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4D8 Prestressed Concrete Exam 2003

Solution

1. First need to find the stresses in the precast beam due to P_1, e_1 & M_g

Top fibre (1)

$$\text{Stress} = \frac{P_1}{A} + \frac{P_1 e_1}{Z_1} - \frac{M_g}{Z_1}$$

$$= \frac{3000}{0.32 \cdot 10^3} + \frac{3000 \cdot 0.1}{-0.425 \cdot 10^3} - \frac{360}{-0.425 \cdot 10^3}$$

$$= 9.4 - 7.1 + 8.5 = 10.8 \text{ N/mm}^2$$

Bottom fibre (2)

$$\text{Stress} = \frac{P_1}{A} + \frac{P_1 e_1}{Z_2} - \frac{M_g}{Z_2}$$

$$= 9.4 + 7.1 - 8.5 = 8.0 \text{ N/mm}^2$$

[8 marks]

These stresses are now fixed.
Magnel diagram needs to be calculated based on changes from this state.

New moduli & Kern points

$I = 0.04\,m^4$

$Z_1' = \frac{-0.04}{(0.80 - 0.575)} = 0.177\,m^3 \qquad -\frac{Z_1'}{A'} = 0.378\,m$

$Z_2' = \frac{0.04}{0.575} = 0.0696\,m^3 \qquad -\frac{Z_2'}{A'} = -0.148\,m$

$Z_3' = \frac{0.04}{(0.95 - 0.575)} = 0.107\,m^3 \qquad -\frac{Z_3'}{A'} = 0.226\,m$

<u>Consider Fibre 2</u> (bottom fibre)

Stresses will be tensile under M_b (1560 − 360 = 1200 kNm) and compressive under zero additional moment.

$$8.0 + \frac{P_2}{A'} + \frac{P_2 e_2}{Z_2'} - \frac{M_b}{Z_2'} \geq 0$$

$$\& \quad 8.0 + \frac{P_2}{A'} + \frac{P_2 e_2}{Z_2'} \leq 20$$

Rearrange

$$\frac{12 Z_2'}{P_2} - \frac{Z_2'}{A'} \geq e_2 \geq \left(\frac{-8 Z_2' + M_b}{P_2}\right) - \frac{Z_2'}{A'}$$

$$\Rightarrow \underline{0.269 \geq e_2 \geq 0.173} \text{ for } P_2 = 2MN.$$

Consider Fibre(1) (Top of Precast)

Compression under M_r ; tension under no load

$$10.8 + \frac{P_2}{A'} + \frac{P_2 e_2}{Z_1'} - \frac{M_r}{Z_1'} \leq 20$$

$$\& \quad 10.8 + \frac{P_2}{A'} + \frac{P_2 e_2}{Z_1'} \geq 0$$

[N.B. Z_1' -ve so inequalities reverse]

$$\frac{9.2 Z_1' + M_r}{P_2} - \frac{Z_1'}{A} \leq e_2 \leq \frac{-10.8 Z_1'}{P_2} - \frac{Z_1'}{A}$$

