ENGINEERING TRIPOS PART IIA

Thursday 2 May 2013 2 to 3.30

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL version

1 (a) Explain the purpose of an equaliser in a baseband digital communication system. Discuss briefly the zero forcing equaliser (ZFE) and the decision feedback equaliser (DFE) comparing their design criteria and problems. [30%]

(b) The z transform of a single received pulse in a baseband digital communication system sampled at the optimum sample instants is given by,

$$p(z) = 1 + 0.2z^{-1} - 0.1z^{-2}$$

Design a 3-tap finite impulse response (FIR) equaliser for this baseband pulse using the zero forcing criterion, i.e., $y_0 = 1$, $y_1 = 0$, $y_2 = 0$. [20%]

(c) For a binary polar data transmission scheme having the pulse response given in part (b) and pulse amplitude $\pm 1\ \text{V}$, determine

(i) The worst case bit error rate (BER) with and without the equaliser from (b).

(ii) The expected BER with the equaliser from part (b) assuming that the input data to the system is equiprobable.

(iii) The BER with an ideal DFE stating any assumptions made.

Assume the presence of additive white Gaussian channel noise with a standard deviation of $0.3\ \text{V}$ and a mean of $0\ \text{V}$.

[50%]

Note: the Gaussian error integral approximation is

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

2 A (16,9) linear binary block code C is constructed as follows. The data are located in code bits 15, 14, 13, 11, 10, 9, 7, 6 and 5. Code bit 12 is an even parity check on code bits 15 to 13, code bit 8 checks code bits 11 to 9, and code bit 4 checks code bits 7 to 5. Code bit 3 is an even parity check on code bits 15, 11 and 7, code bit 2 checks code bits, 14, 10 and 6, code bit 1 checks code bits 13, 9 and 5, and finally, code bit 0 checks code bits 12, 8 and 4.

Note that the left-most code bit is labelled code bit 15 and the right-most code bit is labelled code bit 0 . In addition, assume that the left-most data bit in a data word is labelled data bit 8, while the right-most data bit is labelled data bit 0.

(a) Find the generator and parity check matrices for the code C . [40%]

(b) Show that code C would be unchanged if code bit 0 were calculated as an even parity check on code bits 3 to 1. [10%]

(c) Find the minimum distance of code C. [15%]

(d) Explain how syndromes can be used to implement an error correction decoder for code C. [15%]

(e) Show that code C can be represented as a 4 by 4 array with the data bits located in a 3 by 3 array and even parity checks on each row and column. Hence deduce an alternative to the syndrome based decoder proposed in part (d) for correcting single bit errors. What is the advantage of the alternative approach over the syndrome based decoder? [20%]

3 The digital modulation method known as binary phase-shift keying (BPSK) can be represented by the following phasor waveform:

$$p(t) = e^{j\phi_0} \sum_k b_k g(t - kT_b)$$

where $b_k = \pm 1$, $g(t)$ is a rectangular pulse of amplitude a_0 that is non-zero from $t = 0$ to T_b , T_b is the bit period, and ϕ_0 is a constant phase offset.

(a) Show how a purely real modulated signal $s(t)$, with carrier frequency ω_C , can be generated from this waveform and determine which two carrier phases are produced by this modulation process. [20%]

(b) Signal $s(t)$ is corrupted by additive channel noise such that the receiver of the signal recovers a phasor waveform $r(t) = p(t) + p_N(t)$, where $p_N(t) = (n_1(t) + jn_2(t))e^{j\phi_0}$ represents the noise. Derive an expression for the output of a matched correlation demodulator which, on bit period k , calculates

$$y(k) = G \int_{kT_b}^{(k+1)T_b} \mathrm{Re}\,[r(t) g(t - kT_b) e^{-j\phi_0}]\, dt$$

where G is a constant gain, and show that $y(k)$ depends only on one of the components of $p_N(t)$. [15%]

(c) By separating the expression for $y(k)$ into its signal and noise components, calculate the signal-to-noise voltage ratio of $y(k)$ in terms of E_b , the energy per bit of $p(t)$, and N_0 , the 2-sided noise power spectral density of $p_N(t)$. [30%]

(d) Hence show how the probability of bit error in this receiver may be calculated, assuming additive white Gaussian noise. [15%]

(e) Briefly explain how the bit error performance of quadrature phase shift keying (QPSK) modulation relates to that of BPSK, and why QPSK is usually preferred over BPSK for practical digital transmission systems. [20%]

4 (a) Explain how the autocorrelation function, $r_{pp}(\tau)$, of the phasor waveform $p(t)$ of a modulated digital signal is related to the power spectral density, $P(\omega)$, of the phasor, and hence to the power spectral density, $S(\omega)$, of the modulated signal. [20%]

(b) Hence show that the bandwidth of the main spectral lobe of a linearly modulated signal depends only on the symbol rate of the modulation phasor and does not depend on the number of signalling levels (states) for a multi-level modulation process. [30%]

(c) Derive an expression for how the main-lobe bandwidth of a signal employing M^2-QAM (quadrature amplitude modulation) relates to the bit period, T_b , of the input data and to M^2 , the number of modulation states. You should assume random equiprobable binary data and rectangular full-width pulse shaping in the modulator. [30%]

(d) Explain the trade-offs that affect the choice of M in practical quadrature modulation systems, giving examples of two current systems to illustrate this. [20%]

END OF PAPER