ENGINEERING TRIPOS PART IIA

Wednesday 24 April 2013, 14:00 – 15:30

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

N.B. THE FINAL THREE PAGES OF THIS PAPER ARE SPECIAL DATA SHEETS

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version Final

1 *(a)* For an M/M/1 queuing system, suppose that both arrival rate and service rate are tripled.

(i) How is L, the average number of customers in the system, changed? Support your answer with calculations. [15%]

(ii) How is W, the average time in the system, changed? Support your answer with calculations. [15%]

(iii) How is the steady-state probability distribution changed? Support your answer with calculations. [15%]

(b) An investment analyst has predicted that annual return of S&P 500 will follow a normal distribution with a mean of 10% and a standard deviation of 4% for the year 2014. The investment analyst wants to outperform S&P 500 for the year 2014. She is considering a mutual fund F. The annual return of F is believed to follow a normal distribution with a mean of 9% and a standard deviation of 7%.

(i) Assume that the correlation between S&P 500 and F is expected to be -0.3. Let P be a portfolio with 50% of the investment in S&P 500 and with 50% of the investment in F. Calculate the mean and standard deviation of annual return of portfolio P. [10%]

(ii) Assume that S&P 500 and F are statistically independent. If the investment analyst adopts F, what is the probability that she will outperform the S&P 500? [15%]

(c) A regression equation is constructed for dependent variable Y and independent variable X.

(i) Interpret the correlation coefficient between X and Y. [10%]

(ii) Interpret the R-square statistic. [10%]

(iii) Interpret the slope of the regression equation. [10%]

2 *(a)* At the end of period i, TruckCo observes its inventory level. Then an order may be placed (and is instantaneously received) at the beginning of period i+1 before the demand in period i+1 is observed. We are given the following information: (1) The cost for having excess inventory at the end of a period is £2 per unit. (2) The penalty for each unit of demand not met on time is £3. (3) Placing an order cost is 50p per unit plus a £5 ordering cost. During each period, demand is equally likely to be equal to 1, 2 and 3 units.

TruckCo considers the following ordering policy: At the end of period i, if the inventory is 1 unit or less, order sufficient units to bring the inventory to 4 units at the beginning of period i+1; otherwise order nothing.

(i) What percentage of the time will the inventory level at the end of each period be 4 units? [5%]

(ii) Determine the transition probability matrix. [15%]

(iii) What percentage of the time will the inventory level at the end of each period be 0 units? 1 unit? 2 units? 3 units? [25%]

(iv) Under this ordering policy, determine the following three average costs per period incurred: ordering, inventory, and penalty for not meeting the demand. [25%]

(b) A maker of ink cartridges for colour ink-jet printers has developed a new system for storing the ink. They think the new system will result in a greater capacity product. In order to determine whether this is the case, a test was developed in which a sample of 35 new cartridges was selected. The new cartridges were installed in a printer and test pages were run until the cartridge was empty. The test showed that the mean and standard deviation for the number of pages an ink cartridge can print were 288 and 16.23 respectively. The same test was carried out for a sample of 35 original cartridges. The test for the original cartridges showed that the mean and standard deviation for the number of pages an ink cartridge can print were 279 and 15.91 respectively.

Based on the sample data and a significance level equal to 0.05, determine if the new system is different from the old system in terms of the average number of pages an ink cartridge can print. Be sure to follow the framework of hypothesis testing. [15%]

(c) Explain the relationship and differences between Winter's multiplicative smoothing method and the single smoothing method. [15%]

3 *(a)* The management of Hotel ABC, in Cambridge, wants to better understand demand for fully paid advance bookings. It believes that the number of fully paid advance bookings depends on the `Number of quotes' given via its internet booking system, and the `Competitive price' of rooms. The `Competitive price' is the average of the price, in pounds, of quotes given by five local hotels that compete for customers with Hotel ABC. The hotel uses a revenue management system that sets the price of a room depending on both the number of unfilled rooms and the competitive price, at the time of an advance quote.

