MODULE 4A6 - FLOW INDUCED SOUND AND VIBRATION

IIB CRIBS
2003

Solutions 2003

4A6

1 (a) Write $p'(x,t) = \alpha e^{i\omega t - ikx} + \beta e^{i\omega t + ikx}$,

waves of unknown amplitude going to right + left

$\xrightarrow{\quad} \alpha \qquad \longrightarrow x$

$\xleftarrow{\quad} \beta$

Pressure

$$p' = -\rho_0 \frac{\partial \phi'}{\partial t} \Rightarrow \phi'(x,t) = \frac{i}{\rho_0 \omega}\left[\alpha e^{i\omega t - ikx} + \beta e^{i\omega t + ikx}\right]$$

Velocity.

$$v' = \frac{\partial \phi'}{\partial x} = \frac{k}{\rho_0 \omega}\left[\alpha e^{i\omega t - ikx} - \beta e^{i\omega t + ikx}\right]$$

At $x = 0$, know $p'(0,t) = Pe^{i\omega t} \Rightarrow \alpha + \beta = P \quad \ldots (1)$

$$v'(0,t) = Ve^{i\omega t} \Rightarrow \frac{k}{\rho_0 \omega}(\alpha - \beta) = V \quad \ldots (2)$$

From standard dispersion rel^n for sound, $k = \omega / c_0$

Adding + subtracting (1) + (2) $\Rightarrow$

$$\alpha = \frac{1}{2}(P + \rho_0 c_0 V)$$
$$\beta = \frac{1}{2}(P - \rho_0 c_0 V)$$

[10%]

Qu1 cont.)

Averaging in time: $\overline{p'^2} = \frac{1}{2} \mathrm{Re}\left[\left(\alpha e^{-ikx} + \beta e^{ikx}\right)\left(\alpha^* e^{ikx} + \beta^* e^{-ikx}\right)\right.$

$$= \frac{1}{2}\left[|\alpha|^2 + |\beta|^2\right] + 2\,\mathrm{Re}\left(\alpha^* \beta e^{2ikx}\right)$$

$$= \frac{1}{8}\left((P+\rho_0 c_0 V)^2 + (P-\rho_0 c_0 V)^2 + \frac{2}{8}(P+\rho_0 c V)(P-\rho_0 c V)\cos 2kx\right.$$

since P, V are real

$$= \underline{\underline{\frac{1}{4}\left(P^2 + \rho_0^2 c_0^2 V^2\right) + \frac{1}{2}\left(P^2 - \rho_0^2 c_0^2 V^2\right)\cos 2kx}}$$ [15%]

$P = \rho_0 c_0 V \Rightarrow \overline{p'^2}$ independent of x.

This is a single sound wave propagating in the positive x direction, $\beta = 0$, $\alpha = P$. [5%]

Total [30%]

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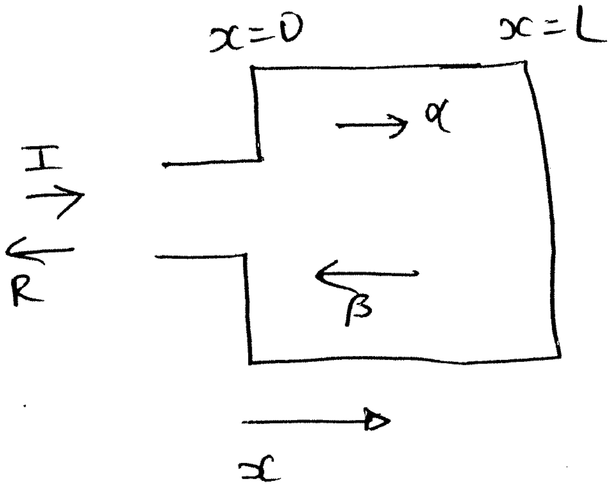
Qu1 cont.)

(b)



In $x<0$, incident wave propagating to right, reflected to left

$$p'(x,t) = Ie^{i\omega t - ikx} + Re^{i\omega t} \quad x<0$$

(reflected)

In $0<x<L$ waves travelling in both directions

$$p'(x,t) = \alpha e^{i\omega t - ikx} + \beta e^{i\omega t + ikx} \quad 0<x<L$$

[15%]

Qu1 cont.)

mass flux continuous across $x=0$

$\Rightarrow \quad A_1 [I-R] = A_2 [\alpha - \beta] \quad \ldots \quad (1)$ [10%]

pressure continuous across $x=0$

$\Rightarrow \quad I+R = \alpha+\beta \quad \ldots \quad (2)$ [10%]

zero velocity on $x=L \Rightarrow \quad \alpha e^{-ikL} = \beta e^{ikL} \ldots \quad (3)$ [10%]

