ENGINEERING TRIPOS PART IIA

Wednesday 1 May 2013 9.30 to 11

Module 3D5

WATER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

The values of relevant parameters are listed at the end of the data book unless otherwise noted in the question.

Attachment: 3D5 Data Book (5 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Three hours of rainfall of intensity 10 mm h^{-1} over a small impervious catchment provides the following discharge variation at the catchment outlet

Duration (h)	0-3	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Discharge (m^3 s^{-1})	4.2	7.8	5.0	3.6	2.2	1.2	0.0	0.0

(i) Calculate the area of the catchment. [15%]

(ii) Estimate the peak discharge produced by two periods of heavy rainfall that occur in quick succession. The first period is between 3 am and 9 am, with intensity 20 mm h^{-1}. The second rainfall happens between noon and 3 pm, and the intensity is 30 mm h^{-1}. [40%]

(iii) Derive the two-hour unit hydrograph of the catchment, *i.e.* the distribution percentages of the outflow on a two-hour basis after a uniform two-hour rainfall. [35%]

(b) Briefly explain the purpose of the coagulation and flocculation process in water treatment. [10%]

2 (a) The water depth in an open channel is 0.99 m and the discharge per unit width is 0.5 $m^2 s^{-1}$. A short length of the channel is excavated so that the channel bed is made deeper by 0.3 m. Calculate the flow velocity in the excavation section. [20%]

(b) Fig. 1 shows a vertical sluice gate built across a wide rectangular channel. The channel bed slope is 0.0003, and the Manning's roughness factor is $n = 0.02$ s $m^{-1/3}$. The flow rate per unit width is 4.646 $m^2 s^{-1}$.

(i) Show that the uniform flow sufficiently upstream of the sluice gate is sub-critical. [15%]

(ii) The depths of flow at section A and section B downstream of the sluice gate are 0.457 m and 0.5 m, respectively. Estimate the distance between the two sections. [20%]

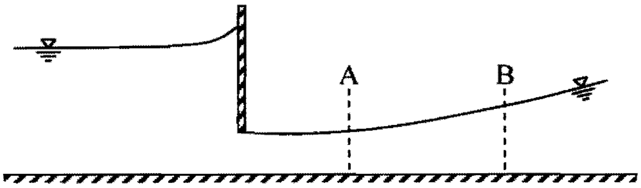


Fig. 1

(c) A long, horizontal and frictionless water tank is initially filled with water to a depth of 0.9 m. A sluice gate suddenly opens at one end so that the water depth immediately in front of the sluice gate drops linearly to 0.4 m in 5 s.

(i) 5 s after the opening of the sluice gate, at what distance upstream of the sluice gate does the water just start to move? [10%]

(ii) 5 s after the opening of the sluice gate, at what distance upstream of the sluice gate is the water depth 0.65 m? [35%]

3 The bed slope of a wide river is 4×10^{-4}. The grain size of the bed material is 0.3 mm. The flow is uniform with depth 2 m and velocity 1 m s^{-1}. Take the grain-related roughness height of the bed to be twice the grain size.

(i) Calculate the total bed shear stress, grain-related shear stress and form-related shear stress. [20%]

(ii) Calculate the roughness height of the bedform based on the information available in the question. [10%]

(iii) Based on the near-bed sediment concentration formula of Zyserman and Fredsøe, estimate the suspended sediment concentration in kg m^{-3} at mid depth of the river. [40%]

(iv) A factory discharges pollutant water at a constant rate from an outfall located at one bank of the river. The pollutant concentration 50 m downstream of the outfall and 1 m away from the bank is measured to be 10 mg litre^{-1}. Estimate the mass of the pollutant discharged by the factory in a day. [30%]

4 (a) A pipeline of diameter 400 mm, roughness height 0.16 mm and length 20 km is designed to convey water from an impounding reservoir to a service reservoir. The water level in the service reservoir is 160 m lower than that in the impounding reservoir. Local head losses, including entry and exit losses, are $10U^2/(2g)$.

