IIB CRIBS
2003

4A11

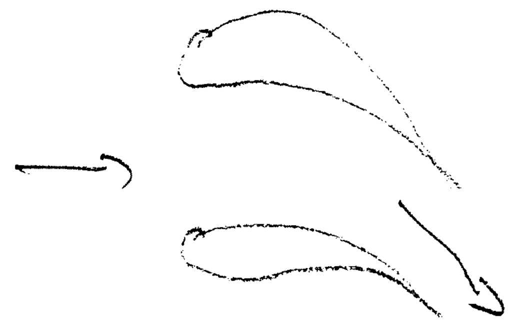
4. $\because$ $M_{2,is} = 1.2$ $\quad$ $s/c = 1.0$

$M_1 = 0.2$ $\quad$ $\beta_1 = 0^\circ$

$\alpha_2 = 70^\circ$

$p_{01} = 101330\,Pa$ $\quad$ $T_{01} = 288.15K$

a) i) assume isentropic



consider mass flow function

$$\frac{\dot{m}\sqrt{c_p T_0}}{A\ p_0}$$

& isentropic $p_{01} = p_{02}$ $\quad$ adiabatic $\Rightarrow T_{01} = T_{02}$

continuity $\Rightarrow \dot{m}_1 = \dot{m}_2$

$$\therefore \left(\frac{\dot{m}\sqrt{c_p T_0}}{A p_0}\right)_1 = F(M_1) = 0.4323$$

$$\left(\frac{\dot{m}\sqrt{c_p T_0}}{A p_0}\right)_2 = F(M_{2is}) = 1.2432$$

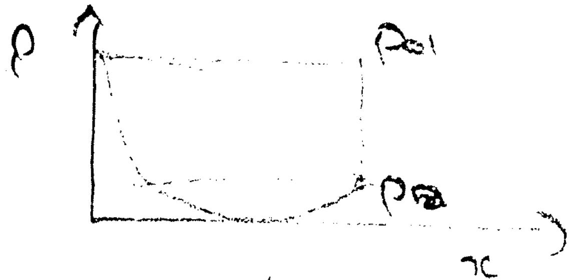
dividing

$$\Rightarrow \frac{s\cos\alpha_{2s}}{s} = \frac{F(M_1)}{F(M_{2is})}$$

$$\Rightarrow \alpha_{2s} = 69.63^\circ$$

3 mins

Zweifel



$$Z = \frac{\dot{m}\Delta V_\theta}{C_x(p_{01}-p_2)} \quad \text{for an axial machine}$$

$$= \frac{\rho_1 s V_1 (V_{\theta 2} - V_{\theta 1})}{C_x(p_{01}-p_2)} \qquad V_{\theta 2} = 0$$

$$= \frac{\rho_1 s V_1 V_{\theta 2}}{C_x(p_{01}-p_2)} \qquad \frac{s}{c} = 1.0$$

have to calculate $\rho_1, V_1, V_{\theta 2}$ & p_2

SFEE

$$\Rightarrow \frac{T_{01}}{T_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)$$

$$\Rightarrow T_1 = 285.86\,K$$

~~use~~ $$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/\gamma-1}$$

$$\Rightarrow p_1 = 98543\,Pa$$

$$\therefore \rho_1 = \frac{p_1}{RT_1} = 1.2011\,kg/m^3$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}} \Rightarrow V_1 = 67.78\,m/s$$

Cascade exit

$T_{01} = T_{02}$

$$\Rightarrow \frac{T_2}{T_{01}} = \frac{1}{\left(1+\frac{\gamma-1}{2}M_2^2\right)} \qquad T_2 = 223.71\,K$$

