ENGINEERING TRIPOS PART IIA

Thursday 25 April 2013 9.30 to 11.00

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

*Answer **one** question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

Supplementary page with partially complete diagrams for Question 5.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Medical Image Acquisition*

*Answer **one** question from this section*

1 (a) Consider the equation

$$\mu(x, y) = \int_0^{\pi} p_\phi(s) * q(s) \; d\phi$$

which is the essence of the filtered backprojection algorithm.

(i) Explain carefully what each term represents. [10%]

(ii) Show how the equation can be approximated as a discrete summation, and hence list the key steps involved in filtered backprojection. [20%]

(b) A student queries whether a filter is strictly necessary: what are the shortcomings of *unfiltered backprojection*? To answer this question:

(i) Consider a point attenuator at the origin. Sketch its sinogram. [10%]

(ii) Hence, sketch the result of unfiltered backprojection of this point attenuator, assuming a large number of projections. [10%]

(iii) Hence, in the limit of infinitely many projections, explain why an arbitrary attenuation distribution would be reconstructed as the correct distribution convolved with a circularly symmetric kernel of magnitude $1/r$. [20%]

(c) Figure 1 shows an attenuation distribution (a) and several reconstructions (b–f) obtained using a virtual CT simulator. The simulator allows control of both the imaging parameters and the reconstruction parameters. Discuss how the parameters likely differed between (b), (c), (d), (e) and (f). [30%]

(cont.

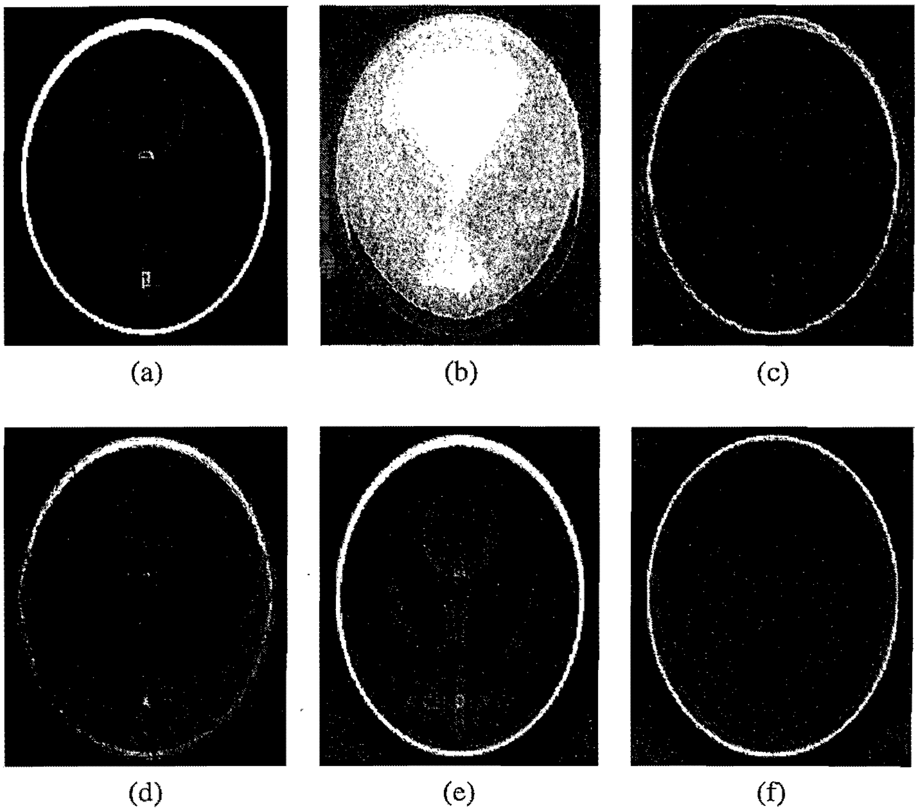

Fig. 1

(TURN OVER

2 (a) What do the acronyms SPECT and PET stand for? List the relative strengths and weaknesses of the two imaging modalities. [20%]