(i) Show that the flow rate is about 240 litre s^{-1}. [20%]

(ii) A control valve is now installed in the pipeline to regulate the flow rate to 100 litre s^{-1}. Calculate the head loss and mechanical power loss at the control valve. [20%]

(b) When operating at 1450 rpm, a variable speed pump has the characteristics tabulated below.

Discharge (litre s^{-1})	0	50	100	150	200
Total head (m)	60	58	52	41	25
Efficiency (%)	0	44	65	64	48

The pump is installed in a pumping station to deliver water from a river, with a water level of 52 m above datum, to a reservoir, with a water level of 85 m above datum. The pipeline is 2000 m long, with a diameter of 350 mm and friction factor λ of 0.0175. Allow $10U^2/(2g)$ for all the local head losses.

(i) Show that the flow rate is about 137 litre s^{-1} at the pump speed of 1450 rpm. [25%]

(ii) Determine the power consumed by the pump when the pump operates at a speed of 1200 rpm. [35%]

END OF PAPER

UNIVERSITY OF
CAMBRIDGE

Engineering Tripos Part IIA

Third Year

Module 3D5: Water Engineering
Data Book (SI units [m, kg, s] unless otherwise noted)

Hydrology

Horton's infiltration model (*f*-capacity)

$$f = f_c + (f_0 - f_c)e^{-K_f t}$$

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

Rational method $Q = CiA$

Boundary Layer

For fully developed boundary layer flow with a free surface (uniform flow in a very wide channel):

Eddy viscosity coefficient $\nu_t = \kappa u_* z\left(1 - \frac{z}{h}\right)$

Velocity in hydraulically smooth regime ($u_* k_s / \nu < 5$) $\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(9.0\frac{z u_*}{\nu}\right)$

Velocity in hydraulically rough regime ($u_* k_s / \nu > 70$) $\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.0 z}{k_s}\right)$

Open Channel Flow

Chézy coefficient in large Reynolds-number flows $C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$

Manning formula $C = \frac{1}{n} \cdot R_h^{1/6}$

Bed shear stress $\tau_b = \rho g R_h S_f = \frac{C_f}{2}\rho \cdot U^2 = \frac{\lambda}{8}\rho \cdot U^2 = \frac{g}{C^2}\rho \cdot U^2 = \frac{g \cdot n^2}{R_h^{1/3}}\rho \cdot U^2 = \rho \cdot u_*^2$

Froude number for rectangular channels $Fr = \frac{U}{\sqrt{gh}}$

Steady flow momentum equation $\sum F = \rho Q(U_{out} - U_{in})$

Uniform flows (Chézy formula) $U = C\sqrt{R_h S_b}$

Gradually varied flows:

$$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

or

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{U^2}{C^2 \cdot R_h}}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$$

Characteristics for unsteady flows in rectangular channels:

$$\frac{d}{dt}\left(U + 2\sqrt{gh}\right) = g\left(S_b - S_f\right) \text{ along } \frac{dx}{dt} = U + \sqrt{gh}$$

$$\frac{d}{dt}\left(U - 2\sqrt{gh}\right) = g\left(S_b - S_f\right) \text{ along } \frac{dx}{dt} = U - \sqrt{gh}$$

Pollutant Transport

Analytical values of the mixing coefficients: $D_{tx} = D_{ty} = 0.15hu_*$, $D_{tz} = 0.067hu_*$, $D_L = 5.86hu_*$

For instantaneous release from origin at $t = 0$ in uniform flows along x direction:

One-dimensional $$\bar{c}(x,t) = \frac{M/A}{\sqrt{4\pi t D_x}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}\right)$$

Two-dimensional $$\bar{c}(x,y,t) = \frac{M/h}{4\pi t\sqrt{D_x D_y}}\exp\left(-\frac{(x-Ut)^2}{4D_x t} - \frac{y^2}{4D_y t}\right)$$

Three-dimensional $$\bar{c}(x,y,z,t) = \frac{M}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}}\exp\left(-\frac{(x-Ut)^2}{4D_x t} - \frac{y^2}{4D_y t} - \frac{z^2}{4D_z t}\right)$$

