4D7 CONCRETE & MASONRY STRUCTURES EXAM CRIB 2003. 1/1

Q1 (a) Define 'limit states' as states beyond which the structure no longer satisfies the design performance criteria.
Classified into ULS & SLS – Ultimate & Serviceability Limit States.
Aim to take into account varying degree of uncertainty in each design parameter by using separate partial safety factors calibrated such that the overall reliability should be roughly the same for different ULS & SLS criteria. These partial factors are used to multiply the loads (or actions) & divide the strengths to reduce the probability of failure to an acceptable 'target' level although there is no clear agreement on what the target level (or levels) should be.

Example: Concrete Bridge (long span)

COLLAPSE/FAILURE – ULS – collapse due to various modes bending, shear etc. stability (e.g. columns)
– these relate to safety of structure & people/users.

PERFORMANCE IN SERVICE – SLS – excessive deflection – public disquiet – may need stiffening.
– vibration – discomfort to users (Millennium Bridge) from wind, traffic
– cracking – implications on safety if corrosion accelerated by cracking ~~&/or~~ if spalling results in accidents
– implications on economic cost for same reasons as above since repair will be required, disruption caused.

Also mention idea of characteristic values at 5% level to increase loading & decrease strengths hence allowing for variability. Then include partial safety factors to get design values.

Require $R_d \geq S_d$ (Resistance R_d, Load S_d) where $R_d = \dfrac{R_k}{\gamma_m}$ (R_k char. strength, γ_m partial factor on strength) (ULS 1.5 conc. 1.15 steel, SLS 1.0)

& $S_d = \gamma_f . S_k$ S_k = char. load
γ_f = partial factor on load
(e.g. ULS 1.4 DL, 1.6 LL)
SLS 1.0 on loads

Q1 (cont.)

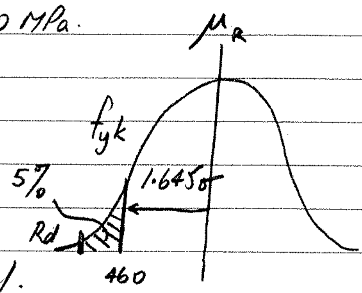
(b) Structural failure is very rare, typical P_f's $\sim 10^{-6}$.

These rare events are assumed to be modelled mathematically by the shape of the tails of the probability distribution curves. The likelihood or probability of such events occurring is determined from the statistical properties of these tails. The validity of this approach can be questioned since there is often little or no data to support the mathetical models in these extreme regions at say, 5 or 6 standard deviations from the mean corresponding to probabilities of failure $\sim 10^{-7}$ or less. An example is that of r/f steel. We specify a characteristic strength of 460 MPa & typically design for ~400 MPa yet actual r/f bar tends to have a characteristic strength of ~480 - 490 MPa & 460 MPa seems to be a proof load value below which no bars are found. If the actual data was modelled in a reliability analysis it would indicate that a normally proportioned r.c. member would be almost impossible to fail under, for example, flexural loading.

It is important to remember that most recorded structural failures result from influences/load effects/causes that are not usually considered in conventional analysis. e.g. most failures of bridges occur due to scour around abutments or impact from ships.

Q1(c) Resistance (strength), R.

(i) $f_{yk} = 460$ MPa. (5% level) $R_d = f_{yd} = \frac{f_{yk}}{\gamma_{ms}} = \frac{460}{1.15} = 400$ MPa.



$CoV = \frac{\sigma}{\mu} = 10\%$.

Find mean value: $z = 1.645$ at 5% level.
(from table in Maths data book)

$$f_{yk} + 1.645\sigma = \mu_R \quad \text{where } \frac{\sigma}{\mu_R} = 0.10.$$

$$\therefore 460 + 1.645(0.1\mu_R) = \mu_R.$$

$$\mu_R(1 - 0.1645) = 460$$

$$\therefore \mu_R = \frac{460}{0.84} = 550.57 \simeq \underline{551 \text{ MPa}}.$$

$$\therefore \sigma_R = 0.1\mu_R = 55.1 \text{ MPa}. \simeq \underline{55 \text{ MPa}}$$

Find probability that bar strength is less than $f_{yd} = 400$ MPa.

$$z = \frac{\mu_R - x_R}{\sigma_R} = \frac{551 - 400}{55} = 2.75$$

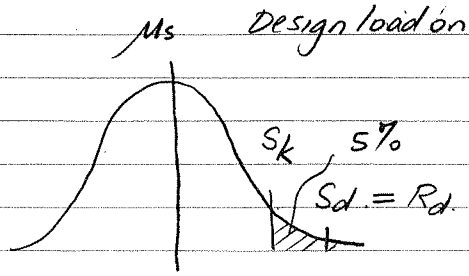
From maths data book for $z = 2.75$ $A = 0.9970$.

$$\Rightarrow P(x_R \leq 400\text{MPa}) = 1 - 0.9970 = \underline{0.003} \quad (3\times10^{-3})$$

(ii) Design value of strength of bars = 400 MPa.

$\phi_{bar} = 16$mm $A_{bar} = \frac{\pi}{4}\times 16^2 = 201.1 \text{mm}^2$

Design load on bars, $S_d = f_{yd} \times A_{bar} = 201.1\times400\times10^{-3} = \underline{80.4\text{k}}$



Load Normal with $\sigma_s = 10$kN.

$S_d = \gamma_f . S_k$ Assume $\gamma_f = 1.4$ for DL.

$$\therefore S_k = \frac{S_d}{\gamma_f} = \frac{80.4}{1.4} = \underline{57.4 \text{ kN}}$$

$$\therefore \mu_s = S_k - 1.645\times\sigma_s = 57.4 - 1.645\times10$$
$$= 40.95 = \underline{41 \text{ kN}}$$

Q 1(c)(ii) cont.