(3) substituted into (1) and (2):

$$\frac{\frac{A_1}{A_2}(I-R)}{I+R} = \frac{\alpha(1-e^{-2ikL})}{\alpha(1+e^{-2ikL})}$$

dividing $\quad \frac{A_1}{A_2}(I-R) = (I+R)\, i \tan kL$

$$\Rightarrow \quad R = \frac{I\left\{1 - \frac{iA_2}{A_1}\tan kL\right\}}{1 + \frac{iA_2}{A_1}\tan kL}$$ [25%]

total [70%]

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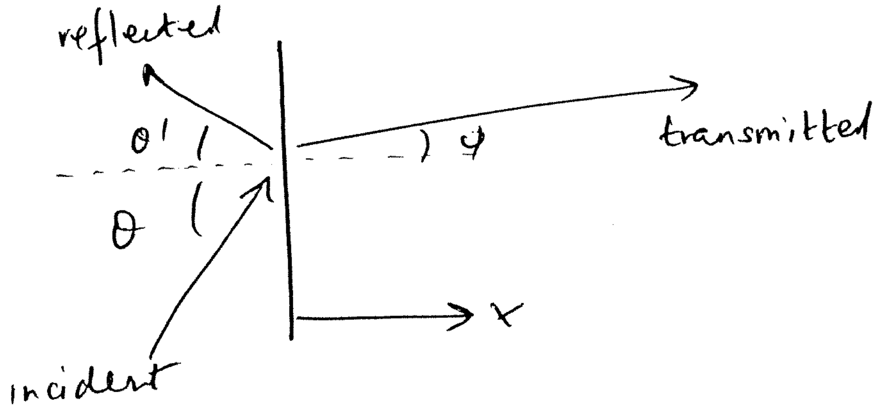
(a)



In $x<0$, reflected + incident waves

$$p'(x,y,t) = I e^{i\omega t - \frac{i\omega x}{c_0}\cos\theta - \frac{i\omega y}{c_0}\sin\theta} + R e^{i\omega t + \frac{i\omega x}{c_0}\cos\theta' - \frac{i\omega y}{c_0}\sin\theta'}$$

<u>N.B.</u> x wavenumber changes sign in reflected wave

In $x>0$ only transmitted wave

$$p'(x,y,t) = T e^{i\omega t - \frac{i\omega x}{c_1}\cos\phi - \frac{i\omega y}{c_1}\sin\phi}$$

<u>NB</u> different sound speed, same frequency.

ontinuity of pressure across $x=0$ for all y necessitates

$$\frac{\sin\theta}{c_0} = \frac{\sin\theta'}{c_0} = \frac{\sin\phi}{c_1}$$

$\Rightarrow$ $\underline{\theta' = \theta}$ and $\underline{\frac{\sin\theta}{c_0} = \frac{\sin\phi}{c_1}}$ Snell's Law [25%]

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Qu2 contd.)

Further, continuity of pressure $\Rightarrow$ $I + R = T$.

Continuity of normal velocity

$$\frac{1}{\rho_0 c_0}(I-R)\cos\theta = \frac{1}{\rho_1 c_1} T \cos\phi$$

eliminate R, $\Rightarrow$ $R = I \dfrac{\left[\dfrac{\rho_1 c_1}{\cos\phi} - \dfrac{\rho_0 c_0}{\cos\theta}\right]}{\left[\dfrac{\rho_1 c_1}{\cos\phi} + \dfrac{\rho_0 c_0}{\cos\theta}\right]}$ [20%]

If $c_1 > c_0$ then have possibility that $\sin\theta \geq c_0/c_1$

$\Rightarrow$ $\sin\phi \geq 1$ $\Rightarrow$ total internal reflection.