For continuous release from origin in uniform flows along x direction:

Two-dimensional $$\bar{c}(x,y) = \frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}}\exp\left(-\frac{y^2}{4D_y\, x/U}\right)$$

Three-dimensional $$\bar{c}(x,y,z) = \frac{\dot{M}}{4\pi x\sqrt{D_y D_z}}\exp\left(-\frac{y^2}{4D_y\, x/U} - \frac{z^2}{4D_z\, x/U}\right)$$

Sediment Transport

Definitions of Shields parameter, non-dimensional grain size and transport stage parameter:

$$\theta = \frac{\tau_b}{g(\rho_s - \rho)d}, \quad d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3}, \quad T = \frac{\tau_b' - \tau_{bc}}{\tau_{bc}} = \frac{\theta' - \theta_c}{\theta_c}$$

Critical Shields parameter $$\theta_c = \frac{0.30}{1+1.2d_*} + 0.055\left[1 - \exp(-0.02d_*)\right]$$

Fall velocity $$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049\cdot d_*^3} - 10.36\right]$$

Shear stress partition: $$C' = 7.8\ln\left(\frac{12.0\cdot R_h}{k_s'}\right), \quad \tau_b' = \rho g\frac{U^2}{C'^2}$$

$$C = 7.8\ln\left(\frac{12.0\cdot R_h}{k_s}\right), \quad \tau_b = \rho g\frac{U^2}{C^2}$$

Volumetric bedload transport rate per unit width:

Meyer-Peter and Müller $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 8\left[\left(\frac{C}{C'}\right)^{1.5}\theta - 0.047\right]^{1.5}$$

van Rijn $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 0.053\frac{T^{2.1}}{d_*^{0.3}}$$

Rouse profile of suspended sediment concentration $$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

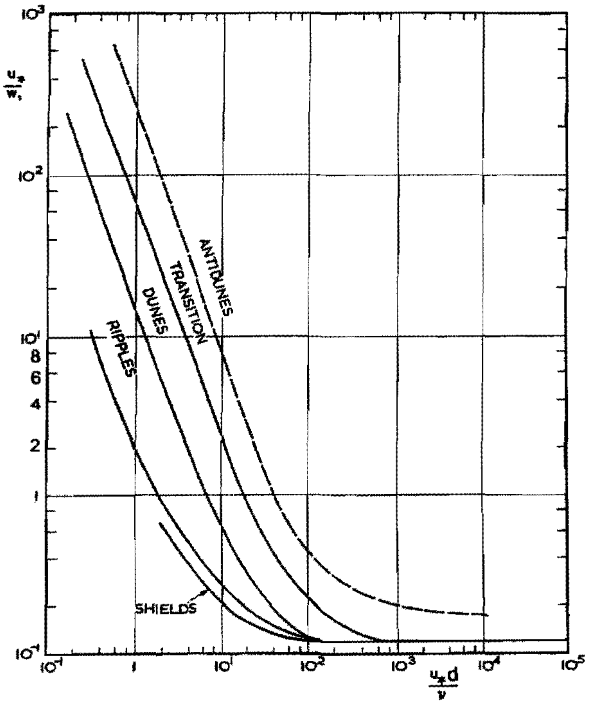
Reference volumetric concentration close to the bed:

Zyserman and Fredsøe $$\bar{c}(2d) = \frac{0.331\cdot(\theta'-0.045)^{1.75}}{1+0.72\cdot(\theta'-0.045)^{1.75}}$$

