2014 IIA 3D5 - Water Engineering Dr D. Liang

1.

(a.i) The total volume of water generated by the 8-hour rain at the catchment outlet is:

$$(250+860+930+520+240+90+20)\times 4\times 3600 = 41{,}904{,}000 \text{ m}^3$$

This should be equal to the volume of the rainfall.

$$A\times(7+14)\times 4\times 10^{-3} = 41{,}904{,}000$$

$$A = 4.99\times 10^{8} \text{ m}^2 \approx 500 \text{ km}^2$$

(a.ii)

According to the unit hydrograph theory:

Duration (h)	0-4	4-8	8-12	12-16	16-20	20-24	24-28	28-32
Total discharge due to the 8-hour rain	250	860	930	520	240	90	20	0
Discharge due to 4-hour 7 mm/h rain	**250**	**360**	210	100	40	10	0.0	0.0
Discharge due to 4-hour 14 mm/h rain	0	**500**	**720**	420	200	80	20	0.0

The first 4-hour discharge, 250 m^3/s, is entirely due to the 7 mm rain, because the 14 mm rain has not fallen yet. For linear catchment, if the rainfall intensity doubles, then the discharge also doubles. Hence, a 4-hour 14 mm rainfall will generates 500 m^3/s discharge in the first 4 hours. Based on this argument, the 14 mm/h rain occurring at 4-8 hours contributes 500 m^3/s discharge to the total discharge at 4-8 hours. The total discharge at 4-8 hours is 860 m^3/s, so the 7 mm/h rain occurring at 0-4 hours contributes (860-500) = 360 m^3/s discharge at 4-8 hours.

(a.iii)

The 7 mm/h rain occurring at 0-4 hours generates 360 m^3/s discharge at 4-8 hours, so the 14 mm/h rain occurring at 4-8 hours shall generate 720 m^3/s discharge at 8-12 hours. Because the total discharge at 8-12 hours is given to be 930 m3/s, the 7 mm/h rain occurring at 0-4 hours shall be responsible for the (930-720)=210 m^3/s discharge.

Following this argument, the total discharge can be split as shown in the above table. Based on the discharge variation due to the first 4-hour rainfall, the 4-hour unit hydrograph is as follows:

Duration (h)	0-4	4-8	8-12	12-16	16-20	20-24	Total
Discharge due to 4-hour 7 mm/h rain	250	360	210	100	40	10	970
Discharge percentages	26	37	22	10	4	1	100

(a.iv)

The infiltration rates after 2 hours:

$$f = f_c + (f_0 - f_c)e^{-K_f t} = 6 + (20-6)e^{-0.4\times 2} = 12.29 \text{ mm/h}$$

If there is plenty of water supply, the infiltration rate will decrease to the following value after 4 hours:

$$f = f_c + (f_0 - f_c)e^{-K_f t} = 6 + (20-6)e^{-0.4\times 4} = 8.83 \text{ mm/h}$$

Both values are bigger than the rainfall intensity in the 2nd 2 hours, so no runoff is generated by the 2nd two-hour rain.

The total infiltration in the first two hours is:

$$\int_0^2 f \cdot dt = f_c(2-0) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f \times 2} - 1\right) = 12 - \frac{14}{0.4}\left(e^{-0.8} - 1\right) = 31.3 \text{ mm}$$