You are given 194 days of data for single room bookings. The price data available to you is restricted to the competitive price for single rooms on each day of the data set, not the competitive price that was used to give an advance price quote on any earlier day. The regression analysis for fully paid advance bookings is shown in Table 1.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.846
R Square	0.716
Adjusted R Square	0.713
Standard Error	7.757
Observations	194

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	28974.504	14487.252	240.747	0.000
Residual	191	11493.667	60.176		
Total	193	40468.170			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	25.344	4.888	5.185	0.000	15.702	34.986
Number of quotes	0.086	0.007	13.258	0.000	0.073	0.099
Competitive price	0.156	0.095	1.635	0.104	-0.032	0.344

Table 1: Regression Summary Output.

(i) Specify the regression equation and explain the variables and parameters in the equation. [10%]

(ii) With a 95% confidence level, what range of bookings would you expect for a particular day when there are 1000 price quotes and the competitive price on that day is £36? How would you present this data to hotel management? [15%]

(iii) Based on the regression equation, what would be the range of values for the change of the number of bookings if `Number of quotes' is increased by 10? [10%]

(iv) How comfortable are you with the regression model summarised in Table 1? How would you explain to hotel management whether the model should be trusted or whether there are deficiencies in the model and what might need to be considered to rectify these. [15%]

(b) Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially, with a mean of 3 minutes.

(i) What is the probability that a person arriving at the booth will have to wait? [10%]

(ii) What is the average length of the queue? [10%]

(iii) The telephone company will install a second booth if it is convinced that an arrival would expect to have to wait at least three minutes for the phone. What would be the average inter-arrival time in order to justify a second booth? [10%]

(c) Use a simple business example to illustrate the flaw of averages. [10%]

(d) Compared with the back-of-envelope approach, what are the merits of what-if analysis (sensitivity analysis)? Compared with Monte Carlo Simulation, what are drawbacks of what-if analysis? [10%]

4 *(a)* The XYZ television network earns an average of $400,000 from a good show and loses an average of $100,000 from a bad show. Of all shows produced by the network, 25% turn out to be good shows and 75% turn out to be bad shows. For $40,000 a market research firm will have an audience watch a pilot of a prospective show and give its opinion about whether the show will be a good one or a bad one. If a show is actually going to be a good one, there is a 90% chance that the market research firm will predict the show to be a good one. If the show is actually going to be a bad one, there is an 80% chance that the market research firm will predict the show to be a bad one.

(i) Determine the expected monetary value for the XYZ television network without the market research. [15%]

(ii) Determine the Expected Value of Perfect Information. [20%]

(iii) Determine the Expected Value of Sample Information. [30%]

Hint: Recall the Bayes' Theorem: $P(A_i \mid B) = P(A_i) \times P(B \mid A_i)/(\Sigma_j P(A_j) \times P(B \mid A_j))$.

(b) There are three types of grocery store in a given community. On January 1 2012, 1/4 of the community shopped at store I, 1/3 at store II, and 5/12 at store III. Each month, store I retains 90% of its customers and loses 10% of them to store II. Store II retains 5% of its customers and loses 85% of them to store I and 10% to store III. Store III retains 40% of its customers and loses 50% of them to store I and 10% to store II.

(i) Determine the transition probability matrix. [5%]

(ii) What proportion of customers will each store retain by February 1, 2012 and by March 1, 2012? [10%]

(iii) Assuming the same pattern continues, what will be the long-run distribution of customers among the three stores? [10%]

(c) Explain the following concepts:

(i) The efficient frontier for the mean-variance diagram in portfolio theory. [5%]

(ii) A 95% confidence interval for the population proportion. [5%]

END OF PAPER

SPECIAL DATA SHEET 1

Formula sheet

Standard errors

$$STEM=\frac{\sigma}{\sqrt{n}}\approx\frac{s}{\sqrt{n}},\quad STEP=\sqrt{\frac{p(1-p)}{n}}\approx\sqrt{\frac{q(1-q)}{n}},$$

$$STEDM=\sqrt{\frac{n_1s_1^2+n_2s_2^2}{n_1+n_2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}.$$

Covariance, Correlation and Regression

Consider data pairs $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$.