Reliability $\beta = \frac{\mu_R - \mu_S}{\sigma_{R-S}}$

A_{bar}

$\mu_R = 551 \times 201.1 \times 10^{-3} = 110.8$ kN

$\sigma_R = 55 \times 201.1 \times 10^{-3} = 11.1$ kN.

$$= \frac{110.8 - 41}{\sqrt{11.1^2 + 10^2}} = 4.67.$$

$\therefore$ In statistics databook for $u = 4.67$ $\phi(u) = 0.9^5 8494$

$$p_f = 1 - \phi(u) = 1 - 0.9^5 8494 = 1.51 \times 10^{-6}$$

(iii) Modified strength

$\mu_R' = 500$ MPa $(100.6$ kN$)$ $f_{yk} = 500 - 1.645 \times 23 = 462$ MPa.

$\sigma_R' = 23$ MPa $(4.63$ kN$)$

$>$ f_{yk} specified of 460 MPa so bars OK

Bars supplied comply with specification.

Modified loads 20% ↑ in μ_S + σ_S over part (ii)

$\therefore \mu_S' = 1.2 \times 41 = 49.2$ kN

$\sigma_S' = 1.2 \times 10 = 12$ kN.

$$\therefore \beta = \frac{\mu_R' - \mu_S'}{\sqrt{\sigma_R'^2 + \sigma_S'^2}} = \frac{100.6 - 49.2}{\sqrt{4.63^2 + 12^2}} = \frac{51.4}{12.9} = 4.0$$

$\therefore$ From statistical data book for $\beta = 4$ i.e. $u = 4$ $\phi(u) = .9^4 6833$

$$\therefore p_f = 1 - \phi(u) = 1 - 0.9^4 6833 = 3.2 \times 10^{-5}$$

(iv) Target $\beta_t = 3.5$ $\sigma_S = 10$ kN $\sigma_R = 11.1$ kN

$\mu_R = 110.8$ kN

$$\therefore \beta_t = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{110.8 - \mu_S}{\sqrt{11.1^2 + 10^2}} = \frac{110.8 - \mu_S}{14.94} = 3.5$$

$$\therefore \mu_S = 110.8 - 3.5 \times 14.94 = 58.5 \text{ kN}$$

④ b.

Q 1(c)(iv) cont.

∴ Characteristic load $S_k = \mu_S + 1.645 \times \sigma_S = 58.5 + 1.645 \times 10 = 74.95$

i.e. $S_k = 75\ kN$

Design load $S_d = \gamma_f \times S_k = 1.4 \times 75 = 105\ kN$

since γ_f for dead load is taken as 1.4.

$Pf\ (\beta = 3.5) = 1 - 0.9^3 7674$ from tables $u = 3.5$

$= 2.3 \times 10^{-4}$ $\phi(u) = .9^3 7674$

Q2(a) Consider 4C's of concrete durability + H_2O.

1. Cement content 2. Cover 3. Compaction 4. Curing + W/c.

Cement content – wish to ensure adequate cement content to ensure strength & also so that sufficient binding matrix is available to hydrate, react & fill voids within composite aggregate/sand/cement mixture.

– also promotes alkaline environment & hence passivation of reinforcing steel to reduce likelihood of corrosion.

- Want cement to be available to promote 'autogeneous healing' of fine cracks.

Cover – provides protective barrier to ingress of deleterious materials (e.g. chlorides, carbonation)

- ensures adequate bond to r/f bars.

(should not be too large or may get spalling).

Compaction – provides dense, impermeable concrete by removing air pockets/bubbles. This again assists in preventing ingress of corrosive materials toward r/f.

(Must avoid segregation from over-compaction).

Curing – essential to prevent drying out of exposed surface of concrete which can result in incomplete hydration reaction. This may inhibit the closing of capillary pores within the matrix, again allowing greater permeability to corrosive materials. Similarly you may get surface cracking from drying shrinkage.

WATER CEMENT RATIO – perhaps the most important parameter to control is W/c ratio. It should be kept to the minimum necessary to permit full hydration & adequate workability (as measured by a slump test) The theoretical minimum (~0.3) ensures the full hydration reaction can occur & hence capillary pores are closed up & high interaction between cement grains promotes strength. This in turn decreases permeability. Too much water results in permeable, low strength concrete e.g. W/c $\gtrsim$ 0.7.

(6)

Q2 (b) Cement replacement materials.

These work by direct replacement of a proportion of the cement added to the concrete mix. e.g. PFA (pulverized fuel ash), silica fume & GBFS (granulated blast furnace slag).

PFA (a pozzolanic material i.e has reactive silica) is usually used for replacing ~20-30% of cement.
The role of these materials is to improve specific properties of the concrete such as:-

1) ↑ workability - has same water demand as cement but improved particle shape
2) Long term gain in strength - it reacts with free lime which would otherwise leach out.
   - depends on fineness of active silica
3) More resistant to sulphate attack. - Amount of C_3A reduced.
* 4) Lower temperature during initial hydration (15°c/100kg c.f. 12°c/100 kg for OPC.)
5) Less bleeding & improved pumpability
6) Reduced shrinkage
7) Inhibits corrosion - thought to fill capillary pores formed by leaching CaO with a pozzolanic product which reduces permeability

GBFS improves concrete in a similar way although it is a cementitious material, not pozzolanic, & hence can act like cement & thus greater % replacement can be used (up to ~80%).

Q2 (c) (i) Chloride induced corrosion

- chlorides from enviroment (e.g. road de-icing salts, saltwater spray in coastal regions) permeates into the concrete by what is thought to be a diffusion process. When chloride ions arrive at the reinforcement surface they seem to act as a catalyst to the corrosion process (but are not consumed in the reaction). Rate of corrosion reaction depends on permeability of cover concrete to oxygen + resistance in the corrosion cell (i.e. resistivity of the concrete) which depends on moisture content.