Excess rainfall in the first two hours is

$$20\times2-31.3 = 8.7 \text{ mm}$$

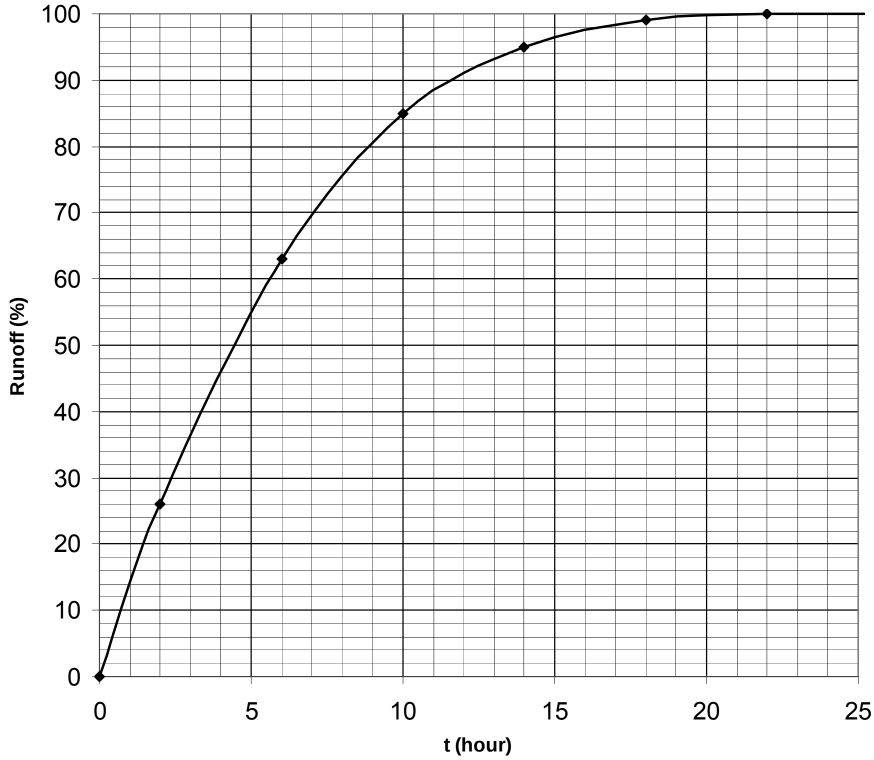
This value times the catchment areas gives the total volume of the runoff:

$$8.7\times10^{-3}\times500\times10^{6} = 4.35\times10^{6} \text{ m}^3$$

Need to construct the 2-hour unit hydrograph of the 4-hour one.
Construct the S curve based on the 4-hour unit hydrograph:

Time (hour)	0	2	6	10	14	18	22	26
Runoff (%)	0	26	63	85	95	99	100	100

Find the runoff proportions due to a two-hour uniform rainfall.



Read from S curve:

Time (hour)	0	1	3	5	7	9	11	13	15	17	19
S-curve value	0	14	36	55	70	81	89	93	96	98	100
Shift S-curve by 2 hours		0	14	36	55	70	81	89	93	96	98
2-hour hydrograph	0	14	22	19	15	11	8	4	3	2	2

The peak flow occurs at 2-4 hours, during which 22% of runoff volume is discharged. The peak discharge is then:

$$\frac{4.35\times10^{6}\times22\%}{2\times3600} = 133 \text{ m}^3/\text{s}$$

(b) In $Q=CiA$, the total rainfall, rather than the excess rainfall, should be used, and Q is the maximum flow rate. The runoff coefficient takes into account the infiltration and retention loss, *etc*. The urban areas are more impervious and smoother than the rural areas, hence less infiltration and faster flow. The urban drainage system also helps carry the rainwater away as fast as possible.

In answering this question, we do not consider the storm water storage facility in cities.

2 (a.i) Wetted perimeter: $P = 8.9 + 2 \times 1.25 \times \sqrt{2} = 12.44 \text{ m}$

Area: $A = 8.9 \times 1.25 + 1.25 \times 1.25 = 12.69 \text{ m}^2$

Hydraulic radius: A/P = 1.02 m

Manning formula: $C = \frac{1}{n} \cdot R_h^{1/6} = \frac{1}{0.025} \cdot 1.02^{1/6} = 40.13$

Chézy formula: $U = C\sqrt{R_h S_b}$, $Q = UA = AC\sqrt{R_h S_b}$

$$S_b = \left(\frac{Q}{AC}\right)^2 \frac{1}{R_h} = \left(\frac{21}{12.69 \times 40.13}\right)^2 \frac{1}{1.02} = 0.00167$$

(a.ii)

Water surface width: $B = 8.9 + 1.25 \times 2 = 11.4 \text{ m}$

Flow speed: $U = \frac{Q}{A} = \frac{21}{12.69} = 1.65 \text{ m/s}$

$Fr = \frac{U}{\sqrt{g\frac{A}{B}}} = \frac{1.65}{\sqrt{9.81 \times \frac{12.69}{11.4}}} = 0.50$ <1, so the flow is subcritical.