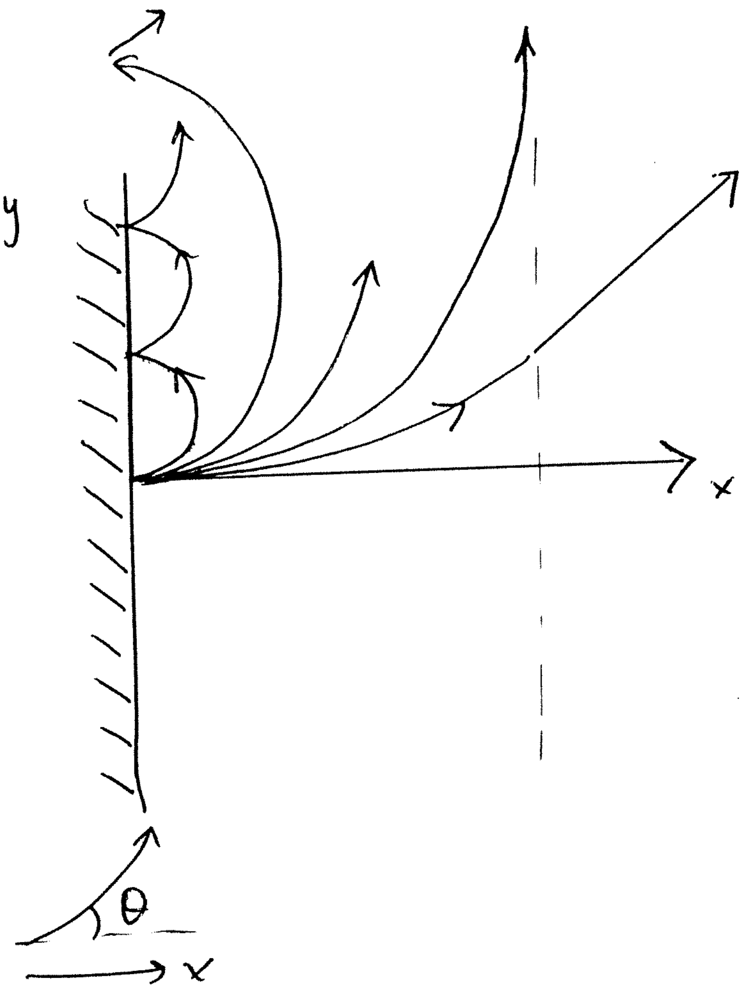
Evanescent wave in $x > 0$.

[5%]

total [50%]

Qu2 cont.)

(b)



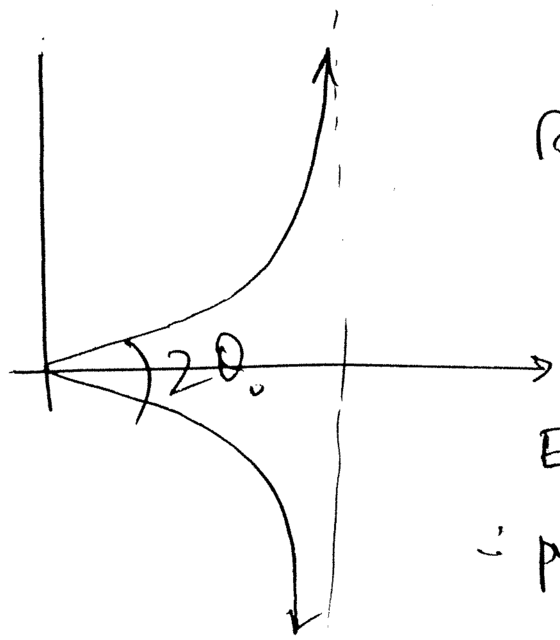
- Rays → straight line beyond $x = H$.
- Rays returning to $x=0$ bounce, angle of reflection = angle of incidence, and bounce forever.
- symmetry about $y=0$

[15%]

The ray which propagates along $x = H$ has $\theta = \frac{\pi}{2}$ when $x = H$. Snell's Law ⇒
$$\frac{\sin \pi/2}{\alpha H + \beta} = \frac{\sin \theta_0}{\alpha . 0 + \beta}$$

∴ This ray is launched at angle $\underline{\underline{\theta_0 = \sin^{-1}\left[\frac{\beta}{\alpha H + \beta}\right]}}$ [15%]

Qu 2 cont.



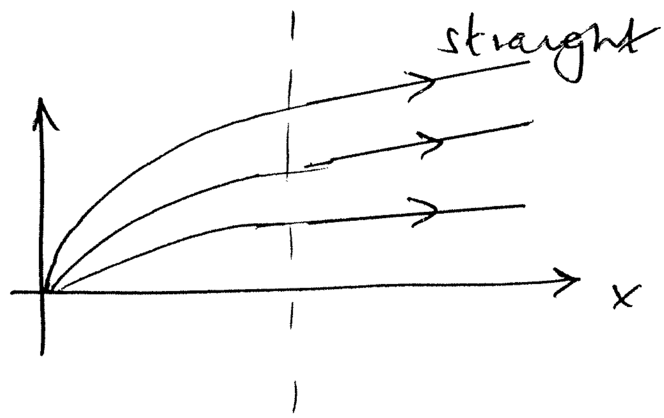
Ray tube carrying energy away from $x=0$ has angle $2\theta_0$

Energy emitted isotropically

$\therefore$ proportion escaping, ie not returning to $x=0$

$= 2\theta_0/\pi$ [10%]