" Carbonation induced corrosion

CO_2 from atmosphere diffuses through the concrete. This is acidic & acts to neutralize the alkalinity in the concrete pore water. As this front reaches the steel, the lower Ph of the pore water results in depassivation of the steel & corrosion can then commence (in the presence of oxygen + water).

The main methods for preventing or minimizing Chloride or Carbonation induced corrosion are to 1) follow 4C's for durable concrete + ensure W/C ratio is low

and 2) provide a barrier to the Cl^- or CO_2.

e.g. - silane coating of surface / waterproofing
- epoxy coated r/f. in original design
- Stainless steel r/f. (may still be corrosion problems + also expensive)

(ii) Identifying corrosion - visual evidence from spalling or surface cracks (particularly aligned with r/f) resulting from expansive nature of rust products &/or visual evidence from brown rust stains leaching out onto surface.

- can also use half-cell potential measure (indirect + gives probability of corrosion), resistivity, some say radar (questionable?), delamination survey (i.e. hammer survey), eddy current measurement, digouts. + more.

Q2(c)(ii) cont.)

Remedy corrosion:

1) Cut out + replace contaminated concrete (e.g. for high Cl^-)
   - replace bars as necessary. Labour intensive + expensive.
   - perhaps only suited to localized areas.
2) CP (Cathodic protection). Use impressed current at bar surface to stop corrosion reaction. Very effective but expensive + involves on-going running costs
3) Sacrificial anode - connect this to r/f - lower potential results in anode corroding before steel.
4) Realkalisation - use an imposed electrical field to drive alkalis back into concrete + result in repassivation of steel. (e.g. for carbonated concrete).
5) Desalination - Cl^- ions "sucked" out from concrete by imposing a potential from a surface mesh to the bars.

Both 4 + 5 are expensive procedures but have been used in practice in the UK.

Cannot easily replace steel with non-ferrous r/f after corrosion has started.

Q 2(d) Decide between two maintenance/repair options on economic grounds. Life of structure is 75 years. Discount rate is 6%.

Option 1 Repair every 25 years @ £250,000.

Net Present Value $NPV = \frac{\Sigma C_i}{(1+r)^{i-1}}$ assuming costs are to be incurred at start of year in which repairs are needed.

$$\therefore \quad NPV = \frac{250 \times 10^3}{(1.06)^{24}} + \frac{250 \times 10^3}{(1.06)^{49}}$$

No costs in last year since at end of 75 year life.

$$= \frac{250 \times 10^3}{4.05} + \frac{250 \times 10^3}{17.38}$$

$$= 61.7 \times 10^3 + 14.39 \times 10^3 = £76.13 \times 10^3 \quad (£76.1k)$$

Option 2 Install Cathodic Protection @ initial cost of £60k plus running costs of £3k per year.

Use continuous discounting

$$A = A_0 \exp(r_c t) \qquad \text{where } 1+r = \exp(r_c)$$

$$1.06 = \exp(r_c)$$

$$\therefore \quad r_c = \ln(1.06) = 0.05827$$

$$\int_0^{75} \frac{3000}{\exp(r_c t)}\,dt = 3000\left[\frac{\exp(-r_c t)}{-r_c}\right]_0^{75} = \frac{3000}{r_c}\left[1 - \exp(-75 r_c)\right]$$

$$= \frac{3000}{0.05827}\left[1 - \exp(-75 \times 0.05827)\right] = 50.83 \times 10^3$$

$$(£50.8k)$$

Must add installation cost in year 1 to running costs.

CP Total cost (option 2) $= 60 + 50.8 = \underline{£110.8k}$

CP much more expensive than original option (45% more).

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2 (d)(ii)

To reduce running costs to match option 1 if installation costs ↓ to £50k

$$\therefore \frac{C_i}{0.05827}\left[1-\exp\left[-75\times 0.05827\right]\right] = 76.13\times10^3 - 50\times10^3$$

$$= 26.13\times10^3$$

C_i = £1542 per annum to match costs of option 1.

2 (d)(ii)

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C_i = £1542 per annum to match costs of option 1.

YQ

(11)

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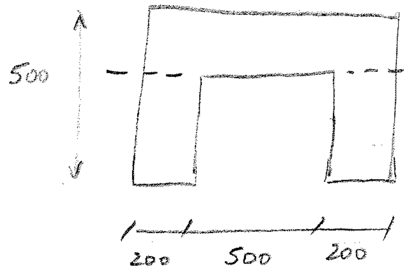
Q3 (a) <u>1st cracking</u>



$\bar{y}$ & I_{UN} given in question

Centroid. $\bar{y} = \dfrac{500\times400\times250 + 500\times200\times400}{500\times(400+200)}$

$= \dfrac{1800}{6} = 300\text{ mm}.$

$I_{un} = \dfrac{900\times200^3}{3} + \dfrac{400\times300^3}{3} = 2.4E9 + 3.6E9$

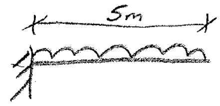
$= 6E9\text{ mm}^4.$

$\sigma_t = \dfrac{My}{I} \Rightarrow M_{cr} = \dfrac{\sigma_t I_{un}}{y} = \dfrac{4\times6E9}{200} = 120E6\text{ Nmm } (120\text{kNm})$

$M_{cr} = 120\text{kNm}$

Maximum moment near support

$M = \dfrac{wL^2}{2}$. $L = 5\text{ m}$.



DL+LL $W_{LL} = 6\text{ kN/m}$

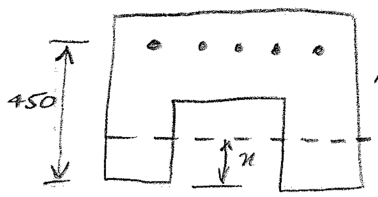
$A_g = 500\times(400+200) = 300E3\text{ mm}^2$

$\therefore w = \dfrac{2M}{L^2} = \dfrac{2\times120}{25} = 9.6\text{ kN/m}.$ $\gamma_{conc} = 24\text{ kN/m}^3$

$\therefore w_{DL} = 24\times0.3 = 7.2\text{ kN/m}$

$W_{DL} = 7.2 \Rightarrow W_{LL} = 2.4\text{ kN/m}$ for 1st cracking

(b) Require I_{cr}.



