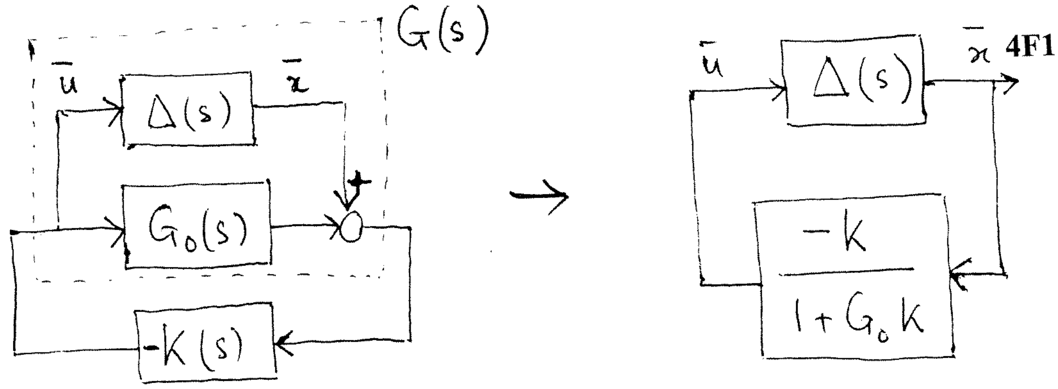
4F1 crib 2003

1 (a)



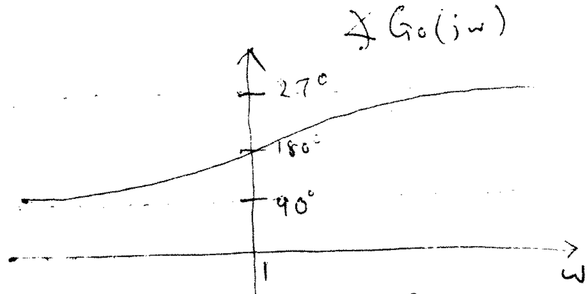
Applying the Small Gain Theorem shows that stability holds for all $|\Delta(j\omega)| \leq h(\omega)$ if and only if

$$\left| \frac{K(j\omega)}{1 + G_0(j\omega)K(j\omega)} \right| < \frac{1}{h(\omega)}$$

(b) (i) Sketch a Bode phase plot first

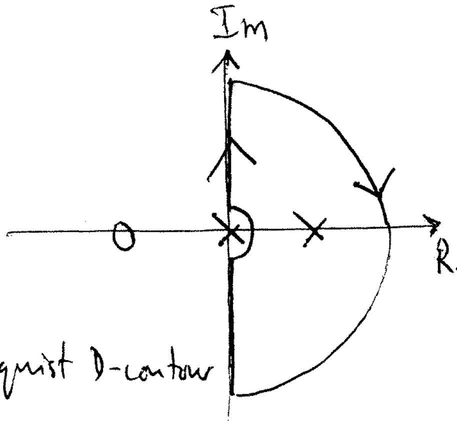
$$G_0(s) = -\frac{2}{s} \cdot \frac{s+1}{1-s}$$

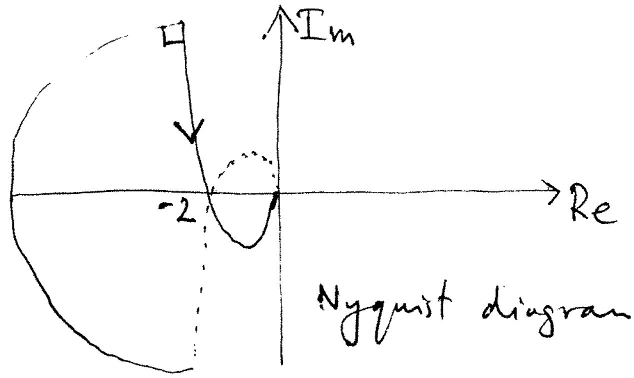
$\uparrow$ $+90°$ $\uparrow$ 2 rising characteristics



$\measuredangle G_0(j1) = +180°$

$|G_0(j1)| = 2$





location of $-\frac{1}{k}$	k-value	# anti-clock. encirc.	#RHP closed loop poles
$-\infty < -\frac{1}{k} < -2$	$0 < k < \frac{1}{2}$	-1	2
$-2 < -\frac{1}{k} < 0$	$\frac{1}{2} < k < \infty$	1	0
$0 < -\frac{1}{k}$	$k < 0$	0	1

