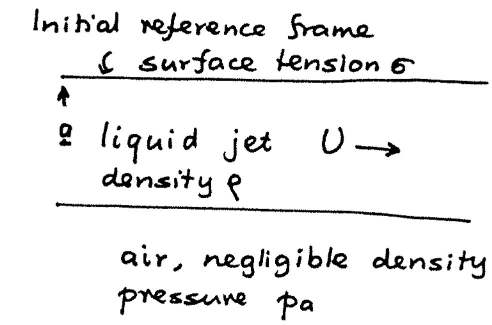
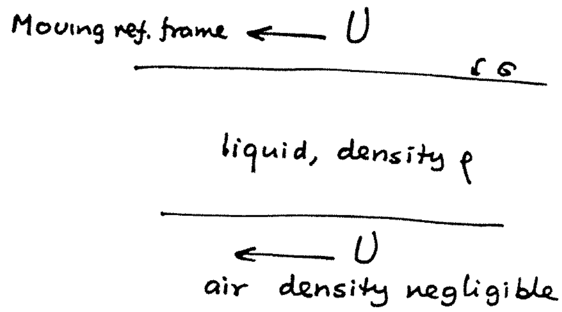
Module 4A10 - FLOW INSTABILITY

Solutions - 2003

Qu1 (i)





Make a coordinate transformation to a frame of reference moving with speed U. In this frame, the liquid jet is at rest and the air moves. However, because the air density is negligible, it has negligible momentum and the boundary condition it imposes on the liquid jet is one of constant (atmospheric) pressure on the jet boundary. U has therefore 'dropped' out of the problem. [10%]

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(ii) Since the liquid is incompressible the volume within a wavelength $2\pi/k$ after the boundary is perturbed must be equal to the volume before.

The volume within a length $2\pi/k = \lambda$ of the perturbed column is

$$= \frac{1}{2}\int_0^\lambda \int_0^{2\pi} r^2 \, d\theta \, dx = \frac{1}{2}\int_0^\lambda \int_0^{2\pi} (\alpha + \beta \cos n\theta \cos kx)^2 \, d\theta \, dx$$

$$= \frac{1}{2}\int_0^\lambda \int_0^{2\pi} (\alpha^2 + 2\alpha\beta \cos n\theta \cos kx + \beta^2 \cos^2 n\theta \cos^2 kx) \, d\theta \, dx$$

$$= \frac{1}{2} 2\pi\lambda\alpha^2 + \frac{1}{2}\lambda\, 2\pi \beta^2 \frac{1}{2}\frac{1}{2} = \pi\lambda\alpha^2 + \frac{\pi}{4}\lambda\beta^2 \quad \text{for } n \neq 0$$

$$= \pi\lambda\alpha^2 + \lambda\pi\frac{\beta^2}{2} \quad \text{for } n = 0$$

$$= \pi\lambda a^2 \text{, the initial volume.}$$

Hence, for $n \neq 0$,

$$\alpha^2 = a^2 - \frac{1}{4}\beta^2$$

$$\alpha = a\left(1 - \frac{1}{4}\frac{\beta^2}{a^2}\right)^{1/2} \doteq \underline{\underline{a\left(1 - \frac{1}{8}\frac{\beta^2}{a^2}\right)}} + O\left(\frac{\beta}{a}\right)^4 \quad (1a)$$

for $n = 0$

$$\alpha = \underline{\underline{a\left(1 - \frac{1}{4}\frac{\beta^2}{a^2}\right)}}$$

(1b)

[30%]

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Qu 1 cont.) We can determine the stability by seeing whether energy is released by a small perturbation (unstable) or whether energy has to be add to make the change.

The energy is potential = $\sigma \times$ area of the liquid-air interface.