$m = \dfrac{E_s}{E_{ce}} = \dfrac{210}{30} = 7.$

$A_s = 1608\text{mm}^2$

Transformed steel area $A_s' = mA_s = 7\times1608 = 11.26\times10^3\text{ mm}^2$

To find n.a. $200\times2\times x\times\dfrac{x}{2} = 11.26\times10^3\times(450-x)$

$200x^2 + 11.26\times10^3 x - 11.26\times10^3\times450 = 0.$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x = \dfrac{-11.26\times10^3 + \sqrt{(11.26\times10^3)^2 + 4.200.11.26\times10^3}}{2\times200}$

$x = 133.5\text{ mm}$ & $d - x = 450 - 133.5 = 316.5\text{mm}$

from parallel axes formula.

$\therefore I_{cr} = \dfrac{400\times133.5^3}{3} + 11.26\times10^3\times316.5^2 = 1.449\times10^9\text{ mm}^4$

$\sigma_c = \dfrac{My}{I_{cr}}$ where $M = \dfrac{wL^2}{2}$ $\gamma_{f_{SLS}} = 1.0$ for DL & LL.

$W_{LL} = 6\text{ kN/m}$.

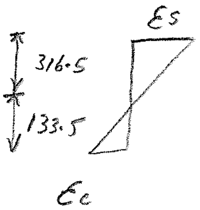
$W_{DL} = 7.2\text{ kN/m}$ from part (a)

Q.3(b) cont. ∴ At SLS $M = 13.2 \times \frac{25}{2} = 165$ kNm. since $W = 13.2$ kN/m

∴ $\sigma_c = \frac{165\times10^3 \times 133.5\times10^{-3}}{1.449\times10^{-3}} = 15.2\times10^6$ N/m^2 i.e. 15.2 MPa

compression at bottom flange of beam.

Steel strain



$\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{15.2}{30\times10^3} = 506.7\times10^{-6}$.

$\varepsilon_s = \frac{316.5}{133.5}\times\varepsilon_c = 0.0012$ $< \varepsilon_y$ NOT YIELDED

where $\varepsilon_y = \frac{f_{yk}}{E} = \frac{460}{210\times10^3}$

$= 0.00219$

(c) (i) Tip deflection - uncracked beam (SLS)

From data book $\delta_{tip} = \frac{WL^3}{8EI}$ where W = total load = wL.

$= \frac{wL^4}{8EI}$

∴ $\delta_{UN} = \frac{13.2\times10^3 \times 5^4}{8\times30\times10^9\times6\times10^{-3}} = 5.7\times10^{-3}$ m i.e. 5.7mm

(ii) Tip deflection - cracked beam (SLS)

$\delta_{cr} = 5.7\times\frac{6}{1.449} = 23.7$mm

(d) $\delta = \xi\delta_{cr} + (1-\xi)\delta_{UN}$ (formula from Data sheet).

$\xi = 1 - \beta_1\beta_2\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$ $\beta_1 = 1.0$ deformed bars

$\beta_2 = 1.0$ short-term loading.

σ_{sr} is stress in steel for loading to cause cracking on cracked section.

$w_{cr} = 9.6$ kN/m from (a) $I_{cr} = 1.449E9$ mm^4 $M_{cr} = 120$ kNm.

$\sigma_c = \frac{120}{165}\times15.2 = 11.05$ MPa

(13)

Q3 (d) cont.

$$\sigma_{sr} = \frac{\overset{\sigma_c}{316.5}}{133.5} \times 11.05 \times \overset{m}{7} = \underline{183.5 \text{ MPa}}$$

σ_s is stress in steel at full load on cracked section.

$$\sigma_s = E_s \varepsilon_s = 210 \times 10^3 \times 0.0012 = \underline{252 \text{ MPa}}$$

$$\therefore \xi = 1 - \left(\frac{183.5}{252}\right)^2 = 0.47$$

$$\therefore \delta = \underbrace{\overset{.53}{\cancel{0.47}} \times 5.7}_{3.02} + \underbrace{\overset{.47}{\cancel{0.53}} \times 23.7}_{11.14} = \underset{14.2}{\cancel{15.2}} \text{ mm}$$

$\therefore \underline{\delta_{tip} = \cancel{15.2} \text{ mm}}$ 14.2. with allowance for tension stiffening.

Actual tip deflection will be less since beam will not be cracked all the way along to the free end & hence curvature in this end region will be less than that assumed above.

(e) Shear capacity

From Data sheet $V_{Rd_1} = b_w d \{\tau_{Rd}\, k(1.2 + 40\rho_1)\}$

$$\text{where } \rho_1 = \frac{A_s}{bd} = \frac{1608}{400 \times 450} = 8.93 \times 10^{-3}.$$

$$\therefore V_{Rd_1} = 400 \times 450 \{0.5 \times (1.6 - 0.45)(1.2 + 40 \times 8.93 \times 10^{-3}) = 161.2 \times 10^3 \text{ N}$$

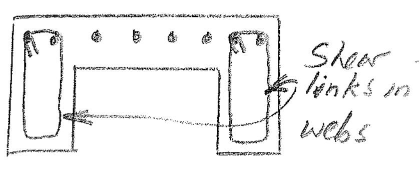
(161.2 kN)

Applied shear force at ULS

$$w_{ULS} = 1.4\, w_{DL} + 1.6\, w_{LL} = 1.4 \times 7.2 + 1.6 \times 6 = 19.7 \text{ kN/m}.$$

$\therefore V_{sd} = 19.7 \times 5 = 98.5 \text{ kN} < V_{Rd_1}$ So OK in shear without shear links.