(b.i)

The flow is non-uniform, but steady. Based on gradually-varied flow equations:

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b}{1 - Fr^2}$$

The Froude number at the entrance is: $Fr = \frac{U}{\sqrt{gh}} = \frac{3.13/0.8}{\sqrt{9.81 \times 0.8}} = 1.40$

So, $dh/dx < 0$ at the entrance. As the water depth decreases, Fr becomes even greater than one.

(On the contrary, if the flow at the entrance is subcritical and the bed is frictionless, then the water depth, as well as the water level, will increase.)

(b.ii)

$$\frac{dh}{dx} = \frac{S_b}{1 - Fr^2} = \frac{0.005}{1 - \left(\frac{3.13/h}{\sqrt{9.81h}}\right)^2} = \frac{0.005}{1 - \frac{1}{h^3}}$$

$$\left(1 - \frac{1}{h^3}\right) dh = 0.005 dx$$

Integrate on both sides: $h + \frac{1}{2h^2} = 0.005x + C$

When x = 0, h = 0.8. So, C=1.58

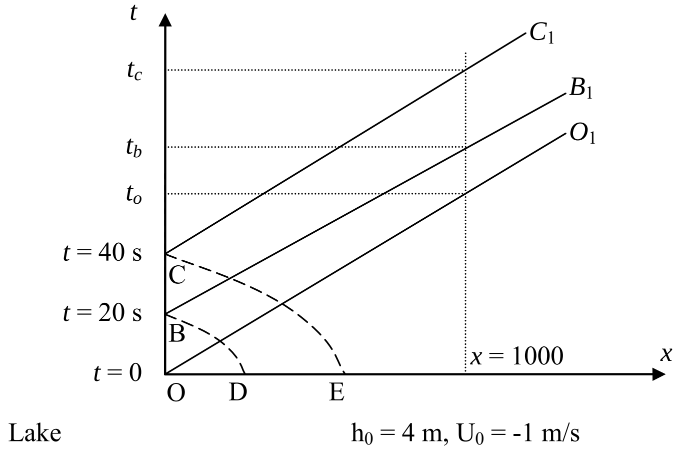
The relationship between h and x is: $h + \frac{1}{2h^2} = 0.005x + 1.58$

This is equivalent to: $h^3 - (0.005 \cdot x + 1.58) \cdot h^2 + 0.5 = 0$

This equation can be derived more easily by using $\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b$.

It can also be derived by writing Bernoulli's equation between the entrance and a downstream section.

(c.i)



The positive characteristic starting from O divides the affected/unaffected regions.

Positive line OO_1 is straight: $\frac{dx}{dt} = U_O + \sqrt{gh_O}$

$$\frac{1000-0}{t_o - 0} = -1 + \sqrt{9.81 \times 4.0} \quad => \quad t_o = 190 \text{ s}$$

Hence, the water depth 1 km upstream of the river mouth remains 4 m at 0-190 s.