If $\alpha<0$



[10%]

no power returns to $x=0$

total [50%]

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Qu 3 a) Equation of mass conservation $\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \underset{\sim}{v} = m$ (1)

Momentum equation $\rho_0 \frac{\partial \underset{\sim}{v}}{\partial t} + \nabla p' = 0$ (2)

Combining $(1) \times \frac{p'}{\rho_0} + (2) \cdot \underset{\sim}{v}$

$$\frac{p'}{\rho_0}\frac{\partial \rho'}{\partial t} + p' \nabla \cdot \underset{\sim}{v} + \rho_0 \underset{\sim}{v} \cdot \frac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v} \cdot \nabla p' = \frac{p' m}{\rho_0} \quad (3)$$

Now $\rho' = p'/c^2$, $\therefore$ $p' \frac{\partial \rho'}{\partial t} = \frac{p'}{c^2}\frac{\partial p'}{\partial t} = \frac{1}{2c^2}\frac{\partial}{\partial t}(p'^2)$

Similarly $\underset{\sim}{v} \cdot \frac{\partial \underset{\sim}{v}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(v^2)$.

Hence equation (3) is equivalent to

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{p'}{\rho_0 c} + \frac{1}{2}\rho_0 v^2\right) + \nabla \cdot (p' \underset{\sim}{v}) = \frac{p' m}{\rho_0}$$

PE — KE — intensity = flux of acoustic energy/ unit area — production of acoustic power/ unit volume

[50%]

$$\frac{\partial}{\partial t}(e_P + e_K) + \nabla \cdot \underset{\sim}{I} = \frac{p' m}{\rho_0}$$

b) Mean power output from point source $= \int \frac{m p'}{\rho_0} dV$

$p' = p_i \sin\omega(t-\tau) + p_s'(\underset{\sim}{x},t)$, where $p_s'(\underset{\sim}{x},t)$ is the pressure due to the source itself.

Mean power output $= \int \frac{m_0}{\rho_0}\delta(\underset{\sim}{x}) \overline{\cos\omega t \; p_i \sin\omega(t-\tau)} \, dV + \int \frac{p_s' m}{\rho_0} dV$

$$= \frac{m_0 p_i}{\rho_0} \overline{\cos\omega t(\sin\omega t \cos\omega\tau - \cos\omega t \sin\omega\tau)} + \frac{\omega^2 m_0^2}{8\pi\rho_0 c}$$

$$= -\frac{1}{2}\frac{m_0 p_i}{\rho_0}\sin(\omega\tau)$$

[15%]

ie the mean power output from the source is changed by an amount $-\frac{1}{2} m_0 p_i \sin(\omega\tau)/\rho_0$

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Qu3 cont.

c) $\overset{m_0}{\bullet} \longleftarrow \ \ell \ \longrightarrow \overset{-m_0}{\bullet}$

Anti-source produce a pressure $\dfrac{\omega m_0}{4\pi\ell} \sin\omega(t-\ell/c)$ at primary source location

Hence $p_i = \dfrac{\omega m_0}{4\pi\ell}$ and $\tau = \dfrac{\omega\ell}{c}$.

i.e the primary source emits sound power $= \dfrac{\omega^2 m_0^2}{8\pi\rho_0 c} - \dfrac{1}{2}\dfrac{m_0 p_i}{\rho_0}\sin(\omega\tau)$

$$= \frac{\omega^2 m_0^2}{8\pi\rho_0 c} - \frac{m_0^2\omega}{8\pi\ell\rho_0}\sin(\omega\ell/c)$$ [30]

$$= \underbrace{\frac{\omega^2 m_0^2}{8\pi\rho_0 c}}_{\text{in isolation}}\left[1 - \underbrace{\frac{c}{\omega\ell}\sin\left(\frac{\omega\ell}{c}\right)}_{\text{modification due to anti-source}}\right]$$

[Note as $\omega\ell/c \to 0$, $\frac{c}{\omega\ell}\sin(\omega\ell/c) \to 1$ and the sound emits no power

as $\omega\ell/c \to \infty$, the 'anti-source' has no effect on the radiated sound power]

The 'anti-source' alters the pressure near the primary source and for small $\omega\ell/c$ prevents it from generating sound. Hence the question 'where has that power gone to' is not sensible - it is never generated [5]