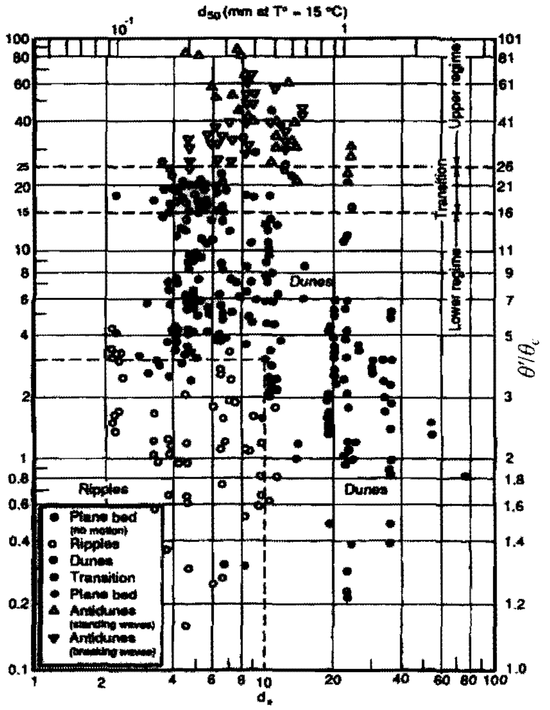
van Rijn $$\bar{c}(a) = 0.015\frac{d\cdot T^{1.5}}{a\cdot d_*^{0.3}}$$

Suspended load per unit width $$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6\cdot u_*\cdot\bar{c}(a)\cdot a\cdot\left[I_1\ln\left(\frac{30h}{k_s}\right)+I_2\right]$$

a/h	$w_s/(\kappa u_*) = 0.2$		$w_s/(\kappa u_*) = 0.6$		$w_s/(\kappa u_*) = 1.0$		$w_s/(\kappa u_*) = 1.5$	
	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113



Liu (1957)



Van Rijn (1984)

Pipeline and Pump

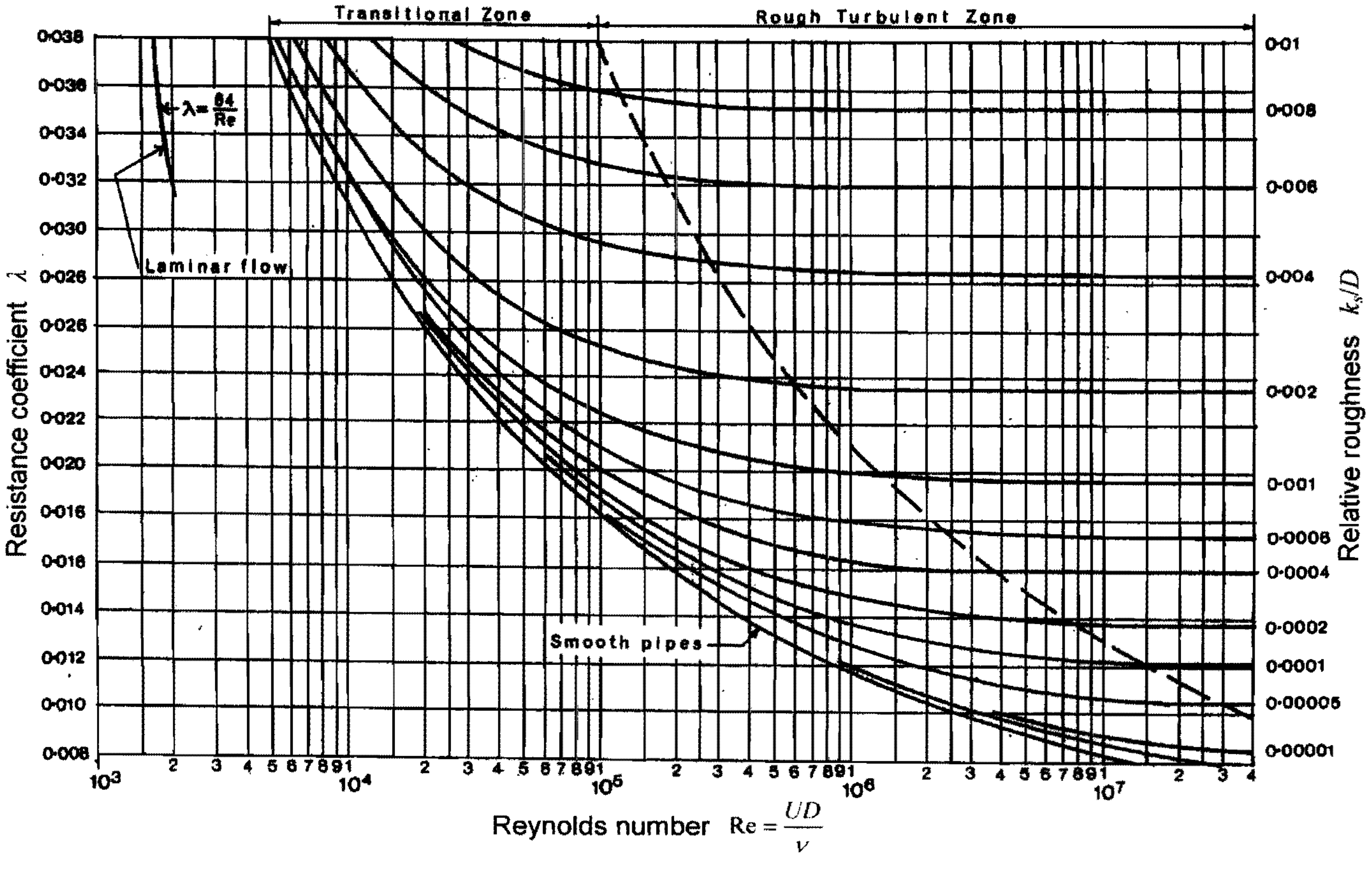
Darcy-Weisbach Equation $H_f = \lambda \dfrac{L}{D}\dfrac{U^2}{2g}$

