CRIBS 4A8

(1) (a) DNS gives all details of the flow and is an exact solution but limited to low Re.
and solutions expensive

LES is less accurate but gives all details at ~~low~~ low wavenumber
Costs less but still expensive

RANS is most dubious but is very cheap.
– only gives time averaged qtys

(b) N grids pts $\sim Re^{\frac{9}{4}}$

N time steps $\sim Re^{\frac{1}{2}}$

Total comp. time $\sim Re^{\frac{1}{2}} Re^{\frac{9}{4}} = Re^{\frac{11}{4}}$

Ratio of times $\left(\frac{6 \times 10^6}{600}\right)^{\frac{11}{4}} = \left(6 \times 10^4\right)^{\frac{11}{4}}$

$= 1.38 \times 10^{13}$

hence total time 1.38×10^{13} hr

= 15 million centurie

4A8 cribs

① (c) $\varepsilon \sim \frac{U_\tau^3}{z}$ $\quad \eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$

$$= \left(\frac{\nu^3 z}{U_\tau^3}\right)^{\frac{1}{4}}$$

$$= \left(\frac{\nu^3(0.018)}{U_\tau^3}\right)^{\frac{1}{4}}$$

$$\frac{\eta}{\delta} = \left(\frac{\nu^3 \; 0.01}{\delta^3 U_\tau^3}\right)^{\frac{1}{4}}$$

$$= \left(\frac{.01}{Re^3}\right)^{\frac{1}{4}}$$

$$= 2.61 \times 10^{-6}$$

(d) for stable flow

$\text{Prod} \sim U_\tau^2 \frac{\partial U}{\partial z}$ which above for a log law gave

$U_\tau^2 \frac{U_\tau}{\kappa z} = \text{dissipation}$

Now in stable condition

$$\frac{dU}{dz} = \frac{U_\tau}{\kappa z}\left(1 + 4.7\left(\frac{z}{L}\right)\right)$$

So dissipation $\sim$ production $= \frac{U_\tau^3}{\kappa z}\left(1 + 4.7 \, z/L\right)$

So η should be smaller

But we expect that U_τ^2 will be smaller as the turbulence is suppressed

1(d) Overall the suppression of turbulence will dominate leading to smaller ε hence larger $\frac{\eta}{\delta}$

(e) Richardson number

$$\frac{\text{Prod due to thermal (buoy effects)}}{\text{Mech production}}$$

$Ri < 0$ Unstable increased turb

$Ri = 0$ Neutral

$Ri > 0$ Stable – reduced turbulence

$Ri > \frac{1}{4}$ – No turbulence.

Monin Obukhov Length, L

Above L buoyancy dominates

below L mech shear dominate

2(a)

$$\frac{d(Uh)}{ds} = u_e = \alpha U$$

$$\frac{d(\rho U^2 h)}{ds} = g(\rho - \rho_a)\, h \sin\Theta$$

$$U h g\left(\frac{\rho - \rho_a}{\rho_a}\right) = \text{constant} = F_0$$

$h = \alpha s$; $U = \text{constant} = B F_0^{1/3}$

a) $h = \alpha s = 0.1 s$

at the village

$h = 0.1 \times 500 = 50$ m.

(b) Assuming all CO_2 at a distance of 10m

Then CO_2 mass flux

$= 1\,m \times 5\,m/s. \times 1.8\,kg/m^3$

$= 9\,kg/s\,/\,m.$ width

At a distance of 500m the total volume flux is $50\,m \times 5\,m/s. = 250\,m^3/s\,/\,m.$ width