(ii) $k = 1 \Rightarrow$ closed-loop stable

$$\frac{k}{1+G_0 k} = \frac{1}{1+\frac{2(s+1)}{s(s-1)}} = \frac{s(s-1)}{s^2+s+2} \quad \text{and} \quad h(w) = \left|\frac{\varepsilon e^{-jw}}{jw+1}\right| = \frac{\varepsilon}{jw+}$$

Result of part (a):

$$\frac{|jw(jw-1)|}{|-w^2+jw+2|} < \frac{|jw+1|}{\varepsilon}$$

$$\Leftrightarrow \quad \varepsilon w < |2-w^2+jw|$$

$$\Leftrightarrow \quad \varepsilon < \sqrt{\left(\frac{2}{w}-w\right)^2+1}$$

(Min. of RHS when $w = \sqrt{2}$)

$\varepsilon = 1$ is largest value allowed.

(iii) when $\Delta(s) = \dfrac{\varepsilon e^{-s}}{s-2}$ and ε very small

Nyquist diagram is essentially unchanged, but there is an extra RHP pole of open loop.

Nyquist stability criterion says closed loop can't be stable.

2(a)

$$0 = \log|S(s_0)| = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sigma_0}{\sigma_0^2 + \omega^2} \log|S(j\omega)|\,d\omega \qquad s_0 = \sigma_0 = 2$$

$$\Rightarrow \quad 0 = \int_0^{\infty} \frac{2}{2^2+\omega^2} \log|S(j\omega)|\,d\omega \leqslant \int_0^1 \frac{2}{2^2+\omega^2} \log\varepsilon\,d\omega + \int_1^{\infty} \frac{2}{2^2+\omega^2} \log 1.5\,d\omega$$

$$= \left[\tan^{-1}\frac{\omega}{2}\right]_0^1 \log\varepsilon + \left[\tan^{-1}\frac{\omega}{2}\right]_1^{\infty} \log 1.5$$

$$= 0.4636 \log\varepsilon + (\pi/2 - 0.4636)\log 1.5$$

$$\Rightarrow \quad \varepsilon \geqslant \exp\left(\frac{0.4636 - \pi/2}{0.4636} \log 1.5\right) = 0.3798$$

alter:

$$0 = \int_{-\infty}^{\infty} \frac{1}{\cosh\nu} \log|S(j2e^{\nu})|\,d\nu \qquad \left(\nu = \log\frac{\omega}{|s_0|}\right)$$

$$\leqslant \int_{-\infty}^{\log 0.5} \frac{1}{\cosh\nu} \log\varepsilon\,d\nu + \int_{\log 0.5}^{\infty} \frac{1}{\cosh\nu} \log 1.5\,d\nu$$

$$= \log\varepsilon\left[\tan^{-1}\sinh\nu\right]_{-\infty}^{\log 0.5} + \log 1.5\left[\tan^{-1}\sinh\nu\right]_{\log 0.5}^{\infty}$$

$$= \log\varepsilon\left[\tan^{-1}\sinh\log 0.5 - \left(-\frac{\pi}{2}\right)\right] + \log 1.5\left[\pi/2 - \tan^{-1}\sinh\log 0.5\right]$$

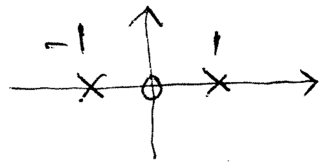
$$\Rightarrow \quad \varepsilon \geqslant \exp\left(-\log 1.5 \; \frac{-0.6435 + \pi/2}{0.6435 + \pi/2}\right) = 0.3798$$

2 (b) (i) RHP or imaginary axis pole-zero cancellations between plant and controller are never allowed because they do not give an internally stable closed-loop. At least one closed-loop transfer function will be unstable: $\frac{K}{1+GK}$ if K has the RHP pole, and

$\frac{G}{1+GK}$ " G " " " "

But, pole-zero cancellations in the LHP, but near the imaginary axis, can also be unsatisfactory, since one of the above transfer functions will be large in magnitude close to the frequency of the pole/zero. Cancellations "deep enough" in the LHP cause no problems.

(ii) $G(s) = \frac{s}{(s-1)(s+1)}$



To stabilise need a RHP in the controller.
Useful to have a LHP zero as well – it is a terminal location for root-locus as gain is increased.

Cancelling pole at $s=-1$ is okay and makes the calculation easier.