Colebrook-White formula $\dfrac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\dfrac{k_s}{3.7D} + \dfrac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$ with $\mathrm{Re} = \dfrac{UD}{\nu}$

Power consumption $P_p = \rho g Q_p H_p / \eta$

Non-dimensional groups $\dfrac{Q_p}{N_p \cdot D_p{}^3}$, $\dfrac{gH_p}{N_p{}^2 \cdot D_p{}^2}$, $\dfrac{P_p}{\rho \cdot N_p{}^3 \cdot D_p{}^5}$

Specific speed $N_S = \dfrac{N_p \cdot Q_p{}^{1/2}}{H_p{}^{3/4}}$



Symbols

A	area
C	runoff coefficient or Chézy coefficient
C_f	shear stress coefficient
c	concentration

D	pipeline or pump diameter
D_L	longitudinal dispersion coefficient
D_x,D_y,D_z	diffusion coefficients in x, y and z directions respectively
D_{tx},D_{ty},D_{tz}	turbulent diffusion coefficients in x, y, and z directions respectively
d	particle diameter
d_*	dimensionless particle diameter
F	force
Fr	Froude number
f	infiltration capacity
f_0	initial infiltration capacity
f_c	equilibrium infiltration capacity
g	gravitational acceleration (= 9.81 m s^{-2})
H	head
h	water depth
i	rainfall intensity
K_f	rate of decrease of f capacity
k_s	roughness height, also called equivalent or Nikuradse's sand roughness height
M	amount of the pollutant released
$\dot{M}$	rate of the pollutant release
N	rotational speed
P	power
Q	discharge
q_b	bedload sediment transport rate
R_h	hydraulic radius
S_b	bed slope
S_f	slope of the total energy line
s	specific gravity, also called relative density or density ratio (= 2.65)
T	transport-stage parameter
t	time
U	mean velocity
u_*	shear velocity
w_s	fall velocity
x,y,z	spatial coordinates
θ	Shields parameter
θ_c	critical Shields parameter
η	efficiency
κ	von Karman constant (= 0.4)
λ	Darcy-Weisbach friction factor
ν	kinematic viscosity coefficient of water (= 10^{-6} m^2 s^{-1})
ν_t	eddy viscosity coefficient
ρ	density of water (= 1000 kg m^{-3})
ρ_s	density of sediment (= 2650 kg m^{-3})
τ_b	bed shear stress
τ_{bc}	threshold bed shear stress for particle motion
—	Reynolds-averaged value
'	effective shear-stress component, also called grain-related shear-stress component
$_p$	pump

D Liang, Jan 2013