Air added is a volume flux of

$(250 - 5) = 245\,m^3/s\,/\,m.$ width

$\therefore$ mass flux of air $= 245 \times 1.2$

$= 294\,kg/s\,/\,m.$ width

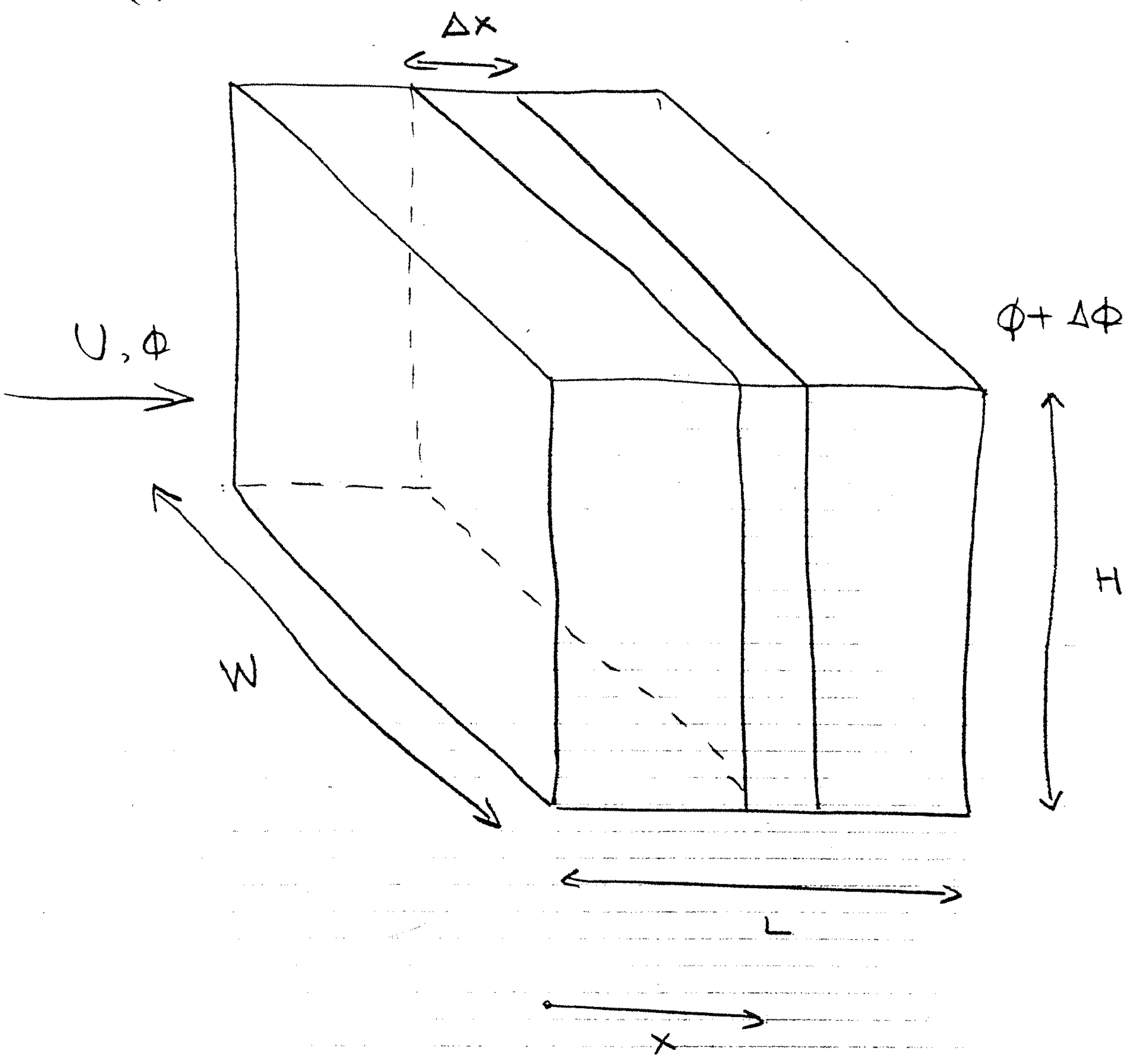
$$\frac{CO_2 \text{ mass flux}}{(CO_2 + \text{air}) \text{ mass flux}} = \frac{9}{294 + 9} = 0.0297$$

mass/mass

(c) The plume velocity is constant and so remains at 5 m/s.

Thus the time taken for the release to get to the village is $500m / 5m/s = 100$ seconds.

3 (a)



$\left(\phi \text{ coming in per s}\right) - \left(\phi \text{ emitted per s}\right) + \left(\phi \text{ from reaction per s}\right) = \left(\phi \text{ coming out per s}\right)$

$$\Rightarrow \quad UHW\phi + q(x)\cdot W\cdot \Delta x + \dot{w}HW\Delta x = UHW(\phi + \Delta\phi)$$

$$\Leftrightarrow \quad q(x)W\Delta x + \dot{w}HW\Delta x = UHW\Delta\phi$$

$$\Leftrightarrow \quad U\frac{\Delta\phi}{\Delta x} = \frac{q(x)}{H} + \dot{w} \quad \Leftrightarrow \quad \boxed{U\frac{d\phi}{dx} = \frac{q(x)}{H} + \dot{w}}$$