The deformed area $= \iint \left[1 + \left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{1}{r}\frac{\partial r}{\partial \theta}\right)^2 \right]^{1/2} r\, d\theta\, dx$

$$= \int_0^{\lambda}\int_0^{2\pi} \left\{ 1 + \beta^2 k^2 \cos^2 n\theta \sin^2 kx + \underbrace{\frac{1}{a^2}}_{\uparrow \text{ neglecting terms } O(\beta^4)} \beta^2 n^2 \sin^2 n\theta \cos^2 kx \right\}^{1/2} (\alpha + \beta \cos n\theta \cos kx)\, d\theta\, dx$$

$$= \int_0^{\lambda}\int_0^{2\pi} \left[1 + \frac{1}{2}\beta^2 k^2 \cos^2 n\theta \sin^2 kx + \frac{1}{2a^2}\beta^2 n^2 \sin^2 n\theta \cos^2 kx \right] (\alpha + \beta \cos n\theta \cos kx)\, d\theta\, dx$$

$$= \alpha\lambda 2\pi + \frac{\alpha}{2}\beta^2 k^2 \frac{2\pi}{2}\frac{\lambda}{2} + \frac{1}{2a^2}\alpha\beta^2 n^2 \frac{2\pi}{2}\frac{\lambda}{2} \quad \text{for } n \neq 0$$

$$= \begin{cases} \alpha\lambda 2\pi + \dfrac{a\beta^2}{4}\left(k^2 + \dfrac{n^2}{a^2}\right)\lambda\pi & \text{for } n \neq 0 \\ \alpha\lambda 2\pi + \dfrac{a\beta^2}{2} k^2 \lambda\pi & \text{for } n = 0 \end{cases}$$

After substituting from (1). We obtain

$\underline{n \neq 0}$ $\quad$ area $= \underbrace{\lambda 2\pi a}_{\text{initial area}} + \dfrac{a\beta^2}{4}\lambda\pi \underbrace{\left(k^2 + \dfrac{n^2 - 1}{a^2}\right)}_{\text{always +ve for } n \neq 0}$ $\quad n \neq 0$

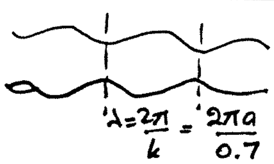
Hence the surface area has increased and the jet is stable to all non-axisymmetric disturbances ($n \neq 0$)

$\underline{n = 0}$ $\quad$ area $= \lambda 2\pi a + \dfrac{a\beta^2}{2}\lambda\pi \underbrace{\left(k^2 - \dfrac{1}{a^2}\right)}_{-\text{ve if } ka < 1}$

Hence the surface area only decreases if $n=0$ (i.e. axisymmetric disturbance) with $ka < 1$.

<u>The liquid surface can only be unstable to long wavelength axisymmetric distances with $ka < 1$.</u> [45%] 9

(iii)



A disturbance with wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi a}{0.7}$ will grow most rapidly and eventually 'neck-off' to form droplets. Their volume will be equal to the volume of liquid within a wavelength, i.e

With d = droplet diameter, $\quad \dfrac{\pi d^3}{6} = \dfrac{2\pi a}{0.7} \times \pi a^2 \Rightarrow d = 3.8a$

droplets whose diameter = <u>3.8 x jet radius</u> [15%] 3

$d^3 = \frac{12\pi}{0.7} a^3$

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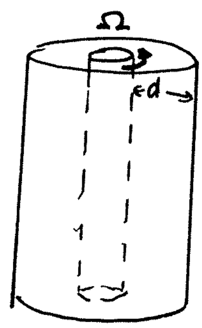
Qu 2



(i) Flow between two cylinders, inner rotating, outer fixed

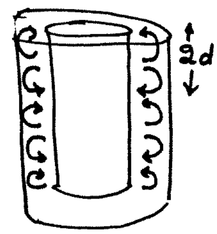
a) Rayleigh's criterion for inviscid, incompressible swirling flows shows that this flow is unstable (Γ^2 decreases with r). The mechanism is due to the pressure gradient $\frac{\partial p}{\partial r} = \frac{\rho V^2}{r}$ required to balance the radial acceleration.