Although shear links are not required for strength most codes specify some minimum area of steel links in beams. (Nominal A_{sv})



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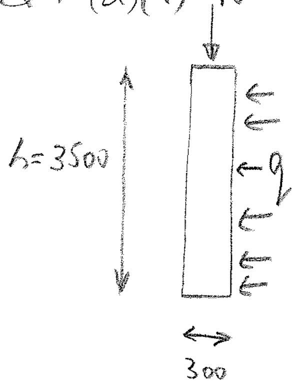
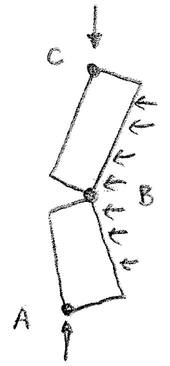
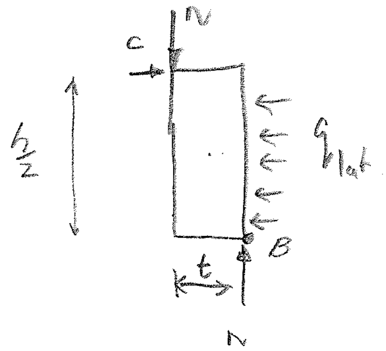
Q4 (a)(i)





Mechanism



FBD of top half BC

Moments about C. $\quad N.t = q_{lat} \cdot \frac{h}{2} \cdot \frac{h}{4}$

$$\therefore \; q_{lat} = \frac{8Nt}{h^2} = \frac{8 \times 900 \times 0.3}{3.5^2} = \underline{176.3 \text{ kPa}}.$$

Adopt Factor of Safety of 2 overall $\Rightarrow q_{lat} = \frac{4Nt}{h^2} = 88.2$ kPa.

(Cannot use normal γ_m since material strength is not involved so effectively have a factor on applied loading.)

(ii) Vertical force = 900 kN

Wall area (per meter length) $A = 0.3 m^2$

$$\sigma = \frac{900 \times 10^3}{0.3} = 3.0 \times 10^6 \, N/m^2 \qquad 3.00 \text{ MPa}.$$

This does not allow for factors of safety & is based on crushing strength of masonry. In practice would wish to apply partial factors on loads & compressive strengths.

Apply $\gamma_f = 1.5$ & $\gamma_m = 3$

$$\Rightarrow \frac{\sigma}{3} = 1.5 \times 3.00 \quad \Rightarrow \sigma = 13.5 \text{ MPa} \quad \text{for masonry.}$$

Bricks to be Class 7 with mortar (i)

or Class 10 " " (ii)

(15)

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Q4(b) Heyman's Plastic Theory for Masonry Arches

Assumptions - tensile strength of masonry is zero (⊘ tension)
- compressive strength is infinite (∞ compression)
- sliding between blocks is prevented by friction (no slip at joints)

Resulting analysis for arches is then purely determined by geometry.

Validity - usually some (small) tensile strength but safe to assume zero.
- compressive strength is not infinite but usually applied (average) stresses are low c.f. stone strength. Need to avoid localized "high points" of concentrated load. Can allow for this finite strength by modifying (reducing) the geometry. e.g. hinge at some distance in from face of stone blocks)

Theorems

1) Statical (Lower Bound Theorem) - a masonry structure will stand if a line of thrust, equilibrating the applied loads, can be found which is entirely within the masonry.

2) Kinematic (upper bound) theorem. If assume failure mechanism with hinges at outer surfaces of masonry, the work equation will give an upper bound on the collapse load. No internal work within the material on these assumptions).

These theories work well for isolated arches but less well for infilled arches e.g. masonry bridges, because of the effect of the infill & the spandrel walls

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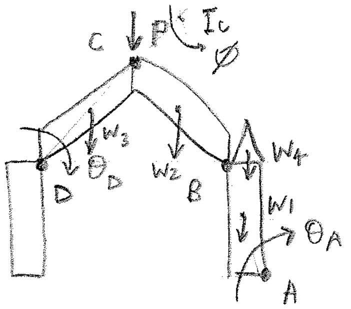
Q 4 (c) Find P



Assume rotation ϕ at instantaneous centre for mechanism with hinges at A, B, C & D.

Scale on diagram $L = 40$mm

Scale distances from diagrams (Fig 3) $(S = r\theta)$

$\delta_c = \frac{\cancel{17}\,18}{40} L\phi$ $\theta = S/r$

$$\theta_D = \frac{\delta_c}{DC} = \frac{\frac{\cancel{17}\,18}{40}L\phi}{\frac{69}{40}L} = \frac{\cancel{17}\,18}{69}\phi = \cancel{0.246}\,0.261\,\phi.$$

$\delta_B = \frac{\cancel{75}\,77}{40} L\phi$

$$\theta_A = \frac{\frac{\cancel{75}\,77}{40}L\phi}{\frac{43}{40}L} = \frac{\cancel{75}\,77}{43}\phi = \cancel{1.744}\,1.7\,\phi.$$

Work equation

$$W_3 \cdot \frac{L}{2} \cdot \theta_D - W_2 \cdot \frac{\cancel{9}\,10}{40}L \cdot \phi - W_1 \cdot 0.2L \cdot \theta_A + P \cdot \frac{\cancel{11}\,10}{40}L \times \phi = 0.$$

Not given γ stone

$W_1 = W_2 = W_3 = W = 0.4L \times L \times \gamma_{stone} = 0.4 \times 25L^2 = 10L^2$ kN

since area of each block is the same.

$$\therefore -WL\phi\left(\frac{\cancel{0.246}\,261}{2} - \frac{\cancel{9}\,10}{40} - 0.2 \times \cancel{1.744}\,1.7\right) = P \cdot \frac{11}{40} L\phi$$

$$P = \frac{40}{\cancel{11}\,10} \cdot \cancel{0.451}\,0.46\,W = \cancel{1.64}\,1.84\,W$$

Assume overall factor of safety = 2 $\Rightarrow P_{allowed} = \frac{\cancel{1.64}\,1.84}{2}W = \boxed{\cancel{0.82}\,0.92\,W}$

(d) Given $W_1' = \frac{W_1}{2} = \frac{W}{2}$ Pinnacle $= W_4$ @ $\left(\frac{29+8}{40}\right)L$ from I_c

Deduced sol^n

NOTE: Centroid of W_4 is at same offset from I_c at centroid of wall stone (W_1).