Draw negative line through line B, according the –ve relationship:

$$U_B - 2\sqrt{9.81 \cdot 4.05} = -1 - 2\sqrt{9.81 \times 4} \quad => U_B = -0.922 \text{ m/s}$$

Positive line BB_1 is straight: $\frac{dx}{dt} = U_B + \sqrt{gh_B}$

$$\frac{1000-0}{t_b - 20} = -0.922 + \sqrt{9.81 \times 4.05} \quad => \quad t_b = 206 \text{ s}$$

So, the water depth rises from 4 m to 4.05 m between 190 s and 206 s.
The duration of this rise is: 16 s

(c.ii)

Draw negative line through line C, according the –ve relationship:

$$U_C - 2\sqrt{9.81 \cdot 4} = -1 - 2\sqrt{9.81 \times 4} \quad => U_C = -1 \text{ m/s}$$

Positive line CC_1 is straight: $\frac{dx}{dt} = U_C + \sqrt{gh_C}$

$$\frac{1000-0}{t_c - 40} = -1 + \sqrt{9.81 \times 4.} \quad => \quad t_c = 230 \text{ s}$$

So, the water depth drops from 4.05 m to 4.0 m between 206 s and 230 s.
The duration of this rise is: 24 s

3 (a.i)

The total roughness height: $k_s = k_s' + k_s'' = 3\times 0.003 + 0.05 = 0.059$ m

Chezy coefficient: $C = 7.8\ln\left(\frac{12.0\cdot R_h}{k_s}\right) = 7.8\ln\left(\frac{12.0\cdot h}{0.059}\right) = 7.8\ln(203.39\cdot h)$

Chezy formula: $U = C\sqrt{R_h S_b}$ => $q = C\cdot S_b^{0.5}\cdot h^{1.5}$

$2 = 7.8\cdot \ln(203.39\cdot h)\cdot 0.004^{0.5}\cdot h^{1.5}$

$h = 0.85$ m is the solution

Velocity: $U = \frac{q}{h} = \frac{2}{0.85} = 2.35$ m/s

Grain-related roughness: $k_s' = 3\times 0.003 = 0.009$ m

Grain-related Chezy factor: $C' = 7.8\ln\left(\frac{12.0\cdot R_h}{k_s'}\right) = 7.8\ln\left(\frac{12.0\times 0.85}{0.009}\right) = 54.86$

Grain-related bed shear stress: $\tau_b' = \rho g \frac{U^2}{C'^2} = 9810\times\frac{2.35^2}{54.86^2} = 18.0$ Pa

Grain-related Shields parameter:

$$\theta' = \frac{\tau_b'}{g(\rho_s - \rho)d} = \frac{18.0}{9.81\times(2650-1000)\times 3\times 10^{-3}} = 0.37$$

$$d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 3\times 10^{-3}\times\left(\frac{9.81\times(2.65-1)}{10^{-12}}\right)^{1/3} = 75.89$$

Critical Shields parameter: $\theta_c = \frac{0.30}{1+1.2d_*} + 0.055[1-\exp(-0.02d_*)] = 0.046$

Transport stage parameter: $T = \frac{\theta' - \theta_c}{\theta_c} = \frac{0.37-0.046}{0.046} = 7.04$

Van Rijn: $\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 0.053\frac{T^{2.1}}{d_*^{0.3}} = 0.053\times\frac{7.04^{2.1}}{75.89^{0.3}} = 0.87$

$q_b = 0.87\sqrt{g(s-1)\cdot d^3} = 0.87\cdot\sqrt{9.81\cdot(2.65-1)\cdot(3\times 10^{-3})^3} = 5.75\times 10^{-4}$ m³/(m·s)

Bedload is: $5.75\times 10^{-4}\times 2650 = 1.5$ kg/(m·s)

(a.ii)

At threshold motion condition: $\theta = \frac{\rho g h S_b}{g(\rho_s - \rho)d} = 0.055$

$$\frac{9810\cdot h\cdot 0.004}{9.81\cdot(2650-1000)d} = 0.055 \Rightarrow h = 22.7\cdot d$$

Chezy coefficient: $C = 7.8\ln\left(\frac{12.0\cdot h}{k_s}\right) = 7.8\ln\left(\frac{12.0\cdot 22.7d}{3d}\right) = 35.17$

