PAPER 4A3 – 2003

Q1

$$P_{02}/P_{01} = 15 \rightarrow T_{02is}/T_{01} = 15^{\frac{\gamma-1}{\gamma}} = 2.168$$

$$\Delta T_{0is} = 290 \times 1.168$$

$$\Delta T_0 = 290 \times 1.168 / .85 = 398.4\ K.$$

$$\psi = 0.4 = \frac{\Delta h_0}{U^2}$$

$$\Delta h_0 = 0.4 \times 250^2 \quad \text{per stage}$$

$$\Delta T_0 = 0.4 \times 250^2 / C_p = 24.87\ K \quad \text{per stage}$$

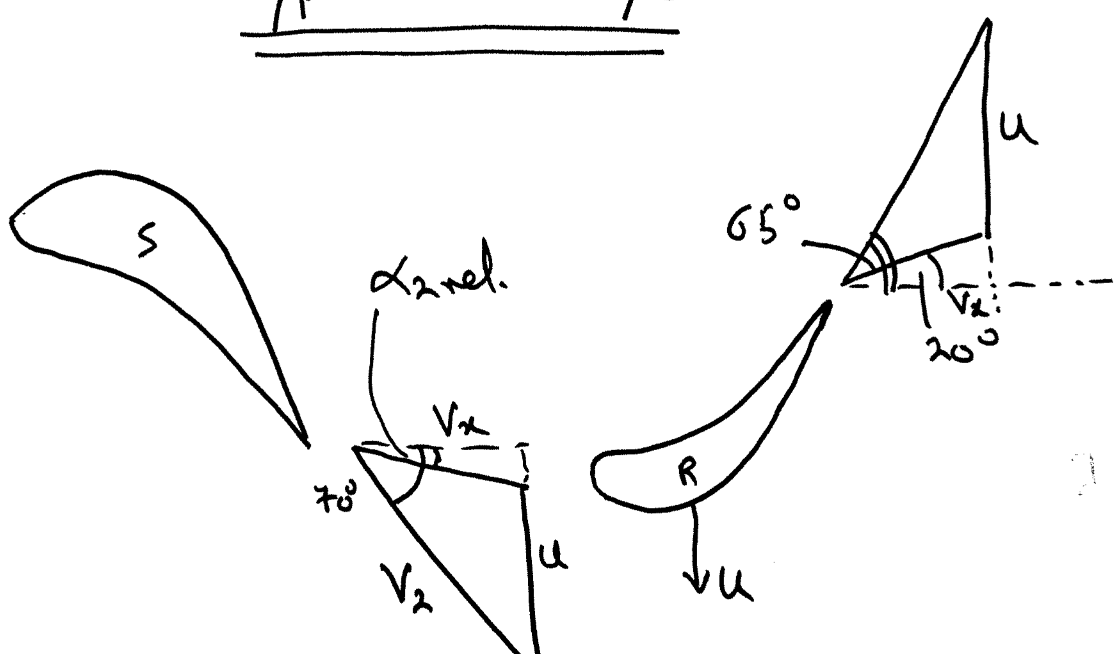
$$\therefore \text{No of stages} = \frac{398.4}{24.87} = \underline{\underline{16.}}$$

$$T_{02} = 290 + 398.4 = 688.4\ K.$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_p}}$$

$$\rightarrow \frac{\gamma-1}{\gamma}\frac{1}{\eta_p} = 0.3192$$

$$\rightarrow \underline{\underline{\eta_p = 89.5\ \%}}$$



$$U = V_x(\tan 65° - \tan 20°) = 1.7805\ V_x.$$

$$V_2 = V_x / \cos 70° = V_x / .342.$$

Q1 Cont'd)

$\tan \alpha_{2rel} = \tan \alpha_2 - \frac{U}{V_x} \rightarrow \alpha_{2rel} = 44^\circ$

But $M_2 = 0.75$, $T_{02} = 1500\,K$

Tables $\rightarrow V_2/\sqrt{C_p T_{02}} = 0.45$

$\rightarrow V_2 = 552.5$ m/s

$\therefore V_x = 188.95$, $U = 336.44$ m/s

$\Delta V_\theta = V_x(\tan\alpha_2 - \tan\alpha_3)$

$= 188.95(\tan 70^\circ + \tan 20^\circ)$

$= 587.9$ m/s.

Euler – $\Delta h_0 = U \Delta V_\theta = 197796$ J/Kg.

$\Delta T_0 = \Delta h_0 / C_p = 196.8$ K.

Total turbine work = total compressor work.

$\therefore N_T \times 196.8 = 398.4$

No. of turbine stages $= 2.02$

Say $= 2$ stages

Because the velocity triangles are the same for the second turbine stage the velocities are the same, but the stagnation temp. is less $\therefore$ speed of sound is less $\therefore$ Mach numbers are higher.

