IIB CRIBS
2003

4D12

Engineering Tripos Part IIB. 2003
Module 4D12. Solutions

1 (a)

$$\Phi = \frac{\omega H \cosh K(y+d)}{2K \sinh Kd} \cos(\omega t - Kx)$$

$$\therefore \quad a = \int u\,dt = \int \frac{\omega H \cosh K(y+d)}{2 \sinh Kd} \sin(\omega t - Kx)\,dt$$

$$\doteqdot -\frac{H \cosh K(y+d)}{2 \sinh Kd} \cos(\omega t - Kx)$$

$$\text{also } b = \int v\,dt = \int \frac{\omega H \sinh K(y+d)}{2 \sinh Kd} \cos(\omega t - Kx)\,dt$$

$$\doteqdot -\frac{H \sinh K(y+d)}{2 \sinh Kd} \sin(\omega t - Kx)$$

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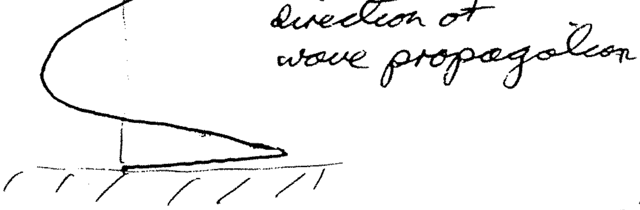
Thus fluid particle orbits are ellipses.

(b) If we neglect viscosity a higher order solution introduces a net drift in the direction of wave propagation.

However, the introduction of viscosity significantly changes the variation of net drift with height. Typical net drift profiles are:



Note strong forward drift in bottom boundary layer; parabolic velocity profile in interior of fluid (debateable). This profile is for a smooth bed in a closed channel. Rough beds can significantly change net drift.

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1 (Contd) (c) Large waves stir sediment into suspension so drift profile in body of fluid is important. With small waves (fine weather) sediment motion is confined to bottom boundary layer. So beaches build up in fine weather and degrade in storms.

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(d) Important currents are:

(1) onshore – offshore drift described above

(2) longshore current due to waves

(3) tidal currents

(4) ocean currents (e.g Gulf Stream)

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2 (a) Bookwork

See any standard text. For example: Ippen "Estuary and Coastline Hydrodynamics" p 58 et seq.

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(b) Assumptions:

(1) Energy is conserved. Energy loss in boundary layers is negligible (reasonably true). There is no wave breaking (large energy loss)

(2) Changes in depth are sufficiently gradual for the solution obtained for constant depth to apply.

(3) The assumptions associated with small-amplitude theory are correct (e.g inviscid, irrotational, $H \ll L$, $HL^2/d^3 < 26$).

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2(c) For depth = 15 m, $L = 50m$

$\therefore \omega = (gK \tanh Kd)^{\frac{1}{2}} = 1.086$

$\therefore T = 5.79$ s

$\therefore \frac{d}{T^2} = 0.448$

From the Data Sheet $\frac{H_{15}}{H_0} = 0.95$

Also $(u)_{y=-d} = \frac{\omega H}{2 \sinh Kd} = 0.17 H_{15}$

(where H_{15} = wave height at $d = 15m$)

For depth = 1 m

$\frac{d}{T^2} = 0.0298$

$\therefore$ From Data Sheet $\frac{H_1}{H_0} = 1.25$

$L/T^2 = 0.52$

$\therefore (u)_{y=-d} = \frac{\omega H}{2 \sinh Kd} = 1.484 H_1$

$\therefore \left(\frac{u_1}{u_{15}}\right)^2 = \left(\frac{1.484 \times 1.25 \times H_0}{0.95 \times 0.17 \times H_0}\right)^2 = 132$

$\therefore \left(\frac{T_{15}}{T_1}\right) = \frac{1}{132}$

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3 Snell's Law: bookwork. See lecture notes or any ~~textbook~~