3 (a) cont'd

If inert, $\dot{\omega} = 0$

$$\therefore \quad U \frac{d\Phi}{dx} = q(x)/H \iff \Phi = \Phi(0) + \frac{1}{UH}\int_0^L q(x)\,dx$$

$$\text{Total emission} = \int_0^L q(x)\,dx \,.$$

(b) Similarly, for unsteady problem with $U=0$:

$$\left(\Phi \text{ at time } t+\Delta t\right) = \left(\Phi \text{ at time } t\right) + \Delta t\left(\Phi_{\text{emitted}} + \Phi_{\text{generated}}\right)$$

$$\Rightarrow \quad (\Phi + \Delta\Phi)(HWL) = \Phi\,(HWL) + \Delta t \cdot \left(q \cdot W \cdot L + \dot{\omega}\,HWL\right)$$

$$\iff \quad \boxed{\frac{d\Phi}{dt} = \frac{q(t)}{H} + \dot{\omega}}$$

If $q(t) = Q\left(1 + \sin\omega t\right)$ & $\dot{\omega} = -K(1+\sin\omega t)\,\Phi$

$$\Rightarrow \quad \frac{d\Phi}{dt} = \frac{Q}{H}(1+\sin\omega t) - K(1+\sin\omega t)\,\Phi$$

$$\iff \quad \frac{d\Phi}{dt} = \frac{Q}{H}(1+\sin\omega t)\left[1 - \frac{KH}{Q}\Phi\right]$$

$$\iff \quad \frac{d\Phi}{1-\frac{KH}{Q}\Phi} = \frac{Q}{H}(1+\sin\omega t)\,dt$$

$$\iff \quad -\frac{Q}{KH}\ln\left(1 - \frac{KH}{Q}\Phi\right) = \frac{Q}{H}t - \frac{Q}{\omega H}\cos\omega t + C$$

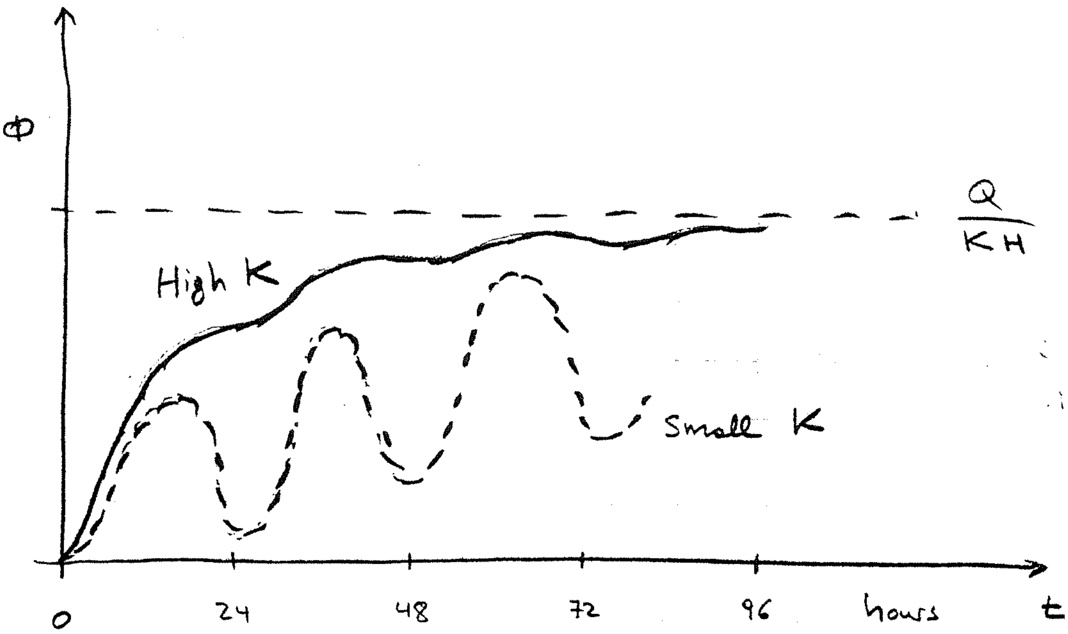
$$\iff \quad 1 - \frac{KH}{Q}\Phi = A \cdot e^{-Kt} \cdot e^{\frac{K}{\omega}\cos\omega t}$$