A particle of fluid displaced to larger r, has more velocity than the surrounding particles and hence the local pressure gradient is not sufficient to restore its position and the particle continues to move to large radius – leading to instability.

(b) viscosity tend to stabilise the flow

(c) $Re = \frac{\Omega R_{inner}}{\nu}$, instability will only occur for $\frac{\Omega R_{inner}}{\nu}$ > critical value

(d) When Re goes just above this critical value, the flow breaks down into steady toroidal vortices, whose pattern repeats with a wavelength ~ 2d along the axis of the cylinder, where d is the gap width between the cylinders



[25%]

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(ii) Boundary layer flow over a flat plate

a) This flow is inviscidly stable by Rayleigh's inflexion point theorem ($U'' \neq 0$) and hence viscosity is the destabilising mechanism. It has the effect of changing the phase relationship between pressure and velocity in linear disturbances and enabling a transfer of energy from the mean flow into some perturbations which then grow.

b) Viscosity can stabilise the flow at very low Reynolds numbers

c) The relevant nondimensional parameter is the Reynold number Re_δ based on the local boundary layer thickness $Re_\delta = \frac{U_\infty \delta(x)}{\nu}$.

d) $\delta(x)$ increases from the leading edge of the plate, proportional to $x^{1/2}$, so Re_δ increases along the plate. The disturbances are Tollmien-Schlichting waves which travel downstream. At low Re_δ (i.e. near the leading edge) all waves decay as they propagate. At a particular axial position a wave of a particular frequency ω starts to grow,

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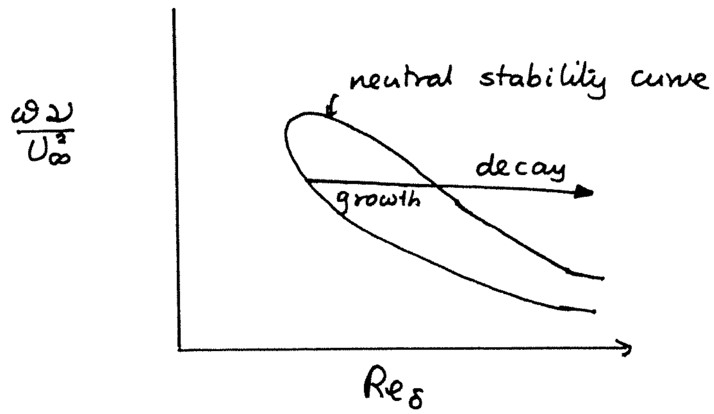
Qu 2 iid continued)



but it propagates downstream to larger Re_δ where it decays. When one of the waves grows to exceed a critical value (and the Re_δ at which this occurs depends on the amplitude of the oncoming flow disturbances) nonlinear and 3D effects become important leading ultimately to turbulence. [25%]

(iii) Temperature gradients in the atmosphere

a) This instability is buoyancy driven. A particle raised adiabatically a height dz will change temperature by dT_p $-\rho g\, dz \left.\frac{\partial T}{\partial p}\right)_s = -\frac{g}{c_p} dz$. If this is hotter than the surroundings it will continue to rise

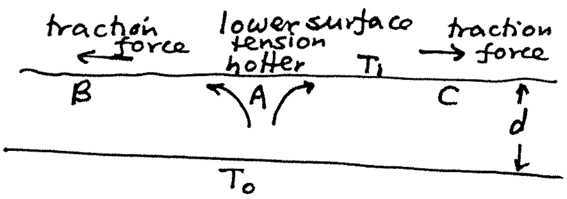
b) Viscosity and thermal diffusion can be stabilising influences as can turbulent mixing.

c) Then main nondimensional parameter is $\frac{c_p}{g}\frac{dT}{dz}$.

d) Thermal plumes and down drafts will occur. [25%]

(iv) a thin layer of liquid with a free upper surface and heated from below

a) There are two mechanisms - the buoyancy effect as in (iii), and an additional second mechanism due to the variation in surface tension with temperature and this is more important when the layer is very thin.