Chezy formula: $U = C\sqrt{R_h S_b}$ => $q = C\cdot S_b^{0.5}\cdot h^{1.5}$

$2 = 35.17\cdot 0.004^{0.5}\cdot h^{1.5}$ => h = 0.93 m

So, the pebble size is: $d = \frac{h}{22.7} = \frac{0.93}{22.7} = 0.041$ m = 4.1 cm

(b.i)

The cross-sectional area is 3 m^2, and the velocity is 1 m/s .
This is a one-dimensional instantaneous-release problem.

$$\bar{c}(x,t) = \frac{M/A}{\sqrt{4\pi t D_x}} \exp\left(-\frac{(x-Ut)^2}{4D_x t}\right) = \frac{M/3}{\sqrt{4\pi t D_x}} \exp\left(-\frac{(x-1\cdot t)^2}{4D_x t}\right)$$

$\bar{c}(x = 10000 \text{ m}, t = 7200 \text{ s}) = 0.02 \text{ g/m}^3$, so:

$$0.02 = \frac{M/3}{\sqrt{4\pi \cdot 7200 \cdot D_x}} \exp\left(-\frac{(10000-7200)^2}{4D_x \cdot 7200}\right) = \frac{M}{902.2 \cdot \sqrt{D_x}} \exp\left(-\frac{272.2}{D_x}\right) \quad (1)$$

$\bar{c}(x = 10000 \text{ m}, t = 10800 \text{ s}) = 2 \text{ g/m}^3$, so:

$$2 = \frac{M/3}{\sqrt{4\pi \cdot 10800 \cdot D_x}} \exp\left(-\frac{(10000-10800)^2}{4D_x \cdot 10800}\right) = \frac{M}{1104.9 \cdot \sqrt{D_x}} \exp\left(-\frac{14.8}{D_x}\right) \quad (2)$$

Divide two equations on the two sides:

$$100 = \frac{902.2}{1104.9} \cdot \frac{\exp\left(-\frac{14.8}{D_x}\right)}{\exp\left(-\frac{272.2}{D_x}\right)} \Rightarrow \exp\left(-\frac{14.8}{D_x}\right) = 122.5 \cdot \exp\left(-\frac{272.2}{D_x}\right)$$

Take natural log on both sides:

$$-\frac{14.8}{D_x} = \ln(122.5) - \frac{272.2}{D_x} \Rightarrow D_x = 53.5 \text{ m}^2/\text{s}$$

From equation (2):

$$2 = \frac{M}{1104.9 \cdot \sqrt{53.5}} \exp\left(-\frac{14.8}{53.5}\right) \Rightarrow \text{M} = 21314 \text{ g} \approx 21 \text{ kg}$$

(b.ii)

$$\tau_b = \rho g R_h S_b = 9810 \cdot \frac{3}{2+3} \cdot 0.001 = 5.886 \text{ Pa}$$

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{5.886}{1000}} = 0.077 \text{ m/s}$$

Analytical solution: $D_L = 5.86 \cdot 1.5 \cdot 0.077 = 0.68 \text{ m}^2/\text{s}$

This is much smaller than the real value, because the analytical formula does not consider the lateral shearing caused by the side walls, the meandering of the river, the effect of secondary flows, etc.

(c)

The drag coefficient for small particles is inversely proportional to the Reynolds number (Stokes flow), while it is a constant for large particles. The drag force is balanced by the submerged weight of particles.

For small particles: $(\rho_s - \rho)d^3 \propto \frac{\nu}{w_s d} \cdot d^2 \cdot w_s^2 \Rightarrow w_s \propto \cdot d^2$

For large particles: $(\rho_s - \rho)d^3 \propto d^2 \cdot w_s^2 \Rightarrow w_s \propto \cdot \sqrt{d}$

4. (i) Apply Bernoulli equation between A and X to find U_x:

$$z_A + \frac{P_A}{\rho g} + \frac{U_A^{\ 2}}{2g} = z_X + \frac{P_X}{\rho g} + \frac{U_X^{\ 2}}{2g} + H_f + H_l$$

