No alterations required. J.M.M.

ENGINEERING TRIPOS PART IIA

Monday 29 April 2013 9.30 to 11.00

Module 3F2

SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: final

1 (a) A system is governed by the nonlinear state-space equations

$$\dot{x} = f(x,u), \qquad y = g(x,u)$$

and has an equilibrium at (x_0, u_0). Explain how a linearised state-space model of the form

$$\delta\dot{x} = A\delta x + B\delta u, \qquad \delta y = C\delta x + D\delta u$$

can be obtained, which approximately describes the system behaviour when it is perturbed slightly from the equilibrium, where $\delta u, \delta x, \delta y$ denote perturbations of u, x, y from their equilibrium values. [30%]

(b) Figure 1 shows a steel ball of mass m which is suspended a distance h below a magnet, by a magnetic force produced by a current u flowing through the winding. The height of the ball is determined by

$$\frac{d^2h}{dt^2} = g - \frac{u}{mh^2}$$

(i) Find the equilibrium height h_0 when the current is steady with value u_0. [10%]

(ii) Show that a linearised state-space model, valid in the vicinity of the equilibrium (h_0, u_0), is given by

$$\delta\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{2g}{h_0} & 0 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ -\frac{1}{mh_0^2} \end{bmatrix} \delta u, \qquad \delta h = \begin{bmatrix} 1 & 0 \end{bmatrix} \delta x$$

for a suitable choice of state vector x. [30%]

(iii) Comment on the stability of this linearised system. [10%]

(iv) What is the transfer function of this system, from δu to δh? [20%]

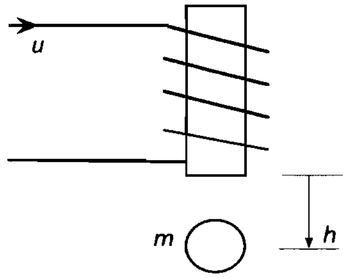

Fig. 1

2 (a) What is meant by the *return ratio* of a feedback system? [10%]

(b) How is the return ratio related to the closed-loop poles of a feedback system? [10%]

(c) 'Every point on a root locus satisfies the *angle criterion*.' Explain what is meant by this statement (for single-input, single-output systems), and give a brief derivation. [20%]

(d) In a power generation system, a power system stabiliser adjusts the set-point of a voltage regulator in response to changes in generated real power. The return ratio of the system 'seen' by the stabiliser is

$$L(s) = k\frac{s^2 + s + \frac{5}{4}}{(s^2 + 25)(s^2 + 2s + 5)}$$

where k is a positive gain.

(i) Find the poles and zeros of this return ratio, and show their locations in the complex plane. [15%]

(ii) Show that the root-locus diagram for this return ratio does not cross the imaginary axis for any $k > 0$. [25%]

(iii) Sketch the root-locus diagram for this return ratio. [20%]

3 Figure 2 shows a cart of mass m which can move smoothly along a straight rail. A bob of mass M hangs on the end of a light rigid rod of length ℓ, which is pivoted smoothly at the cart. The position of the cart along the rail is z, and the angle of the rod to the vertical is θ, as shown in Fig. 2. A force u acts on the cart, parallel to the rail. A camera measures the position y of a point on the rod, a distance a from the cart. For small angles θ the equations of motion of the cart and rod are given by

$$m\ell\ddot{\theta} = u - (m+M)g\theta \quad \text{and} \quad m\ddot{z} = u - Mg\theta$$

and the measurement is given by

$$y = z - a\theta$$

(a) Define a suitable state vector and write these equations in state-space form. [20%]

(b) Show that this system is not observable if [30%]

$$a = \frac{M}{m+M}\ell$$

(c) Suggest a physical explanation for the loss of observability at this value of a. [10%]

(d) Suppose that a is not exactly at the value found in part (b) but is close to it. The system will then be observable but close to being unobservable. Explain how 'closeness to unobservability' can be defined precisely. [30%]

What is the practical consequence of placing the camera at such a value of a? [10%]

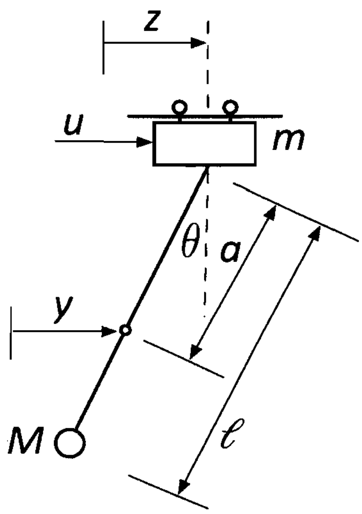

Fig. 2

4 (a) Define *controllability* of a linear state-space system. [10%]

(b) Give a standard test for the controllability of a linear state-space system. [10%]

(c) Two linear systems, with state vectors x_1 and x_2, respectively, have the same input u, and are governed by the *same* equations:

$$\dot{x}_1 = Ax_1 + Bu, \qquad \dot{x}_2 = Ax_2 + Bu$$

Show that the combined system, with state vector $[x_1^T, x_2^T]^T$, is not controllable. [20%]

(d) Suppose that the first of these systems has output $y = B^T x_1$, and that this signal becomes the input to the second system. Show, by constructing a simple example, that it is possible for the combined system to be controllable in this case. [30%]

(e) Three identical integrators, each with transfer function k/s $(k > 0)$, are connected in series. Each one is realised as

$$\dot{x}_i = \sqrt{k}u_i, \quad y_i = \sqrt{k}x_i, \quad \text{for } i = 1,2,3.$$

Design a state-feedback scheme that places all the closed-loop poles at -1. [30%]

END OF PAPER