(b) Explain why iterative algorithms are generally preferred to filtered backprojection when reconstructing SPECT and PET images. [20%]

(c) For SPECT/PET, sketch approximate relationships between: (i) image signal-to-noise ratio (SNR) and measurement time; and (ii) image SNR and radionuclide dose. In each case, justify your answer. [20%]

(d) In *time-of-flight PET*, the difference between the detection times of the two photons is used to infer where, on the line between the two detectors, the photons were emitted. Suppose this time difference can be measured with precision $\pm\Delta t$.

(i) Deduce an expression for the uncertainty $\pm\Delta x$ with which the emitter's location (along the line) can be measured. State any assumptions you make. [20%]

(ii) If $\Delta t = 250\,\text{ps}$, calculate Δx. In the light of your answer, discuss the likely benefits of time-of-flight PET. [20%]

THIS PAGE DELIBERATELY LEFT BLANK

(TURN OVER

SECTION B *Extracting Information from 3D Data*

*Answer **one** question from this section*

3 (a) Define *interpolation* and *approximation*, and explain their respective advantages and disadvantages in the context of medical imaging data. [20%]

(b) A one-dimensional data set has values $F(x) = (0, 0, 0, 1, 0, 0, 0)$ at locations $x = (-3, -2, -1, 0, 1, 2, 3)$. Sketch both linear and nearest-neighbour interpolants of this data in the range $-2 < x < 2$. [10%]

(c) A piecewise parametric function S is fitted to the data F in (b), such that each segment S_x of S from x to $x + 1$ is defined by

$$S_x(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \mathbf{M} \begin{bmatrix} F(x-1) \\ F(x) \\ F(x+1) \\ F(x+2) \end{bmatrix}, \quad x \in \{-2, -1, 0, 1\}$$

where $0 \le t \le 1$. The matrix $\mathbf{M}$ is a function of two parameters a and b:

$$\mathbf{M} = a \begin{bmatrix} -1 & -1 & 1 & 1 \\ 2 & 1 & -2 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \frac{b}{6} \begin{bmatrix} -1 & -9 & 9 & 1 \\ 3 & 12 & -15 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & -2 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(i) What are the conditions on a and b for S to interpolate F? [15%]

(ii) By considering the gradient at $t = 0$ and at $t = 1$, or otherwise, show that S has C^1 continuity. [15%]

(iii) What are the conditions on a and b for S to have C^2 continuity? [20%]

(iv) Sketch S in the range $-2 < x < 2$ for each of the following conditions: $\{a = 0, b = 1\}$, $\{a = \frac{1}{2}, b = 0\}$ and $\{a = 0, b = 0\}$. [20%]

4 (a) Briefly discuss the processes involved in extracting a surface from a 3D medical data set, highlighting any difficulties and commenting on any particular challenges posed by CT and ultrasound data. [30%]

(b) It is required to define the normals to an iso-surface within a CT data set.

(i) The iso-surface is represented with a polygonal mesh composed of triangles. Explain how to calculate consistent surface normals at each vertex. [10%]

(ii) The iso-surface is now represented with a set of cubic parametric B-spline patches $P(s,t) = [x(s,t) \;\; y(s,t) \;\; z(s,t)]$, where

$$x(s,t) = [s^3 \;\; s^2 \;\; s \;\; 1]\mathbf{M}Q_x\mathbf{M}^T[t^3 \;\; t^2 \;\; t \;\; 1]^T$$

and similarly for $y(s,t)$ and $z(s,t)$. Show how to calculate the normal to this surface at any surface location (s,t), in terms of $\mathbf{M}$, s, t and $Q_{\{x,y,z\}}$. [20%]

(iii) What do the elements of $Q_{\{x,y,z\}}$ in (ii) represent, and how might you derive these from the 3D data for one surface patch $P(s,t)$? [10%]