P_A is atmospheric pressure at reservoir = 0, and U_A is negligible.
Local losses at entry and at two bends between A and X:

$$H_l = \sum \zeta \frac{U^2}{2g} = (0.5 + 0.25 + 0.25) \times \frac{U_X^{\ 2}}{2g} = \frac{U_X^{\ 2}}{2g}$$

Pressure head at X: $\dfrac{P_A}{\rho g} = \dfrac{120000}{9810} = 12.23$ m

Relative roughness: $k_s/D = 0.27/450 = 0.0006$

Assuming fully turbulent flow, the Moody Diagram gives λ = 0.0173, thus

$$100 + 0 + 0 = 75 + 12.23 + \frac{U_X^{\ 2}}{2g} + \left(0.0173 \times \frac{500}{0.45}\right) \times \frac{U_X^{\ 2}}{2g} + (0.5 + 0.25 \times 2)\frac{U_X^{\ 2}}{2g}$$

$$U_X = 3.43 \text{ m/s}$$

Check the Reynolds number: $\mathrm{Re}_X = \dfrac{U_X D}{\nu} = \dfrac{3.43 \times 0.45}{1.14 \times 10^{-6}} = 1.35 \times 10^6$

From Moody Diagram: λ = 0.0178
Re-compute with this revised friction factor:

$$100 + 0 + 0 = 75 + 12.23 + \left(0.0178 \times \frac{500}{0.45} + 2\right) \times \frac{U_X^{\ 2}}{2g}$$

$$U_X = 3.39 \text{ m/s}$$

Check the Reynolds number: $\mathrm{Re}_X = \dfrac{U_X D}{\nu} = \dfrac{3.39 \times 0.45}{1.14 \times 10^{-6}} = 1.34 \times 10^6$

From Moody Diagram: λ = 0.0178 (accept the previous value)

$$Q = \frac{\pi D_X^{\ 2}}{4} \cdot U_X = \frac{3.14 \times 0.45^2}{4} \times 3.39 = 0.54 \text{ m}^3\text{/s}$$

Full mark will be given if simply testing Q in the solution.

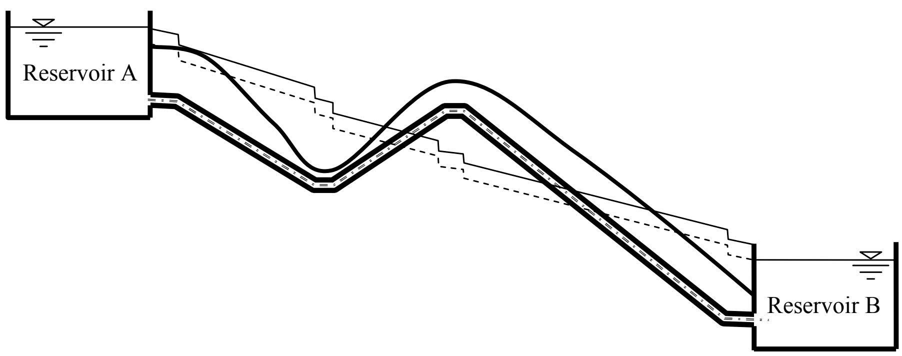
(ii) Apply Bernoulli between reservoirs:

$$z_A = z_B + H_f + H_l$$

$$100 = z_B + \left(0.0178 \times \frac{1500}{0.45} + 0.5 + 6 \times 0.25 + 1\right) \times \frac{3.39^2}{2g}$$

$z_B = 63.5$ m, which is 36.5 m below z_A.

(iii) The sketch below qualitatively shows the positions of the energy grade line and the hydraulic grade line.