$$0.160 \leq e_2 \leq 1.338 \text{ m}$$

Consider Fibre (3) Top of insitu

Compression under M_r ; tension under no load

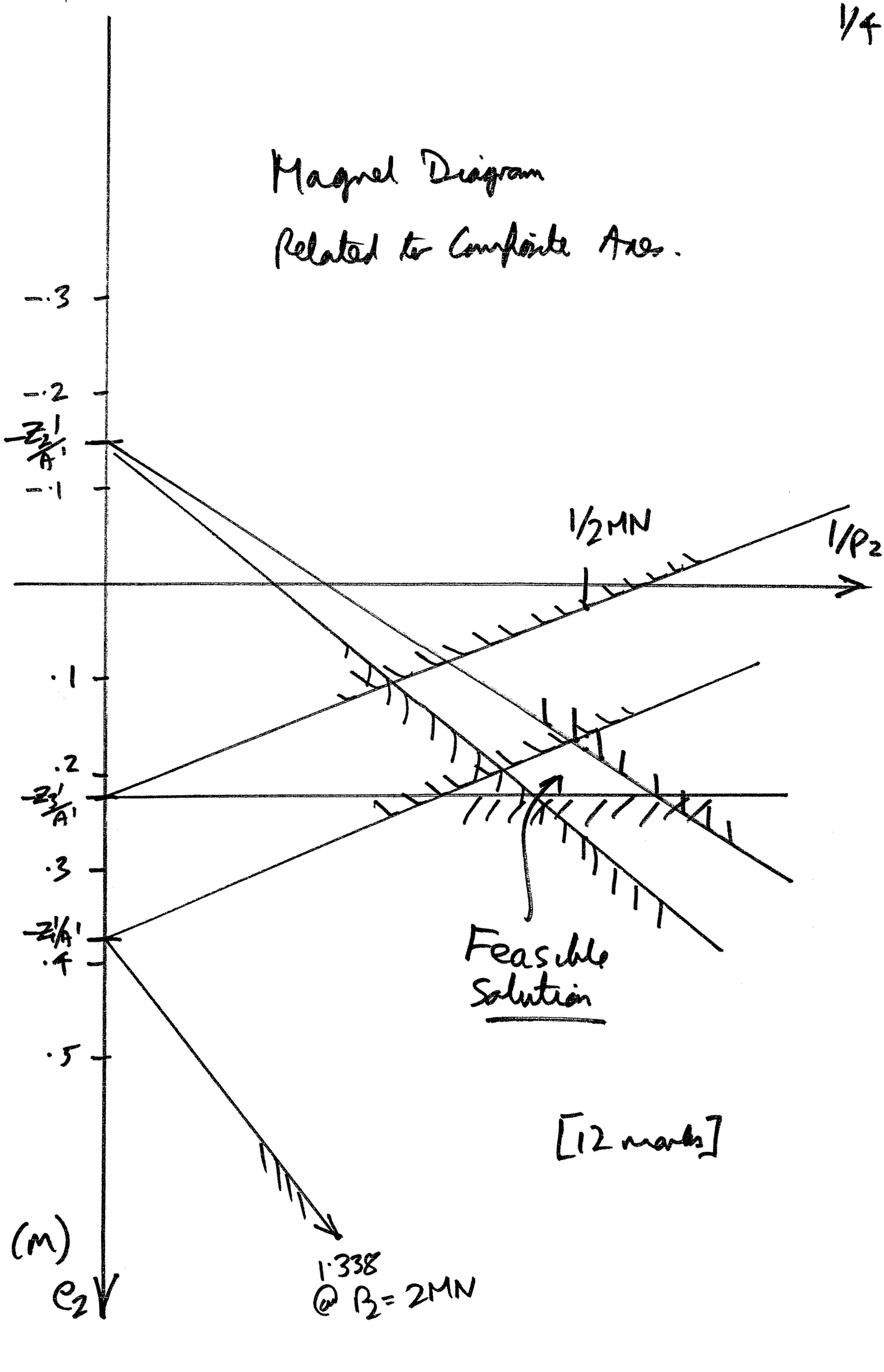
$$\frac{P_2}{A'} + \frac{P_2 e_2}{Z_3'} - \frac{M_r}{Z_3'} \leq 15$$

$$\frac{P_2}{A'} + \frac{P_2 e_2}{Z_3'} \geq 0$$

$$\Rightarrow \quad 0.226 \geq e_2 \geq 0.026 \text{ m}$$

[8 marks for stress conditions
8 marks for inequalities]

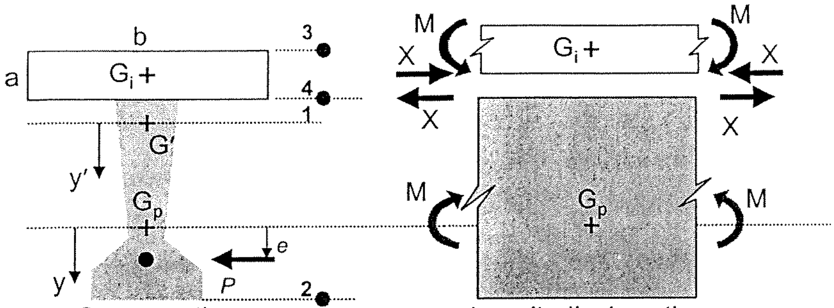
Magnel Diagram
Related to Composite Axes.



[12 marks]

Qu 2. (a) Bookwork - taken from notes but they should provide more on derivation.