(iv) Suggest how you could calculate surface normals directly from the data, without extracting a specific iso-surface representation. [10%]

(c) Contrast the properties of the surface normals derived from the three schemes in (b) (i), (ii) and (iv), paying particular attention to the effects of noise in the CT data. [20%]

(TURN OVER

SECTION C *3D Graphical Rendering*

*Answer **one** question from this section*

5 (a) Explain what is meant by the computer graphics term *rasterisation*. [10%]

(b) A triangle has vertices A, B and C, as shown in Fig. 2. This is the triangle's true shape: it lies parallel to the view plane and its vertices just happen to map precisely to pixel centres in the frame buffer. The triangle is processed by a standard surface rendering pipeline. Non-horizontal edges are rasterised as follows:

$$
\begin{aligned}
&m = (x_2 - x_1)/(y_2 - y_1);\\
&\texttt{for } y = y_1 \texttt{ to } y_2 \texttt{ do } \{\\
&\quad x = x_1 + m(y - y_1);\\
&\quad \textit{output}(\textit{round}(x), y);\\
&\}
\end{aligned}
$$

where (x_1, y_1) and (x_2, y_2) are the edge's integer end points, $y_2 > y_1$ and the function *round()* rounds a real number to the nearest integer (and upwards when the fractional part is 0.5). Two copies of Fig. 2 and two extra pixel grids can be found on a supplementary page, for you to annotate and hand in with your answer.

(i) Indicate on grid (a) of the supplementary page which pixels are output by the edge rasterisation algorithm. [10%]

(ii) On grid (b), indicate which pixels are subsequently shaded. [10%]

(iii) The (monochrome) intensities at A, B and C are 30, 40 and 50 respectively. Assuming Gouraud shading, calculate the intensity at pixel P. [10%]

(cont.

(c) The rendering is part of an animated sequence, in which the view coordinate system is subsequently rotated 45° around the z_v axis, which passes directly through the centre of pixel P. The effect is that objects in the scene appear rotated 45° clockwise around $(200, 200)$. Find the new locations of the triangle's vertices, round them to the nearest pixels and hence sketch the rendered appearance of the rotated triangle. Use grids (c) and (d) on the supplementary page as you see fit. Also find the new rendered intensity at pixel P. [30%]

(d) With reference to (b) and (c), comment on the nature of any rendering artifacts you have identified and discuss how they might be suppressed. [30%]

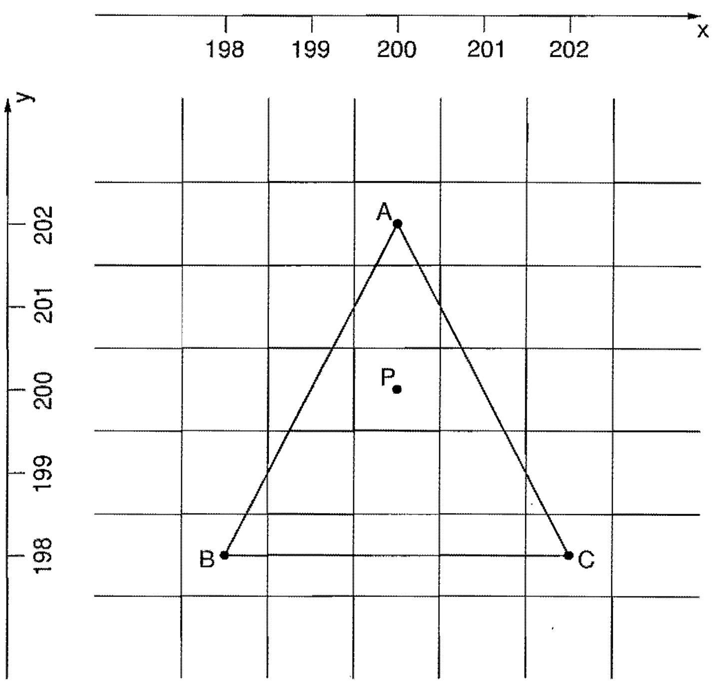

Fig. 2

6 (a) Describe briefly the key features of the *shadow z-buffer* algorithm. [30%]