$\therefore$ The pinnacle rotates same amount. $\therefore$ Loss of WD in wall stone must be balanced by addition work done by pinnacle. $\Rightarrow$ Wt loss of wall stone = Wt of pinnacle.

$\Rightarrow$ Area of pinnacle must equal half area of wall stone (since density of wall stone is $\frac{\gamma_{stone}}{2}$ & of pinnacle is γ_{stone})

$\therefore$ Area pinnacle $= \frac{1}{2} \times b \times H = \frac{1}{2} \times 0.4L \times H = \frac{1}{2}\,0.4L^2$ $\therefore H = L$.

$\Rightarrow \boxed{H = L}$ Pinnacle height must equal wall height.

Calculated sol^n to Q.4(d)

Want work done by W4 equal to loss of work done by reduced density of W_1.

$$\Rightarrow \quad W_4 \cdot \cancel{0.2L} \cdot \cancel{\theta_A} = \frac{W_1}{2} \cdot \cancel{0.2L} \cdot \cancel{\theta_A}$$

$$\therefore W_4 = \frac{W_1}{2} = \frac{W}{2} \qquad W_4 = \cancel{\gamma_{stone}} \cdot \cancel{\tfrac{1}{2}} \cdot \cancel{0.4L} \cdot H = \frac{\cancel{\gamma_{stone}}}{\cancel{2}} \cdot \cancel{0.4L} \cdot L$$

$$\therefore \underline{H = L}$$

(18)

4/5

ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Thursday 1 May 2003 2.30 to 4

Module 4D7

CONCRETE AND MASONRY STRUCTURES

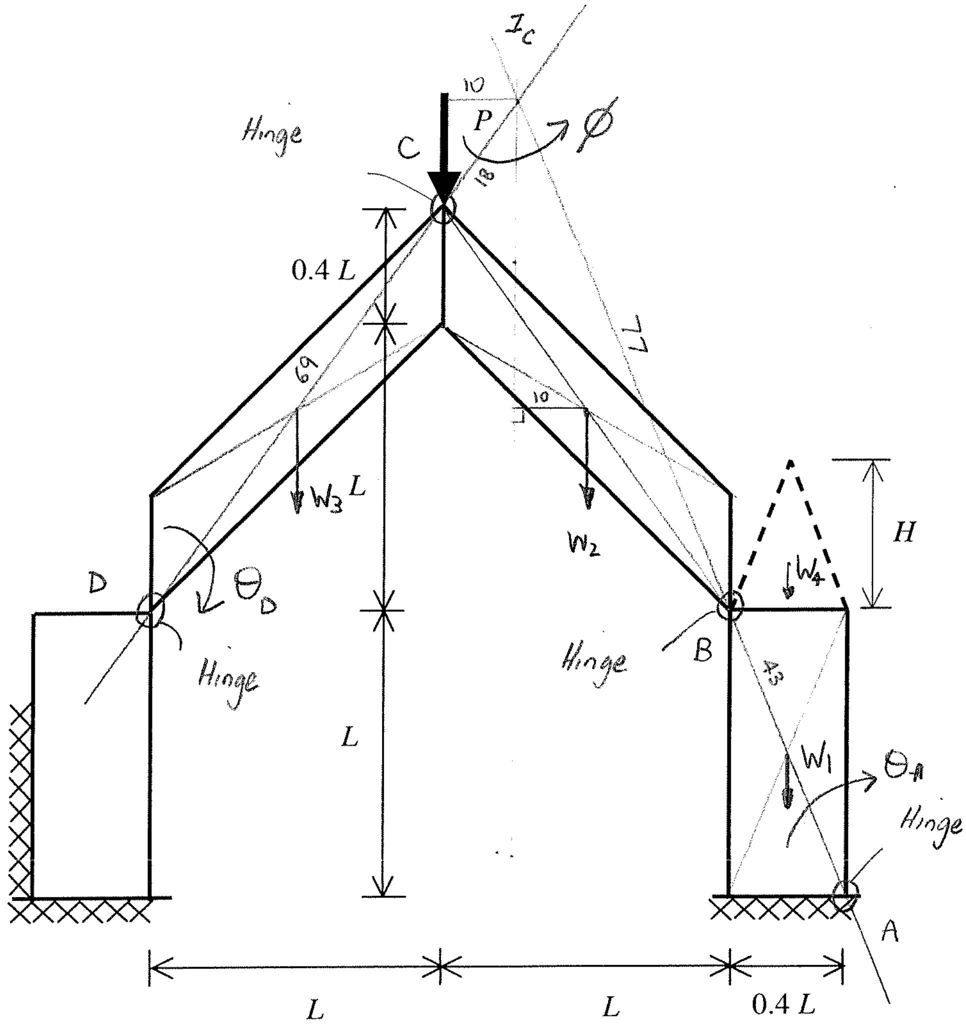
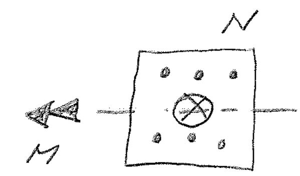


Fig. 3

Scale $L \equiv 40$mm

Working sheet for Question 4
(may be handed in with your script)