T_0 entering stage 2 $= 1500 - 196.8 = 1303.2$ K

$\therefore V_2/\sqrt{C_p T_0} = .4828$ for second stator

Tables $\rightarrow M_2 = 0.81$ for second stator.

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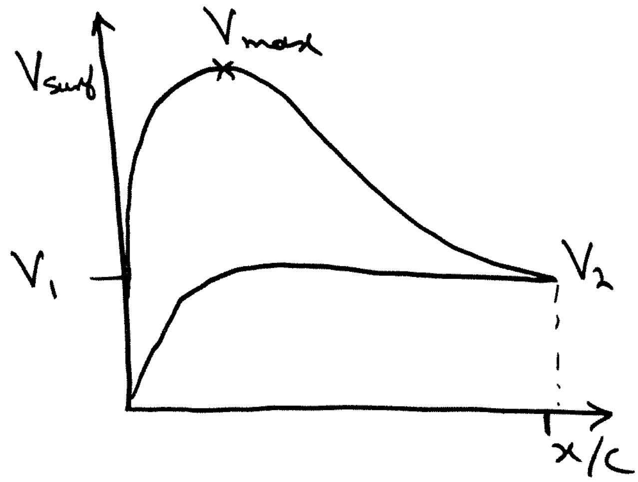
Q2



$$D.F = \frac{V_{MAX} - V_2}{V_{MAX}}.$$

Limiting value for stall ~ 0.6

D.F = Fcn (P/c) so setting a safe value for D.F. enables the pitch : chord ratio to be chosen.

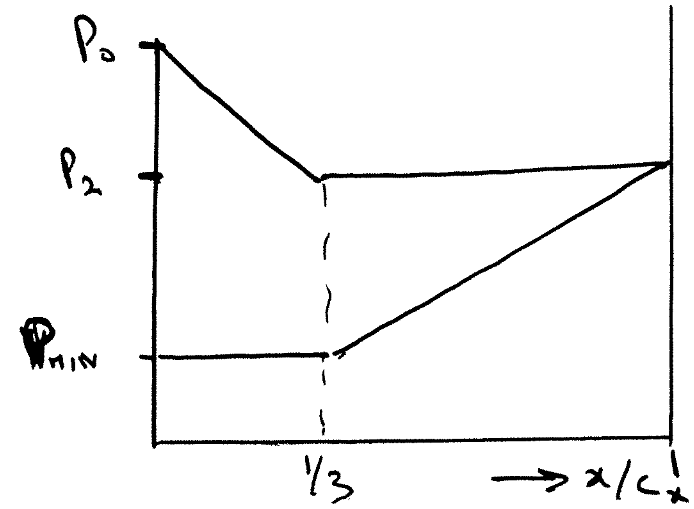
Treating the blade - blade passage as a 1 dimensional channel. Compressible flow relationships show.

$$\frac{dV}{V} = - \frac{1}{1-M^2} \frac{dA}{A}$$

So for a given area change in the passage ($\frac{dA}{A}$ constant) the velocity change increases with increasing Mach No

Hence $\frac{(V_{max} - V_2)}{V_{max}}$ = D.F. increases + M.

Q2 Cont'd



Over 1st 1/3 of blade, average ΔP

$$= \tfrac{1}{2}(P_0 + P_2) - P_{MIN}.$$

Over rear 2/3 of blade, average ΔP

$$= \tfrac{1}{2}(P_2 - P_{MIN})$$

$\therefore$ Total tangential force on blade

$$= \left[\tfrac{1}{3}\left[\tfrac{1}{2}P_0 + \tfrac{1}{2}P_2 - P_{MIN} \right] + \frac{2}{3}\left[\frac{1}{2}P_2 - \frac{1}{2}P_{MIN} \right] \right] C_x$$

$$F_\theta = \left[\tfrac{1}{6}P_0 + \tfrac{1}{2}P_2 - \tfrac{2}{3}P_{MIN.} \right] C_x$$

$$F_\theta = \left[\frac{2}{3}(P_0 - P_{MIN}) - \frac{1}{2}(P_0 - P_2) \right] C_x$$

$$= \left[\frac{2}{3} \times \frac{1}{2}\rho V_{MAX}^2 - \frac{1}{2} \cdot \frac{1}{2}\rho V_2^2 \right] C_x$$

$$= \rho V_2^2 \left[\frac{1}{3}\left(\frac{V_{MAX}}{V_2}\right)^2 - \tfrac{1}{4} \right] C_x.$$

$$D.F. = \frac{V_{MAX} - V_2}{V_{MAX}} = 1 - \frac{V_2}{V_{MAX}}.$$

$$\therefore \quad \frac{V_{MAX}}{V_2} = \left[\frac{1}{1-D} \right]$$

$$\therefore \quad F_\theta = \rho V_2^2 C_x \left[\frac{1}{3}\left(\frac{1}{1-D}\right)^2 - \tfrac{1}{4} \right] \quad ||$$

Q2 cont'd.

