollers at support A is crucial to the calculation of the elastic critical load. Describe how such a support condition could be achieved in practice in a multi-storey building. Explain how your approach to the problem would change if supports A and C were both on rollers. [20%]

(d) Assuming the column is made of steel with 355 MPa yield stress, compare the elastic buckling load computed in part (a) with the axial plastic capacity of the column. Describe briefly how the elastic and plastic capacities may be combined to produce a design axial load capacity, and without further calculation, provide a rough estimate for the design axial load capacity of the column. [20%]

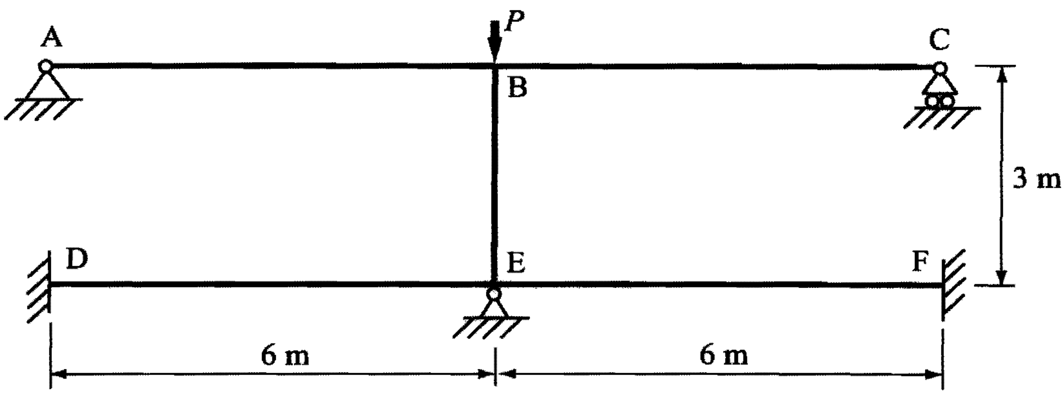


Fig. 6

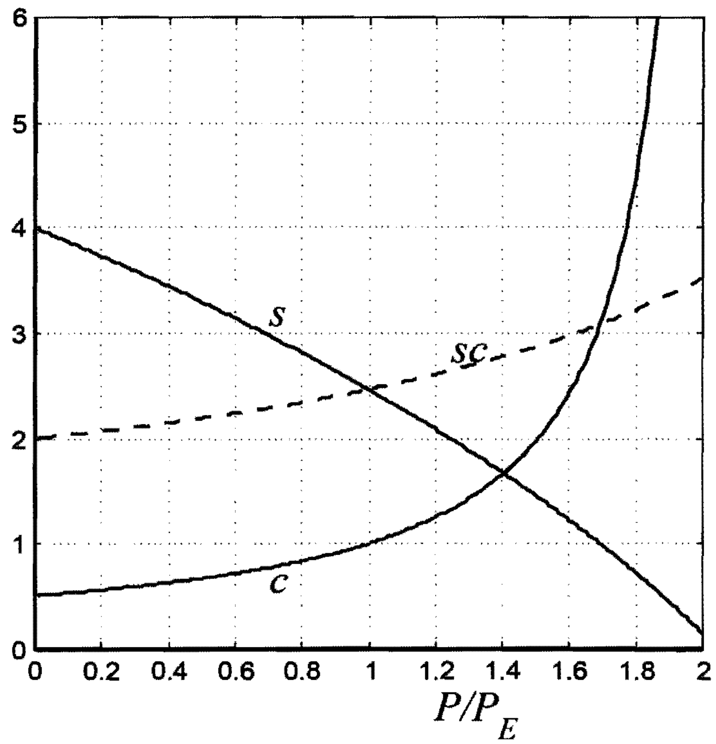


Fig. 7: Graphs of s and c stability functions. P is the axial load and P_E is the Euler load. The product sc is also shown (dashed).

END OF PAPER

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