Let m_X and m_Y denote the respective means of the X and Y data.

Let s_X and s_Y denote the respective standard deviations of the X and Y data.

Covariance between X and Y is given by

$$\mathrm{cov}(X,Y)=\frac{\sum_{i=1}^{n}(X_i-m_X)(Y_i-m_Y)}{n}=\frac{\sum_{i=1}^{n}X_iY_i}{n}-m_Xm_Y$$

The correlation coefficient between X and Y is given by

$$\mathrm{correl}(X,Y)=r=\frac{\mathrm{cov}(X,Y)}{s_Xs_Y}.$$

The line of best fit is given by

$$Y-m_Y=\frac{rs_Y}{s_X}(X-m_X).$$

Variance of a portfolio

Consider three random variables x, y and z with means m_x, m_y, and m_z, respectively; variances Var(x), Var(y), and Var(z), respectively; and covariance between x and y, for example, given by the formula above. Given any numbers α_x, α_y, α_z, let $v=\alpha_x x+\alpha_y y+\alpha_z z$. Then the variance of v is given by

$$\begin{aligned}\mathrm{Var}(v)=&\alpha_x{}^2\mathrm{Var}(x)+\alpha_y{}^2\mathrm{Var}(y)+\alpha_z{}^2\mathrm{Var}(z)\\&+2\big(\alpha_x\alpha_y\mathrm{cov}(x,y)+\alpha_y\alpha_z\mathrm{cov}(y,z)+\alpha_x\alpha_z\mathrm{cov}(x,z)\big)\end{aligned}$$

Time Series Forecasting (Winters' multiplicative smoothing method)

$$E_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(E_{t-1} + T_{t-1})$$
$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$
$$S_t = \gamma \frac{X_t}{E_t} + (1-\gamma)S_{t-c}$$

$$F_{t+k} = (E_t + kT_t)S_{t+k-c}$$

Markov Chains (calculate probabilities for first passage time and expected first passage times)

$$f_{ij}(1) = P_{ij}$$
$$\vdots$$
$$f_{ij}(n) = P_{ij}^{(n)} - f_{ij}(1)P_{jj}^{(n-1)} - \ldots - f_{ij}(n-1)P_{jj}^{(1)}.$$

$$E(H_{ij}) = 1 + \sum_{k \neq j} E(H_{kj})P_{ik}, \forall i.$$

Queueing Theory (Poisson distribution, exponential distribution, performance metrics for the M/M/s queue, the M/M/1 queue is a special case of the M/M/s queue)

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0,1,\ldots$$

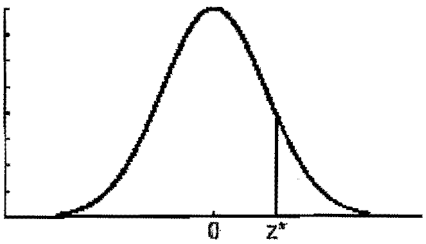
$$P(X \le t) = 1 - e^{-\mu t}, \quad \forall t \ge 0.$$

$$p_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$$p_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} p_0 & \text{if } 0 \le n \le s \\ \frac{(\lambda/\mu)^n}{s!s^{n-s}} p_0 & \text{if } n \ge s \end{cases}$$

$$L_q = \left(\frac{(\lambda/\mu)^{s+1}}{(s-1)!(s-\lambda/\mu)^2}\right) p_0.$$

SPECIAL DATA SHEET 2
Standard Normal Distribution Table
(Areas under the standard normal curve beyond z*, i.e., shaded area)



z*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010