Equating F_θ to the change in tangential momentum

$$F_\theta = \rho V_x p . V_x (\tan\alpha_1 - \tan\alpha_2)$$

$$V_x = V_2 \cos\alpha_2$$

$$\therefore \quad F_\theta = \rho V_2^2 (\tan\alpha_1 - \tan\alpha_2) p \cos^2\alpha_2$$

$$\therefore \quad \frac{p}{C_x} = \frac{\frac{1}{3}\left(\frac{1}{1-D}\right)^2 - 1/4}{(\tan\alpha_1 - \tan\alpha_2)\cos^2\alpha_2}$$

If $\alpha_2 = 0$, then for a repeating stage

$$u = V_x \tan\alpha_1$$

$$\Delta h_0 = u . F_\theta / \rho V_x . p$$

$$\frac{\Delta h_0}{u^2} = \frac{F_\theta}{\rho V_x^2 \tan\alpha_1 \cdot p}$$

But $V_2 = V_x$ when $\alpha_2 = 0$.

$$\therefore \quad \frac{\Delta h_0}{u^2} = \left[\frac{1/3\left(\frac{1}{1-D}\right)^2 - 1/4}{\tan\alpha_1}\right]\frac{C_x}{p}$$

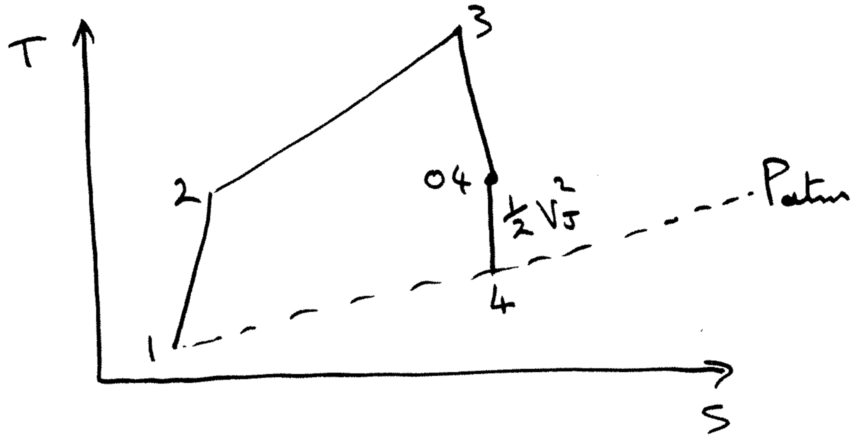
$\therefore$ Need low $\frac{p}{C_x}$ and low α_1 to achieve high loading at a given value of diffusion factor.

Q3
P1

At 10,000 m, Tables → $P_1 = 26.5$ kPa, $T_1 = 223.3$K

$$\frac{T_1}{T_{01}} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-1} \rightarrow T_{01} = 257.06\text{K}$$

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_{01} = 43.41 \text{ KPa.}$$



a)
$$\frac{T_{02}}{T_{01}} = (1.8 \times 20)^{\frac{\gamma-1}{\gamma \eta_p}} \rightarrow T_{02} = 801.81\text{K}.$$

$$\Delta T_{0comp} = T_{02} - T_{01} = 544.75.$$

$$C_{pg} = 287 \times \frac{1.3}{0.3} = 1243.7 \text{ J/Kg.}$$

Turbine work = compressor work

$$\therefore \quad T_{03} - T_{04} = \frac{1005}{1243.7} \times 544.75 = 440.2\text{K}.$$

$$\therefore \quad T_{04} = 1200 - 440.2 = 759.8\text{K}.$$

$$\frac{P_{04}}{P_{03}} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{\gamma}{(\gamma-1)\eta_p}} = .1107$$

$$\therefore \quad P_{04} = .1107 \times 36 \times 43.41 = 173.1 \text{ KPa}$$

$$P_4 = P_1 = 26.5 \text{ KPa.}$$

$$T_4/T_{04} = \left(\frac{P_4}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{26.5}{173.1}\right)^{\frac{.3}{1.3}} = 0.6485$$