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Qu 4 From the data card:

jet noise is generated by quadrupoles of strength T_{ij}

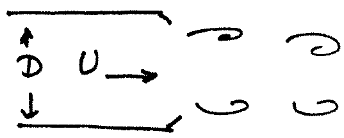
$$= \rho v_i v_j + (p' - c^2 \rho')\delta_{ij} - \tau_{ij}$$

the pressure field generated by quadrupoles is

$$p'(\underset{\sim}{x}, t) = \frac{\partial^2}{\partial x_i \partial x_j} \int \frac{T_{ij}(\underset{\sim}{y}, t - |\underset{\sim}{x} - \underset{\sim}{y}|/c)}{4\pi |\underset{\sim}{x} - \underset{\sim}{y}|} d^3\underset{\sim}{y}$$

which in the far-field simplifies to

$$p'(\underset{\sim}{x}, t) = \frac{x_i x_j}{4\pi |\underset{\sim}{x}|^3 c^2} \frac{\partial^2}{\partial t^2} \int T_{ij}\left(\underset{\sim}{y}, t - \frac{|\underset{\sim}{x}|}{c} + \frac{\underset{\sim}{x} \cdot \underset{\sim}{y}}{|\underset{\sim}{x}| c}\right) d^3\underset{\sim}{y}.$$



For a jet of diameter D, velocity U, a typical frequency in the jet $\sim \frac{U}{D}$

Hence compactness ratio $\frac{\omega D}{c} \approx \frac{U}{D}\frac{D}{c} = M$ jet Mach number.

The jet is compact and we can neglect retarded time variations

$$p'(\underset{\sim}{x}, t) = \frac{x_i x_j}{4\pi |\underset{\sim}{x}|^3 c^2} \frac{\partial^2}{\partial t^2} \int T_{ij}\left(\underset{\sim}{y}, t - \frac{|\underset{\sim}{x}|}{c}\right) d^3\underset{\sim}{y}$$

Now $\frac{\partial}{\partial t} \sim \frac{U}{D}$

$T_{ij} \sim \rho_0 U^2$

Hence $p'(\underset{\sim}{x}, t) \sim \frac{1}{4\pi |\underset{\sim}{x}| c^2} \frac{U^2}{D^2} \rho_0 U^2 D^3 = \frac{D \rho_0}{4\pi |\underset{\sim}{x}|} \frac{U^4}{c^2}$

Sound pressure scales on U^4 and hence <u>sound power on U^8.</u> [40%]

a) When the jet blows near and parallel to a very large rigid plane surface, the quadrupole sources have image sources in the surface of equal magnitude. Some images have the same sign as the primary source (eg T_{11}) and others the opposite sign. The net effect is only a modest change in amplitude, still U^8 scaling and a modified directivity. [20%]

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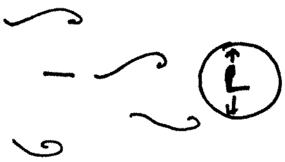
Qu4 cont. b) When the jet blow over a small, fixed and rigid body, it generates an addition dipole sound field



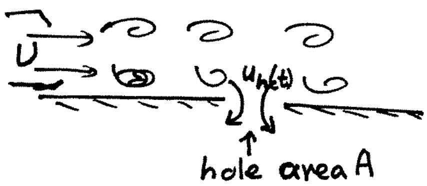
$$= \frac{x_i}{4\pi|\underset{\sim}{x}|c} \frac{\partial}{\partial t} F_i(t - |\underset{\sim}{x}|/c)$$

where $\underset{\sim}{F}$ is the force on the body.

For a body of size $\underset{\sim}{F}$, $\frac{\partial \underset{\sim}{F}}{\partial t} \approx \frac{U}{L} C_f \rho_0 U^2 L^2$

and this far-field pressure $\sim \frac{M C_f \rho_0 U^2}{4\pi} \frac{L}{|\underset{\sim}{x}|}$ where $M = U/c$

We now have dipole sound with different directivity ($\cos\theta$ where θ is from direction of $\frac{\partial F}{\partial t}$). The amplitude is a factor $\sim\left(\frac{L}{DM}\right)^2$ larger than the jet noise - potentially a large amplification in a low Mach number flow. [20%]



When there is a hole in the large, plane surface pressure fluctuations in the jet will drive an unsteady air flow through the hole. Denote this velocity by $u_h(t) \propto U$

Then is therefore additional monopole source

$$p' = 2 \times \frac{A}{4\pi|\underset{\sim}{x}|} \frac{\partial}{\partial t} \left(\rho_0 u_h\left(t - \frac{|\underset{\sim}{x}|}{c}\right)\right) \sim \frac{A}{2\pi|\underset{\sim}{x}|D} \rho_0 U^2$$

×2 due to image in rigid surface

This sound field is omni-directional, and ~~the~~ pressure perturbation is order $A/(D^2 M^2)$ larger than the jet noise. [20%]

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