$p_{02} = p_{01}$

$$\Rightarrow \frac{p_2}{p_{01}} = \frac{1}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}} \qquad p_{2is} = 41786\,Pa$$

$$\rho_{2is} = \frac{p_{2is}}{RT_2} = 0.6508\,kg/m^3$$

$$V_{2s} = M_{2s}\sqrt{\gamma R T_2} = 359.77\,m/s$$

check flow angle

continuity

$$\Rightarrow \rho_1 S V_1 = \rho_2 S \cos\alpha_2 V_{2S}$$

$$\Rightarrow \cos\alpha_2 = \frac{\rho_1}{\rho_2}\frac{V_1}{V_{2s}}$$

$$\Rightarrow \alpha_{2s} = 69.65^\circ$$

& $\delta_s = \chi_2 - \alpha_{2s} = 0.346^\circ$

$V_{\theta 2s} = V_2 \sin\alpha_2$

& Zweifel, $Z_s = 0.461$

iii) repeat (i) with $\frac{p_{02}}{p_{01}} = 0.95$

$\Rightarrow p_{02} = 96263\,Pa$

keep $M_{2s} = 1.2$

now $\frac{p_{01}}{p_2} = \left(1 + \frac{\gamma-1}{2} M_{2s}^2\right)^{\gamma/\gamma-1}$ is unchanged

so p_2 is still $41786\,Pa$

& $\frac{T_2}{T_{02}} = \left(\frac{p_2}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}$

$\Rightarrow T_2 = 227.02K$

& M_2 is obtained from $\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$

$\Rightarrow M_2 = 1.1603$

$V_2 = M_2 \sqrt{\gamma R T_2} = 350.43\,m/s$

$\rho_2 = \frac{p_2}{R T_2} = 0.6413\,kg/m3$

continuity

$\Rightarrow \rho_1 V_1 = \rho_2 V_2 \cos\alpha_2$

$\Rightarrow \alpha_2 = 68.76°$

& $\delta_2 = 1.238°$

$Z = \frac{\rho_1 V_1 V_2 \sin\alpha_2}{p_{01} - p_2} = 0.446$

$$Z = \frac{\rho_2 V_2 \cos\alpha_2 V_2 \sin\alpha_2}{p_{01} - p_2}\left(\frac{s}{c}\right)$$

if loss is fixed $\Rightarrow p_2$ fixed

and M fixed $\Rightarrow V_2$ fixed & hence ρ_2

$$\Rightarrow Z = \frac{\rho_2 V_2^2}{p_{01}-p_2}\,\frac{s}{c}\,\frac{1}{2}\sin 2\alpha_2$$

use $2\sin\alpha_2\cos\alpha_2 = \sin 2\alpha_2$

max when $2\alpha_2 = 90^\circ \Rightarrow \underline{\alpha_2 = 45^\circ}$

$$\sigma = \frac{p_{02}}{p_{01}}$$

$$p_{01} - p_2 = \frac{p_{02}}{\sigma} - p_2$$

$$= p_2\left(\frac{p_{02}}{p_2}\frac{1}{\sigma} - 1\right)$$

$$= p_2\left(\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}\frac{1}{\sigma} - 1\right)$$

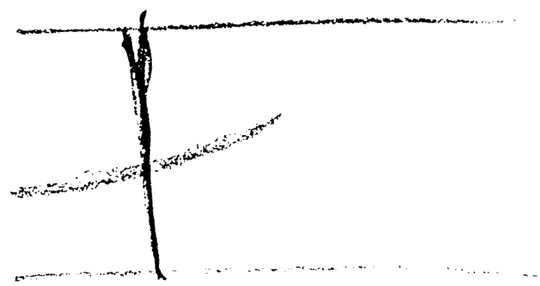
$$Z = \frac{\rho_2 V_2^2 \cos\alpha_2 \sin\alpha}{p_{01}-p_2}\,\frac{s}{c}$$

$$\rho_2 V_2^2 = M_2^2 \gamma p_2$$

$$\therefore Z = \gamma M_2^2\,\frac{1}{2}\sin 2\alpha\,\frac{s}{c}\,\frac{1}{\left[\frac{1}{\sigma}\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1} - 1\right]}$$

and all is dependent on M_2, gas props, $\frac{s}{c}$ as expected.