Apply Bernoulli equation between X and Y to find z_Y (noting U_X and U_Y):

$$z_X + \frac{P_X}{\rho g} + \frac{U_X^2}{2g} = z_Y + \frac{P_Y}{\rho g} + \frac{U_Y^2}{2g} + H_f + H_l$$

$$75 + 12.23 = z_Y - 10.1 + \left(0.0178 \times \frac{300}{0.45} + 0.25 + 0.25\right) \times \frac{3.39^2}{2g}$$

$$z_Y = 90.1 \text{ m}$$

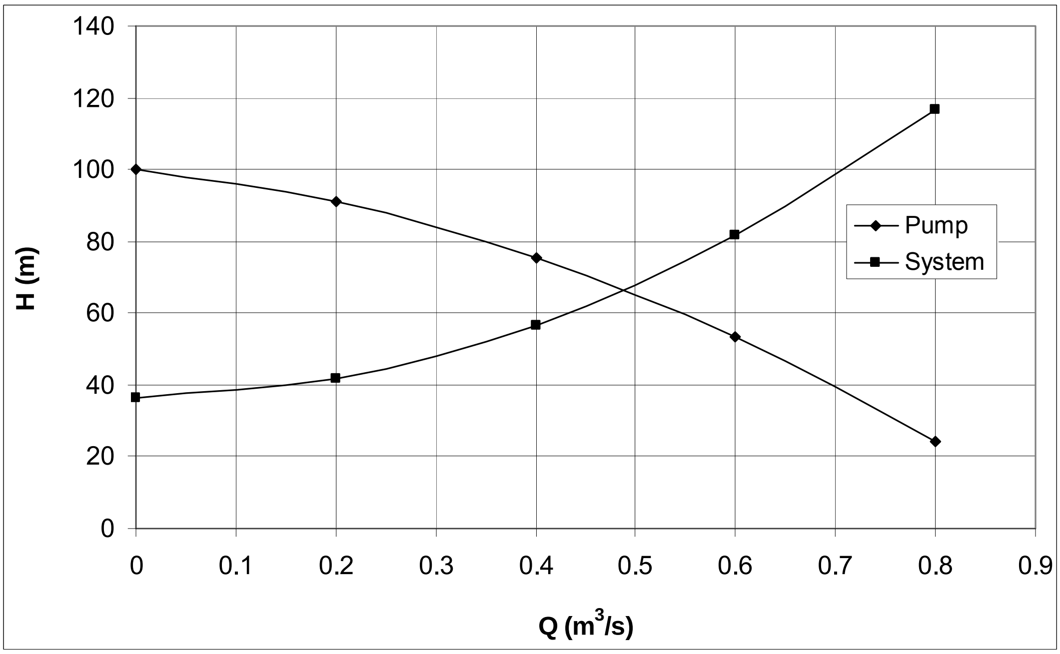
The minimum excavation depth is: 93-90.1 = 2.9 m
Negative pressure is avoided, because it may induce the pollutants in the ground water to be sucked into the pipeline at joints and cracks.

(iv) Develop the system curve.
Static head = 100 – 63.5 = 36.5 m

$$\text{Losses in head} = \left(\lambda \times \frac{1500}{0.45} + 0.5 + 6 \times 0.25 + 1\right) \times \frac{U^2}{2g}$$

λ is determined according to $\mathrm{Re} = \frac{UD}{\nu}$ and $k_s/D = 0.0006$

Q (m^3/s)	0	0.2	0.4	0.6	0.8
U (m/s)	0.00	1.26	2.52	3.77	5.03
Re	0.0E+00	5.0E+05	9.9E+05	1.5E+06	2.0E+06
λ		0.0182	0.0179	0.0178	0.0177
Losses	0.00	5.14	20.22	45.26	80.04
System	36.50	41.64	56.72	81.76	116.54



From the diagram: Q = 0.49 m^3/s.

(v) If the pump is installed close to reservoir A, then the position of the pump is high and at the downstream end of the pipe. Then, water is mainly sucked into Reservoir A, rather than being pushed. Cavitation will occur on the inlet side of the pump and along the pipeline upstream of the pump, which actually prevents flow from taking place.