Q3 Cont'd P2

$\therefore \; T_4 = 492.73 \text{ K}.$

$\frac{1}{2} V_4^2 = C_{pg}(T_{04} - T_4) = 1243.7(759.8 - 492.7)$

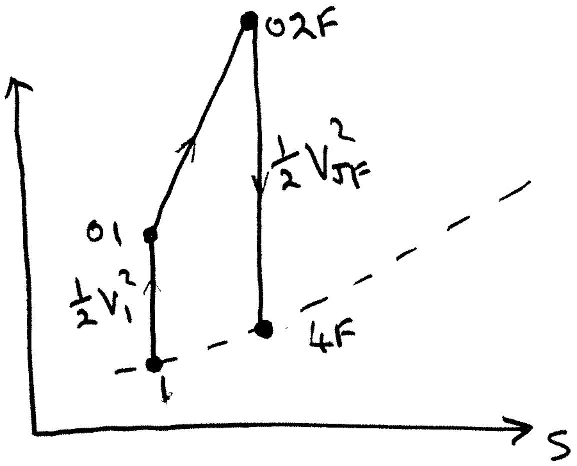
$\rightarrow V_4 = \underline{\underline{815.1 \text{ m/s}}}$

$V_1 = \sqrt{2C_p(T_{01} - T_1)} = 260.65 \text{ m/s}.$

$\eta_P = \dfrac{2V_1}{V_1 + V_4} = \dfrac{2 \times 260.65}{260.65 + 815.1} = \underline{\underline{0.485}}$

b)

For the fan.



$P_{02F} = P_{01} \times r_{fan}$. – check that $r_{fan} = 1.41$

$T_{02F} = T_{01}\left(\dfrac{P_{02F}}{P_{01}}\right)^{\frac{\gamma-1}{\eta_p \gamma}}$, $\quad T_{4F} = T_{02F} \times \left(\dfrac{P_1}{P_{02F}}\right)^{\frac{\gamma-1}{\gamma}}$.

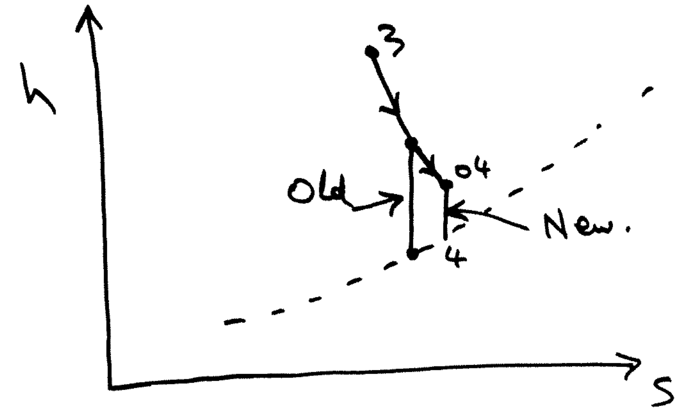
$\frac{1}{2} V_{JF}^2 = C_p(T_{02F} - T_{4F})$ given $V_{JF} = 350 \text{m/s}$.

$\underline{\underline{\text{If}}}$ $r_{fan} = 1.41$, $T_{02F} = 286.68 \text{K}$, $T_{4F} = 225.6 \text{K}$

$\rightarrow \frac{1}{2} V_{JF}^2 = C_p(286.68 - 225.6)$

$\rightarrow V_{JF} = \underline{350.4 \text{ m/s}}$ $\;$ $\underline{\text{so O.K.}}$

Q3 Contd c) Consider the expansion through the new turbine



If we can assume T_4 stays const - as given then extra work per Kg of core flow

$= \frac{1}{2}(815.1^2 - 260.65^2)$

$= 298.22$ KJ/Kg.

Fan work per Kg of fan flow

$= C_p(T_{02F} - T_{01}) = 29.77$ KJ/Kg.

$\therefore \dot{m}_{core} \times 298.22 = \dot{m}_{FAN} \times 29.77$

$\rightarrow \dfrac{\dot{m}_{FAN}}{\dot{m}_{CORE}} = \underline{\underline{10.}}$

d) Thrust $= \dot{m}_a(V_J - V_1)$

$\therefore$ Original $\dot{m}_a = \dfrac{120}{4} \times \dfrac{10^3}{(815.1 - 260.65)} = \underline{54.1 \text{ Kg/s}}$

New $\dot{m}_a = \dfrac{120}{2} \times \dfrac{10^3}{(350 - 260.65)} = \underline{\underline{671.5 \text{ Kg/s}}}$

Ratio of mass flow rates $= \underline{12.41}$

Q3 contd P4

d) cont.

When tested on the ground under dynamically similar conditions

$$\frac{\dot{m}\sqrt{T_{01}}}{P_{01}} = \text{same as in flight}$$

$$\frac{\dot{m}_{ground}}{\dot{m}_{flight}} = \frac{P_{0g}}{P_{0f}}\sqrt{\frac{T_{0f}}{T_{0g}}}$$

$$= \frac{1.013}{.434}\sqrt{\frac{257.06}{288.15}}$$

$$= \underline{\underline{2.204}}$$

$$\dot{m}_{ground} = \underline{\underline{1480}} \text{ Kg/sec for new engines.}$$