2.



radial momentum eq^n

$$\frac{\partial p}{\partial r} = \frac{\rho V_\theta^2}{r}$$

Use $T\,ds = dh - \frac{dp}{\rho}$

$$\Rightarrow \frac{\partial p}{\partial r} = \cancel{\rho}\frac{\partial h}{\partial r} - \cancel{\rho} T\frac{\partial s}{\partial r} = \frac{\cancel{\rho} V_\theta^2}{r}$$

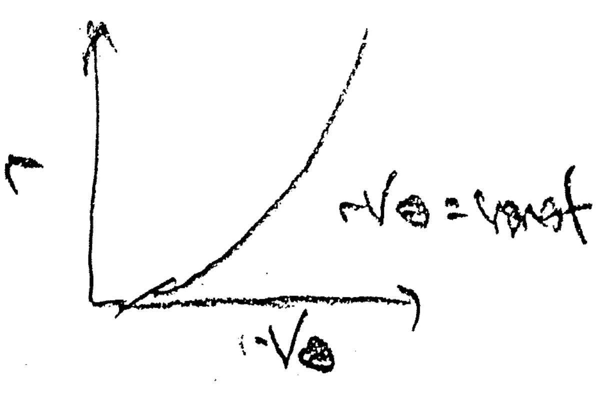
$$\Rightarrow \frac{\partial h}{\partial r} + \frac{\partial(\frac{1}{2}V^2)}{\partial r} - \frac{\partial(\frac{1}{2}V^2)}{\partial r} - \frac{T\partial s}{\partial r} = \frac{V_\theta^2}{r}$$

$$\Rightarrow \frac{\partial h_0}{\partial r} - \frac{\partial(\frac{1}{2}V^2)}{\partial r} - \frac{T\partial s}{\partial r} = \frac{V_\theta^2}{r}$$

$$\Rightarrow \frac{\partial h_0}{\partial r} - \frac{T\partial s}{\partial r} = V_m\frac{\partial V_m}{\partial r} + V_\theta\frac{\partial V_\theta}{\partial r} + \frac{V_\theta^2}{r}$$

$$= V_m\frac{\partial V_m}{\partial r} + \frac{V_\theta}{r}\frac{\partial}{\partial r}(rV_\theta)$$

b) i)



$$R = \frac{\Delta h_r}{\Delta h_{stage}}$$

(ii) if the reaction at the mid-span is 0·60 and the reaction at the hub is 0·20 what is the radius ratio r_{hub}/r_{casing}? $V_r \simeq 0$

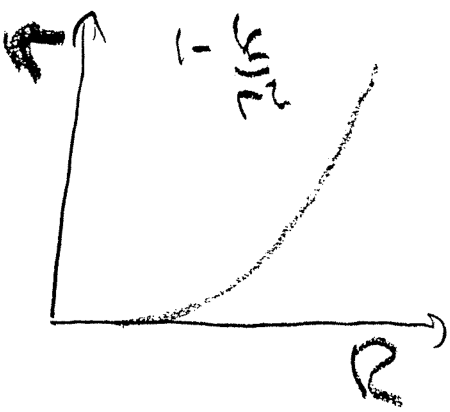
$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}}$$

$\Delta h_{stage} = \Delta h_0$ as $V_{\theta 1} = V_{\theta 3} = 0$ & $V_x =$ const

$$= \Omega r V_{\theta 2}$$

$$\Delta h_{rotor} = \Delta h_0 - \Delta(V^2)$$
$$= \Omega r V_{\theta 2} - V_{\theta 2}^2$$

$$\therefore \quad R = 1 - \frac{V_{\theta 2}}{\Omega r}$$
$$= 1 - \frac{(r V_{\theta 2})}{\Omega} \frac{1}{r^2}$$



mid span & hub values

$$\Rightarrow \quad 0{\cdot}6 = 1 - \frac{k}{r_m^2} \quad \& \quad 0{\cdot}2 = 1 - \frac{k}{r_h^2}$$

$$\therefore \quad \underline{k = 0{\cdot}4 r_m^2}$$

$$\Rightarrow \quad \frac{0{\cdot}4 r_m^2}{r_h^2} = 0{\cdot}8$$

$$\Rightarrow \quad \frac{r_m}{r_h} = \sqrt{2}$$

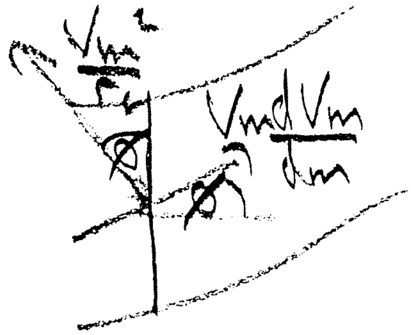
$$r_m = \frac{r_h + r_c}{2} \quad \Rightarrow \quad 1 + \frac{r_c}{r_h} = 2\sqrt{2}$$

$$\Rightarrow \quad \frac{r_c}{r_h} = 2\sqrt{2} - 1 = 1{\cdot}8$$

∴ c) require streamline curvature term resolved along QO (a radial line in this case)