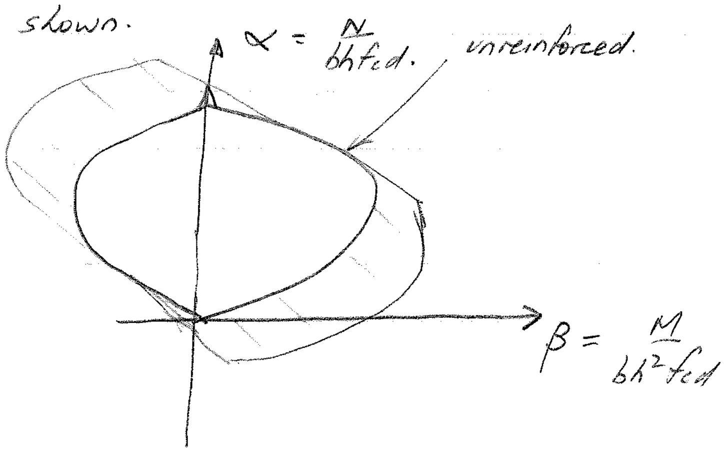
Q5 (a) Interaction diagram.



Assume: 1) zero tensile strength for concrete
2) linear strain diagram.
3) Simple stress/strain curves for steel + concrete.

Firstly consider an unreinforced beam & derive non-dimensional value for axial load and moment at ULS. Because of presence of axial load the moment will have a different value depending on which axis it is measured about, hence it is important to agree on a convention to use as the reference axis – this is taken as the fixed mid-depth axis. (It is not feasible to choose the neutral axis as it can take any position, & it is not initially known)

Then plot values of non-dimensional values α, β for different assumed positions of the neutral axis. This generates an initial interaction diagram as shown.



Then consider the addition of reinforcement. Find that ratio of β/α for reinforcement is only dependent on location of steel, not stress in the steel. $\Rightarrow$ Can add vectorially to diagram to represent effects of steel. Can then allow for other layers of steel by adding vectors to the graph.

Design charts with interaction diagrams for a given section geometry & r/f position can be generated, with various curves plotted depending on amount of reinforcement ratio ρ.

20

S/2

Q5 (b) Importance of $\varepsilon_y/\varepsilon_{cm}$.

If $\varepsilon_y/\varepsilon_{cm} \to 0$ all steel yields at failure & curves will be as explained in part (a) which assumes failure occurs when concrete in compression reaches ultimate strain capacity ε_{cm} (irrespective of steel position) so we get a fully plastic solⁿ.

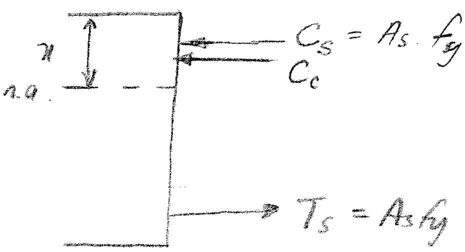
If $\varepsilon_y/\varepsilon_{cm}$ is not zero, the length of the added vector for a steel layer depends on how close it is to the n.a. – a non-plastic solⁿ which can significantly alter the interaction diagram shape.

Will get critical value for position of n.a. above which section will behave as over-reinforced & steel stress will be less than yield at failure.

Q5(c)

(i) Assume initially that axial stress is zero & that all steel is yielded.

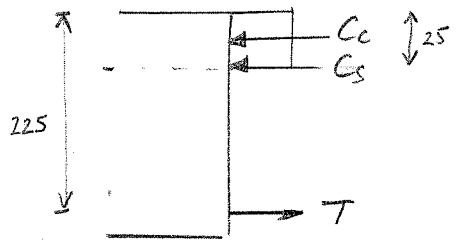
Ultimate moment



$= 3 \times \frac{\pi}{4} 10^2 \times 400 = \underline{94.3\ kN}$

Have equal top & bottom steel thus for T = C MUST have C_s on n.a. & not yielding since otherwise do not have equilibrium

$\Rightarrow$ N.A. at depth of top steel.



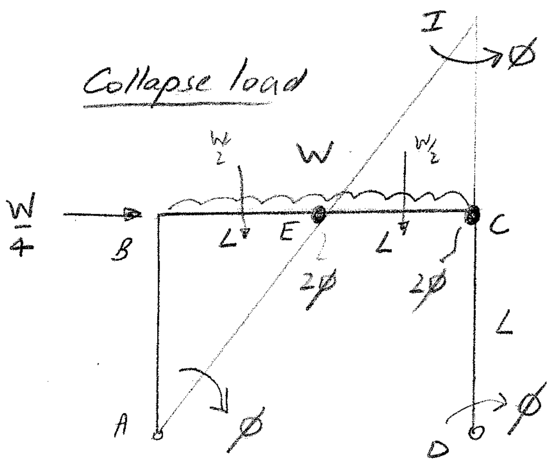
Assume concrete stress block with max $0.6 f_{cd}$.

$C_c = 0.6 \times 30 \times 150 \times 25 = \underline{67.5\ kN}$

$\Rightarrow T_s = C_s + C_c \Rightarrow C_s = 94.3 - 67.5 = \underline{26.8\ kN}$

$\therefore M_u = [67.5(212.5) + 26.8 \times 200] \times 10^{-3} = \underline{19.7\ kNm}$

Collapse load



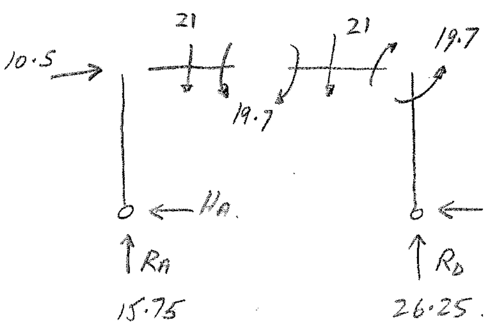
$\frac{W}{2} \cdot \frac{L}{2} \phi + \frac{W}{2} \cdot \frac{L}{2} \cdot \phi + \frac{W}{4} \cdot L\phi = M_u \cdot 2\phi \cdot 2$

$\frac{3WL}{4} = 4 M_u$

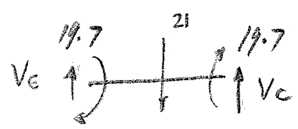
$W = \frac{16 M_u}{3L} = \frac{16 \times 19.7}{3 \times 2.5} = 42\ kN$ (total load)

$\left(\frac{W}{4} = 10.5\ kN\right)$

(ii) Axial forces



Consider FBD of beam section EC.