At $t=0,\ \Phi=0 \Rightarrow A = e^{-\frac{K}{\omega}}$

$$\Rightarrow \quad \Phi = \frac{Q}{KH}\left[1 - e^{-Kt} \cdot e^{\frac{K}{\omega}(\cos\omega t - 1)}\right]$$

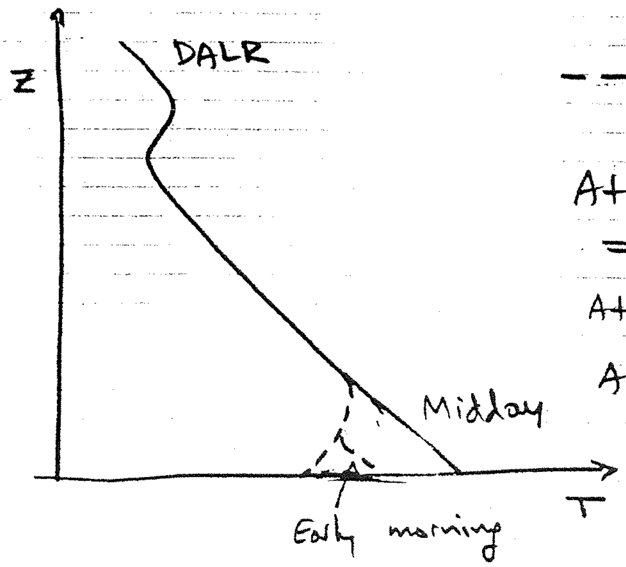
Maximum possible?

3 (b) Cont'd



Depending on K, some pollutants may oscillate during diurnal cycles, others reach equilibrium quickly.

(c)



--- Midnight

At night; stable ⇒ no mixing
⇒ pollution stays low

At early morning, H is low } due to progressive heating
At midday, H is high }
which causes unstable & then neutral b.l
⇒ Φ higher at morning, but emission is less

4. (a)

$$\bar{\Phi}(x,y) = \frac{Q/L}{U\sqrt{2\pi}\,\sigma} e^{-y^2/2\sigma^2} \quad , \sigma = Cx$$

$$\sigma^2 = 2\frac{x}{U}K$$

Eqtn for variance, g, in 2-D:

$$U\frac{\partial g}{\partial x} + \underbrace{\cancel{V\frac{\partial g}{\partial y}}}_{(V=0)} = \left(K\frac{\partial^2 g}{\partial x^2} + K\frac{\partial^2 g}{\partial y^2}\right) + 2K\left[\left(\frac{\partial\bar{\Phi}}{\partial x}\right)^2 + \left(\frac{\partial\bar{\Phi}}{\partial y}\right)^2\right] - \frac{2g}{T_{turb}}$$

- $$\frac{\partial\bar{\Phi}}{\partial x} = \frac{\partial\bar{\Phi}}{\partial\sigma}\frac{\partial\sigma}{\partial x} = C\cdot\left[-\frac{\bar{\Phi}}{\sigma} + \bar{\Phi}\left(\frac{y^2}{\sigma^3}\right)\right] = C\cdot\frac{\bar{\Phi}}{\sigma}\left(\frac{y^2}{\sigma^2}-1\right)$$

$$\Rightarrow \left(\frac{\partial\bar{\Phi}}{\partial x}\right)^2 = C^2\frac{\bar{\Phi}^2}{\sigma^2}\left(\frac{y^2}{\sigma^2}-1\right) \quad \text{(A)}$$

- $$\frac{\partial\bar{\Phi}}{\partial y} = -\frac{2y}{2\sigma^2}\bar{\Phi} = -\frac{y}{\sigma^2}\bar{\Phi} \Leftrightarrow \left(\frac{\partial\bar{\Phi}}{\partial y}\right)^2 = \frac{\bar{\Phi}^2}{\sigma^2}\cdot\frac{y^2}{\sigma^2} \quad \text{(B)}$$

Comparing (A) & (B): $o\left[\left(\frac{\partial\bar{\Phi}}{\partial x}\right)^2\right] \sim C^2\frac{\bar{\Phi}^2}{\sigma^2}$