(b) A scene comprising several smooth, continuous surfaces is rendered using the shadow z-buffer algorithm. There is a single light source. For the viewpoint, the relationship between homogeneous 3D screen and local coordinates is given by

$$\begin{bmatrix} wx_s \\ wy_s \\ wz_s \\ w \end{bmatrix} = \begin{bmatrix} 0.005 & 0 & 0 & 0 \\ 0 & 0.005 & 0 & 0 \\ 0 & 0 & -0.01 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

For the light source, the relationship is

$$\begin{bmatrix} wx'_s \\ wy'_s \\ wz'_s \\ w \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -50 \\ 0 & 1 & 0 & 0 \\ -1.25 & 0 & 0 & 75 \\ -1 & 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

Both the viewpoint's frame buffer and the light source's shadow z-buffer are 400×400 pixels. The viewpoint's rasterisation algorithm outputs a point A with device coordinates $(200, 200)$ and corresponding 3D screen coordinates $(x_s, y_s, z_s) = (0, 0, 0.5)$.

(i) By inspection of the two transformation matrices, describe in broad terms the key properties of the viewing arrangement and the light source. [15%]

(ii) Calculate point A's 3D screen coordinates (x'_s, y'_s, z'_s) as seen from the light source. [15%]

(iii) Hence calculate by how much point A's intensity should be attenuated to allow for shadowing. The relevant portion of the shadow z-buffer is shown in Fig. 3, with each z'_s value represented as an 8-bit integer. The dots in Fig. 3 are the centres of the eight pixels neighbouring point A, as output by the viewpoint's rasteriser and mapped to the shadow z-buffer. [20%]

(iv) Suppose you had found the answer to (ii) to be $(0, 0, -0.1)$ (though this is not what you should have found). What would this tell you about the likely efficacy of the shadow z-buffer algorithm in this instance? [20%]

(cont.

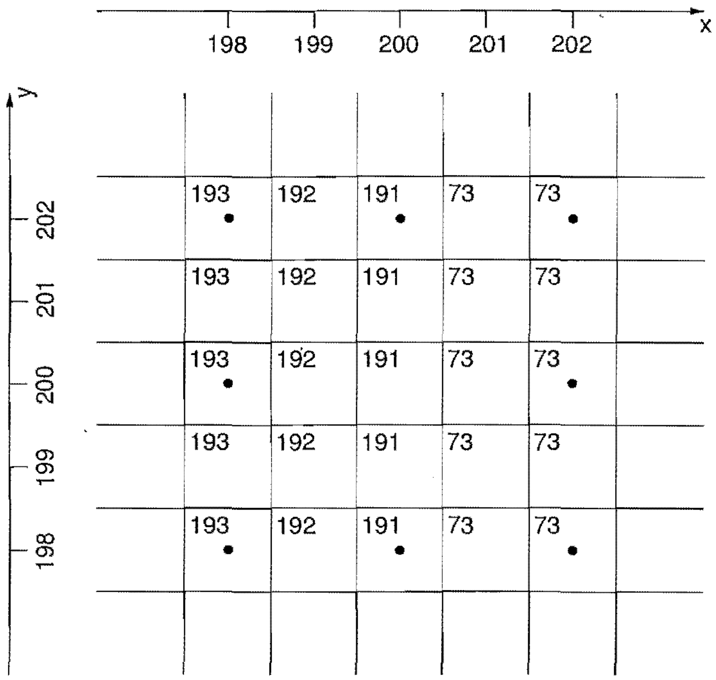

Fig. 3

END OF PAPER

Part IIA 2013

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

2. (d) (i) $\Delta x = c\Delta t/2$, (ii) 3.75 cm

5. (b) (iii) 37.5

   (c) 36.25

6. (b) (ii) $(0, 0, 0.75)$, (iii) 25%