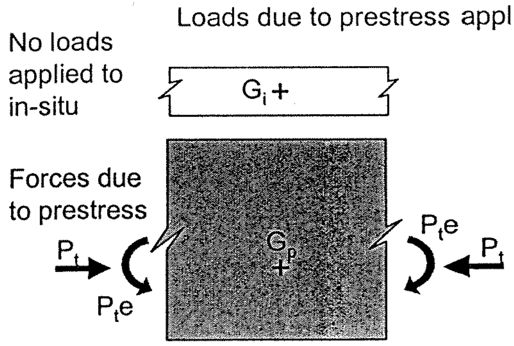
Principle of analysis – separate in-situ and precast elements and impose self equilibrating forces (X) and moments (M)



Cross section   Longitudinal section

Note – this is an application of St. Venant's principle – the self-equilibrating moments must be caused by vertical forces between the in-situ and precast but we do not need to know the details

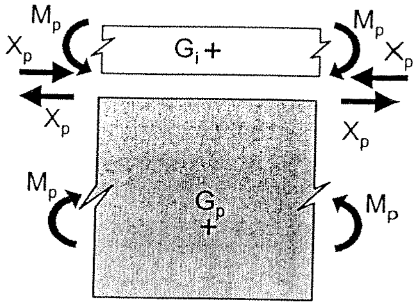
Loads due to prestress applied to the precast section alone



Must be rigorous with signs

- External moments and all curvatures positive sagging
- External forces positive in compression
- Self-equilibrating forces and moments +ve as shown on previous page

	At t = 0	
In-situ	$M_0 = 0$	$N_0 = 0$
Precast	$M_0 = -P_t e$	$N_0 = P_t$



Additional self-equilibrating moments and forces after creep has taken place

	At t = ∞	
In-situ	$M_\infty = - aX_p/2 - M_p$	$N_\infty = X_p$
Precast	$M_\infty = - P_t e + y_1 X_p + M_p$	$N_\infty = P_t - X_p$

Remember y_1 is -ve

Change in curvature in the precast section will thus be:-

$$\Delta\kappa_p = \frac{1}{E_{p\infty} I_p}(-P_t e + X_p y_1 + M_p) - \frac{1}{E_p I_p}(-P_t e)$$
$$= \frac{1}{E_p I_p}\left[-\varphi P_t e + (1+\varphi)(M_p + X_p y_1)\right]$$

Similarly, for the in-situ concrete $\Delta\kappa_i = -\frac{(1+\varphi)}{E_i I_i}\left(M_p + \frac{aX_p}{2}\right)$

These must be equal to ensure compatibility, so

$$(1+k)M_p + \left(y_1 + \frac{ak}{2}\right)X_p = \frac{\varphi}{1+\varphi}P_t e \quad \text{where } k = \frac{E_p I_p}{E_i I_i}$$

Similarly, the change in compressive interfacial strain in the precast section

$$\Delta\varepsilon_p = \frac{y_1}{E_p I_p}\left(\varphi P_t e - (1+\varphi)\left[M_p + y_1 X_p\right]\right) + \frac{1}{E_p A_p}\left(\varphi P_t - (1+\varphi) X_p\right)$$

Must equal the interfacial strain in the in-situ concrete

$$\Delta\varepsilon_i = \frac{(1+\varphi)}{E_i I_i}\frac{a}{2}\left(M_p + \frac{a X_p}{2}\right) + \frac{(1+\varphi)}{E_i A_i} X_p$$

After equating these expressions and rearranging, the equations for curvature change and strain change can be rearranged to give:-

$$\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix}\begin{bmatrix} M_p \\ X_p \end{bmatrix} = \frac{\varphi}{1+\varphi}\begin{bmatrix} P_t e \\ \left(y_1 e + r_p^2\right) P_t \end{bmatrix}$$

where $\alpha = 1 + k$ and $r_p^2 = I_p / A_p$

$$\beta = y_1 + \frac{ak}{2} \qquad r_i^2 = I_i / A_i$$

$$\gamma = y_1^2 + r_p^2 + k\left(\frac{a^2}{4} + r_i^2\right)$$

Solve for M_p and X_p

3. (a) Creep and shrinkage will be important since presumably stressed soon after casting. Tendon probably curved, so friction important. Long tendons so wedge slip not important. Elastic shortening high.

(b) Concrete precast so most of creep and shrinkage will have happened before stressing. Some tendons will be short, so wedge slip important. Friction important but cables will be fairly straight.

(c) Creep and shrinkage not important. Friction will only occur at deviators so no wobble losses. Elastic shortening low because of presence of a lot of untensioned rebar.

4 Untensioned rebar adds to ultimate moment capacity. Restrains creep and shrinkage, so loss of tendon force is lower but effect on concrete is much higher. Prestress in concrete will thus be less so cracking more likely. Crack widths will need to be checked but should be small if detailed properly. Implications for durability because exposing steel to air and/or water. But, this may not be the prestressing steel so may not be critical.