$o\left[\left(\frac{\partial\bar{\Phi}}{\partial y}\right)^2\right] \sim \frac{\bar{\Phi}^2}{\sigma^2}$

because $y \sim o(\sigma)$ $\Rightarrow \left(\frac{\partial\bar{\Phi}}{\partial x}\right)^2 \ll \left(\frac{\partial\bar{\Phi}}{\partial y}\right)^2$

- $$o\left(\frac{\partial^2 g}{\partial y^2}\right) = \frac{g}{\sigma^2}$$

$$o\left(\frac{\partial^2 g}{\partial x^2}\right) = \frac{g}{x^2} = \frac{g\cdot C^2}{\sigma^2} \quad \Rightarrow \frac{\partial^2 g}{\partial x^2} \ll \frac{\partial^2 g}{\partial y^2}$$

$\Rightarrow$ axial diffusion & production negligible

NOTE: Key concept here is that $y = o(\sigma)$

$$- \; U\frac{g}{x} \sim K\frac{g}{\sigma^2} \Rightarrow U\frac{g}{x} \sim \sigma^2\frac{U}{x}\frac{g}{\sigma^2}$$

$$\Rightarrow \text{same order } \frac{Ug}{x}$$

$$- \; K\left(\frac{\partial\bar{\Phi}}{\partial y}\right)^2 \sim o\left(\sigma^2\frac{U}{x}\cdot\frac{\bar{\Phi}^2}{\sigma^2}\right)$$

$$= o\left(\bar{\Phi}^2\frac{U}{x}\right)$$

$\Rightarrow$ g must be of the same order of magnitude as $\underline{\underline{\bar{\Phi}^2}}$

$$- \; \frac{2g}{T_{turb}} \sim o\left(\frac{g}{T_{turb}}\right) \Rightarrow \underline{\underline{T_{turb} \sim \frac{x}{U}}}$$

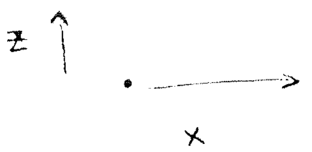
<u>NOTE:</u> Usually, we write the dissipation term as $\frac{2g}{T_{turb}}$, with T_{turb} the large-eddy timescale, taken as <u>constant</u>. More properly, the dissipation term is modelled as $\frac{2g}{T_{diss}}$, with T_{diss} a dissipation timescale. Then, $T_{diss} \sim o\left(\frac{x}{U}\right)$

$$\Leftrightarrow T_{diss} \sim o\left(\frac{Cx}{CU}\right)$$

$$\Leftrightarrow T_{diss} \sim o\left(\frac{\sigma}{u'}\right) \sim o\left(\underbrace{\frac{L_{turb}}{u'}}_{T_{turb}}\cdot\frac{\sigma}{L_{turb}}\right)$$

If width of plume increases in parallel to turbulence lengthscale our $2g/T_{turb}$ model is OK. If width of plume has its own history, then $2g/T_{turb}$ model is not that good and a different timescale needed.

4(b)



$$\bar{\Phi}(x,z) = \frac{Q/L}{U\sqrt{2\pi}\;\sigma(x)}\left[\exp\left(-\frac{(z-0)^2}{2\sigma^2}\right) + \underbrace{\exp\left(-\frac{(z+0)^2}{2\sigma^2}\right)}_{\text{image source at } z=0}\right]$$

$$\Rightarrow \bar{\Phi}(x,z) = \frac{2\;Q/L}{U\sqrt{2\pi}\;\sigma}\,e^{-z^2/2\sigma^2}$$

$$\Rightarrow \boxed{\Phi(x,0) = \frac{2\;Q/L}{U\sqrt{2\pi}\;\sigma(x)}}$$

Q/L : kg of pollutant per s per m

$U = 5$ m/s

$\sigma(x) = 0.1\,x$

To find Q/L kg of pollutant per s. per m. of motorway

$$= \frac{\text{cars}}{\text{m}} \cdot \frac{\text{kg. of poll.}}{\text{car s}} = \frac{\text{cars}}{\text{s}} \cdot \frac{1}{U_{car}}\left(\frac{\text{kg of poll}}{\text{m.}}\,U_{car}\right)$$

$$= 1 \cdot \frac{0.1}{1000} = 1 \times 10^{-4} \text{ kg/s.m.}$$

$$\bar{\phi}(2000, 0) = 7.98 \times 10^{-8} \text{ kg/m}^3$$