In deep water $\omega = (gK)^{\frac{1}{2}}$ $\therefore C_0 = \frac{\omega}{K} = 15.61\ m/s$

$\alpha_0 = 45°$

(a) At depth $= 7m$ $d/T^2 = 0.07$

From Data Sheet $L/T^2 = 0.795$ $\therefore L = 79.5m$

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$\therefore c = \frac{L}{T} = 7.95$

From Snell's Law $\alpha = \sin^{-1} \frac{7.95}{15.61} \sin 45° = 21.1°$

(b) $\frac{H}{H_0} = 1.03$ at $d/T^2 = 0.07$ (From Data Sheet)

also $K_r = \left(\frac{\cos 45}{\cos 21.1}\right)^{\frac{1}{2}} = 0.8706$

$\therefore H = 1.03 \times 0.8706 \times 6 = 5.38\ m$

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(c) Waves break at about $H = 0.78d = 5.46m$. This is slightly greater than 5.38m, which suggests that the waves have not yet broken. However,

(i) the calculation is based on linear theory

(ii) the effects of bottom slope on the breaking point are usually significant

(iii) wind stress and interactions with other wave components may also be significant

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4. $F = C_D \rho \frac{u|u|D}{2} + C_M \rho \frac{\pi D^2}{4} \frac{du}{dt}$

$$\quad\;\; \overset{\shortparallel}{F_D} \qquad\qquad\qquad \overset{\shortparallel}{F_I}$$

2) Substituting for u and $\frac{du}{dt}$ from the Data sheet gives

$$F_D = \int_{-d}^{0} \rho \frac{C_D D \omega^2 H^2 \cosh^2 k(y+d)}{8 \sinh^2 kd} \sin(\omega t - kx) |\sin(\omega t - kx)| \, dy$$

$$= \frac{C_D \rho D \omega^2 H^2}{16 \sinh^2 kd} \left(d + \frac{\sinh 2kd}{2k} \right) \sin(\omega t - kx) |\sin(\omega t - kx)|$$

and $F_I = \int_{-d}^{0} C_M \rho \frac{\pi D^2}{4} \, \omega^2 \frac{H \cosh k(y+d)}{2 \sinh kd} \cos(\omega t - kx) \, dy$

$$= C_M \frac{\rho \pi D^2 \omega^2 H}{8k} \cos(\omega t - kx)$$

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Total Force $F = F_D + F_I$.

) Denote the ratio of the amplitudes of F_D and F_I by R

i.e. $R = \frac{C_D \cdot H}{2 C_M \pi D} \left(\frac{d + \frac{\sinh 2kd}{2k}}{\sinh^2 kd} \right)$

<u>For shallow water</u> $(kd \to 0)$

$$R \to \frac{C_D H}{2 C_M \pi D} \frac{2d}{(kd)^2} = \frac{C_D H}{C_M \pi D kd}$$

The surface velocity $u_M = \frac{\omega H \cosh kd}{2 \sinh kd} \to \frac{\omega H}{2kd}$

K6. (Contd)

$\therefore$ Keulegan - Carpenter n° $KC = \frac{U_M T}{D} = \frac{\omega H}{2Kd} \frac{2\pi}{\omega D} = \frac{H\pi}{Kd\, D}$

$\therefore R = \frac{C_D}{C_M} \frac{KC}{\pi^2}$ which is small since $C_D \approx 1$, $C_M \approx 2$ and $KC < 1$

For deep water $(Kd \to \infty)$

$$R \to \frac{C_D H}{2 C_M \pi D} \left(\frac{Kd + \frac{e^{2Kd}}{4}}{\frac{e^{2Kd}}{4}} \right) = \frac{C_D H}{2 C_M \pi D}$$

and $U_M \to \frac{\omega H}{2}$

$\therefore R = \frac{C_D\, KC}{2 C_M \pi^2}$ which is again small.

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