Moments about E $\qquad 21 \times \frac{2.5}{2} + 19.7 \times 2 = V_C \times 2.5$

$\therefore V_C = \underline{26.26\ kN}\ (= R_D)$

$R_A = 42 - 26.26 = \underline{15.74\ kN}$

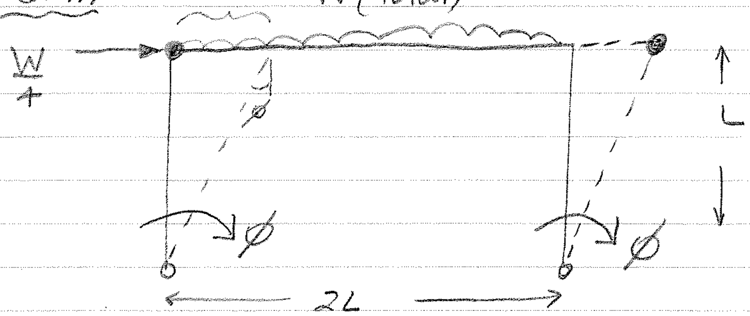
(Quicker method – moments about A

$42 \times 2.5 + 10.5 \times 2.5 = R_D \times 5 \Rightarrow R_D = \frac{52.5}{2} = \underline{26.25}$

Q5(c)(i) cont.

Check other failure modes are not critical

SWAY



$\tan\phi \simeq \phi = \frac{\delta}{L}$

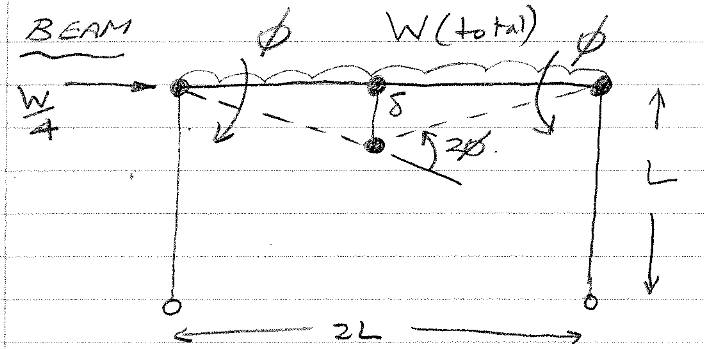
$\therefore \delta = L\phi$

$WD = \frac{W}{4}.\delta = \frac{WL\phi}{4}.$

$ED = M_u.\phi.2$

$\therefore$ WD = ED $\qquad \frac{WL\phi}{4} = M_u\phi 2 \qquad \therefore W_2 = \frac{8M_u}{L} = \frac{8\times 19.7}{2.5} = \underline{63\ kN}$

BEAM



$\tan\phi \simeq \phi = \frac{\delta}{L} \qquad \therefore \delta = \phi L.$

$WD = W \times \frac{\delta}{2} = \frac{W\phi L}{2}$

$ED = M_u(\phi + 2\phi + \phi) = 4\phi M_u.$

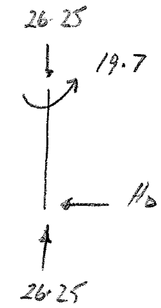
WD = ED $\qquad \frac{W\phi L}{2} = 4\phi M_u \qquad \therefore W_3 = \frac{8M_u}{L} = \underline{63\ kN.}$

$\Rightarrow$ 1st combined mode on previous page is critical mode with

$W_1 = 5.3\frac{M_u}{L} = \underline{42\ kN}$

Q 5(c)

(ii) cont. FBD Section CD.

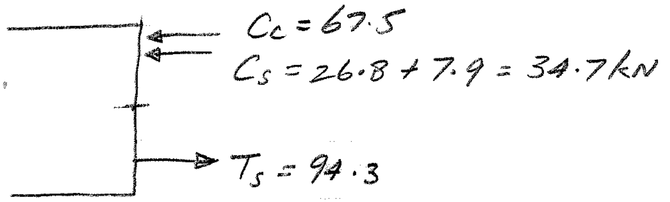


$H_D = \frac{19.7}{2.5} = 7.88\ (7.9)$ kN

$\therefore H_A = 10.5 - 7.9 = \underline{2.6}$ kN

$\therefore$ Axial load in beam BC is 7.9 kN c.f. axial load in columns AB & CD of 26.3 kN. Expect column strength to be enhanced more than beam due to beneficial effect of axial force on moment capacity (provided it is not too high relative to moment).

$\therefore$ Expect hinges to form in beam not columns, with axial load of 7.9 kN.



All axial force must be carried by compression steel which has not yielded since tension steel has already yielded & concrete is at limit.

Modified M_u. Take moments about centre of section. (since must define axis for moments when additional axial force is considered).

$$M_u = 94.3 \times 0.1 + 67.5 \times 0.1125 + 34.7 \times 0.1$$
$$= 20.5 \text{ kNm.}$$

(only 4 % increase in M_u)

New estimate for W $\quad W = \frac{16 M_u}{3L} = \frac{16 \times 20.5}{3 \times 2.5} = 43.7$ kN.

c.f. 42 kN earlier

(i.e. 4% ↑)