Upwelling fluid is hotter when it meets the surface. It therefore has less surface tension than the fluid at B and C. Surface traction therefore pulls the fluid away from A, so amplifying the motion.

b) Viscosity and diffusion are stabilising influences

c) The relevant nondimensional number is the Maragoni number

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Qu 2 iv c continued)

$$Ma = -\frac{d\sigma}{dT}\frac{(T_0 - T_1)d}{\rho\nu\kappa} \qquad \sigma = \text{surface tension}$$

d) The fluid breaks up into steady hexagonal cells, where the upwelling liquid rises in the centre of the hexagonal and sinks around the edges. [25%]

3./ Various examples covering

buffetting
vertex shedding
torsional divergence
galloping
flutter (of various kinds)
wave excited
turbulence excited

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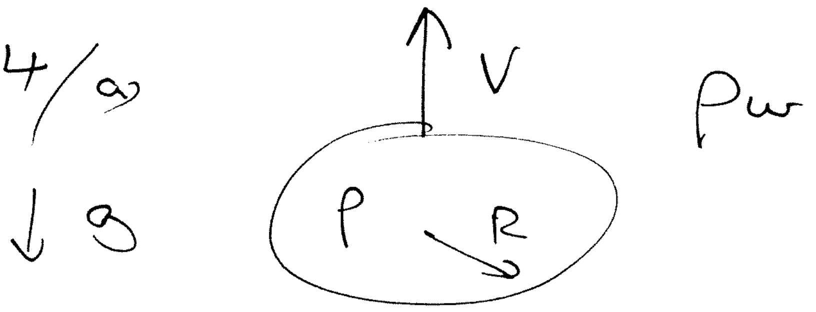
4/a)



$$C_D \frac{1}{2}\rho_w V^2 \cancel{\pi} R^2 = (\rho_w - \rho)\frac{4}{3}\cancel{\pi} R^3 g$$

$$V^2 = \frac{8}{3}\left(\frac{\rho_w - \rho}{\rho_w}\right) g \frac{R}{C_D} \quad ; \quad V = \left(\frac{8}{3}\left(\frac{\rho_w-\rho}{\rho_w}\right)\frac{gR}{C_D}\right)^{\frac{1}{2}}$$

[20%]

b) Turbulence intensity v' (or v'/V)

Turbulence spectrum $\phi_{vv}(\omega)$

Turbulence length scale L which could be seen as a single measure of the spectrum.

The turbulence is really a spatial variation of velocity through which the capsule moves at a velocity V. The spatial spectrum is described in terms of the wavenumber k which converts to a frequency by $\omega = \frac{2\pi k}{V}$?

[30%]

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Qu 4 cont.)

c) Provided the turbulence scale is large compared with the capsule size

$$F = \frac{1}{2}\rho C_D (V+v)^2 \pi R^2$$

$$\overline{F} = \frac{1}{2}\rho C_D (V^2 + \cancel{\overline{v^2}}) \pi R^2$$

$$f = \frac{1}{2}\rho C_D 2Vv (\pi R^2)$$

$$\overline{f^2} = (\rho C_D V \pi R^2)^2 \overline{v^2}$$

[30%]

d) By knowing the spectrum of the ~~force~~ velocity $\Phi_{vv}(\omega)$ and the ~~tran~~ aerodynamic admittance $|H_{vf}|^2$ $(\rho C_D V \pi R^2)^2$

then $\Phi_{ff}(\omega) = |H_{vf}|^2 \Phi_{vv}(\omega)$

Then need to spectrally link the forcing to the acceleration

integrate to velocity

integrate to displacement.

[20%]