+ meridional acc^n resolved along QO

ie acceleration is



$$\therefore \text{ need } \frac{V_m^2}{r_c}\cos\phi$$

+ a blade force term per unit mass flow resolved along QO

$$\frac{F_\phi}{\rho}$$

$$\therefore -\frac{F_r}{\rho} + \frac{\partial h_0}{\partial r} - T\frac{\partial s}{\partial r} = V_m\frac{\partial V_m}{\partial r} + \frac{V_\theta}{r}\frac{\partial (rV_\theta)}{\partial r} - \frac{V_m^2}{r}\cos\phi - V_m\frac{\partial V_m}{\partial r}$$

3. Panel method

- fast, accurate
- subsonic only
  no loss

Streamline curvature

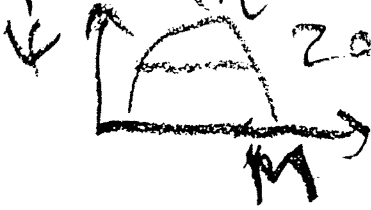
- periodicity difficult to satisfy
- easy for inverse design
- supersonic patches possible
- inaccurate L.E.

Velocity potential

- fast, irrotational flow
- can handle supersonic & weak shocks
- iterative (Kutta)
- no losses
- generally replaced by Euler + NS codes

Stream function

- no widely used
- not really for supersonic flows

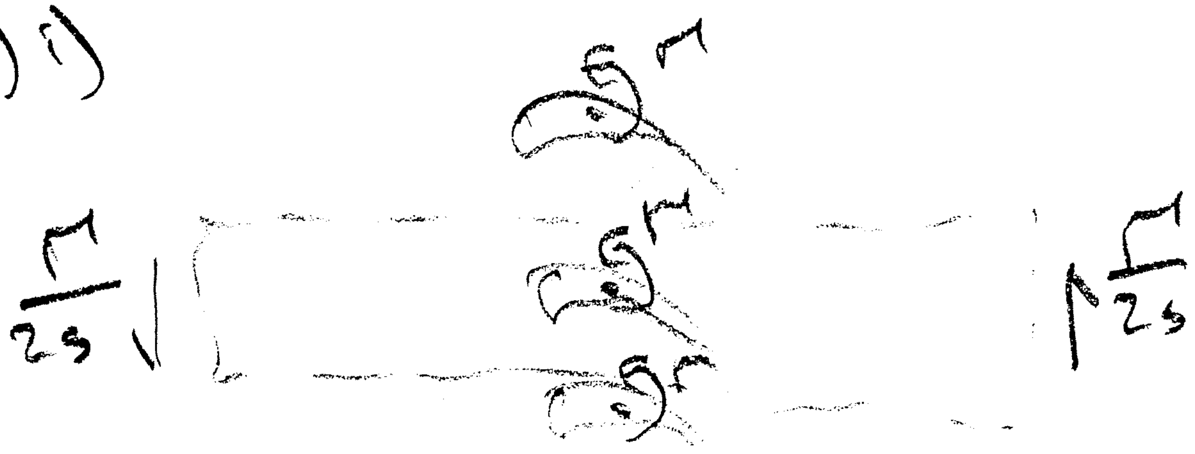


Euler + NS

- general
- losses included in NS
- expensive to run
- low M difficult

+ a few details of each method would be given etc

(b) i)



$$V_\theta = \frac{\Gamma}{2s} = 1.375 V_x$$

ii) blend 2 solutions 1 @ const V_x at inlet
other @ V_θ at inlet
blend to allow for induced terms.

c) Lean: lecture notes