Hence choose: $k(s) = \frac{k(s+1)}{s-1}$

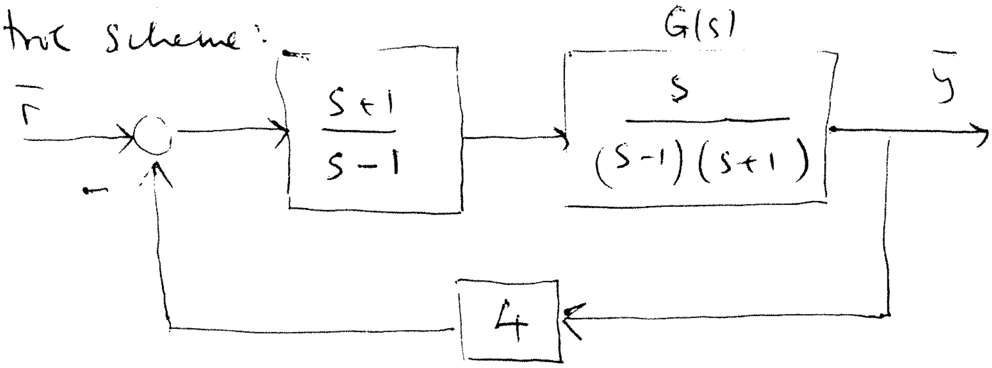
$$G(s)k(s) = \frac{s}{(s-1)\cancel{(s+1)}} \cdot \frac{k\cancel{(s+1)}}{s-1} = \frac{ks}{(s-1)^2}$$

Closed-loop poles: $(s-1)^2 + ks \equiv s^2 - 2s + 1 + ks$

Setting $k=4$ puts closed-loop poles at -1

2 (cont.)

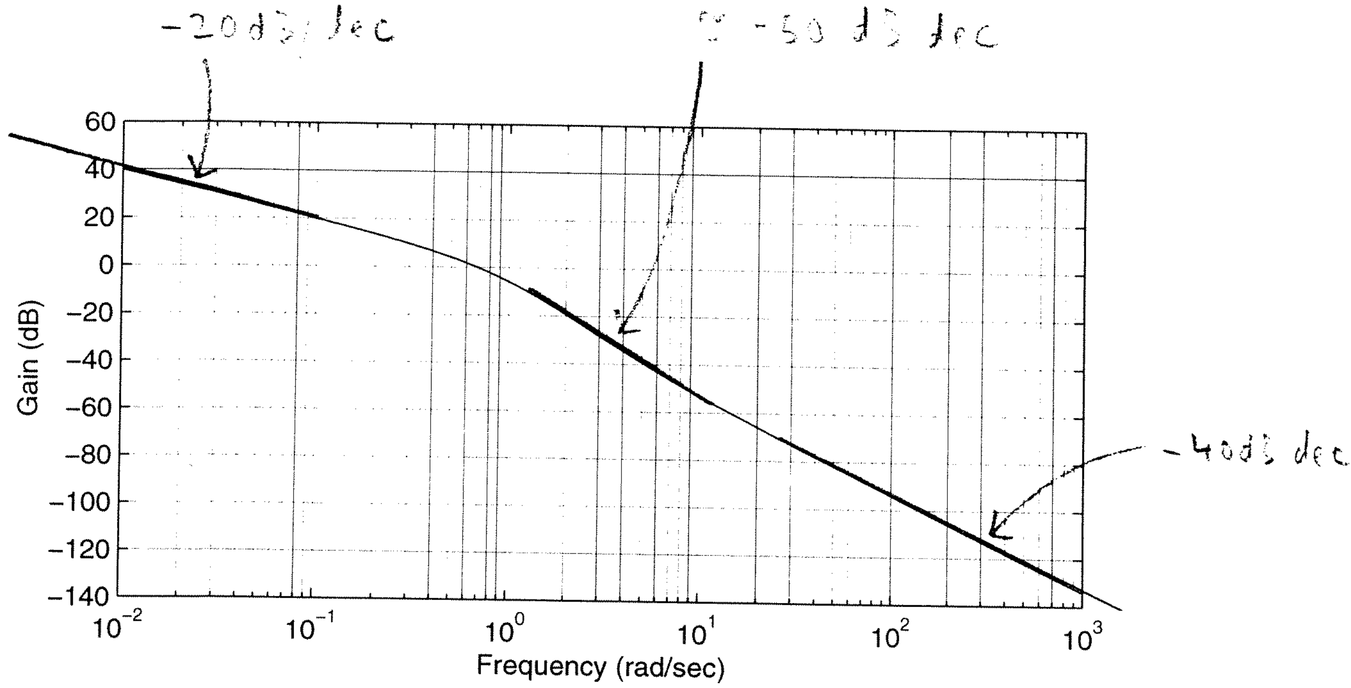
Control scheme:



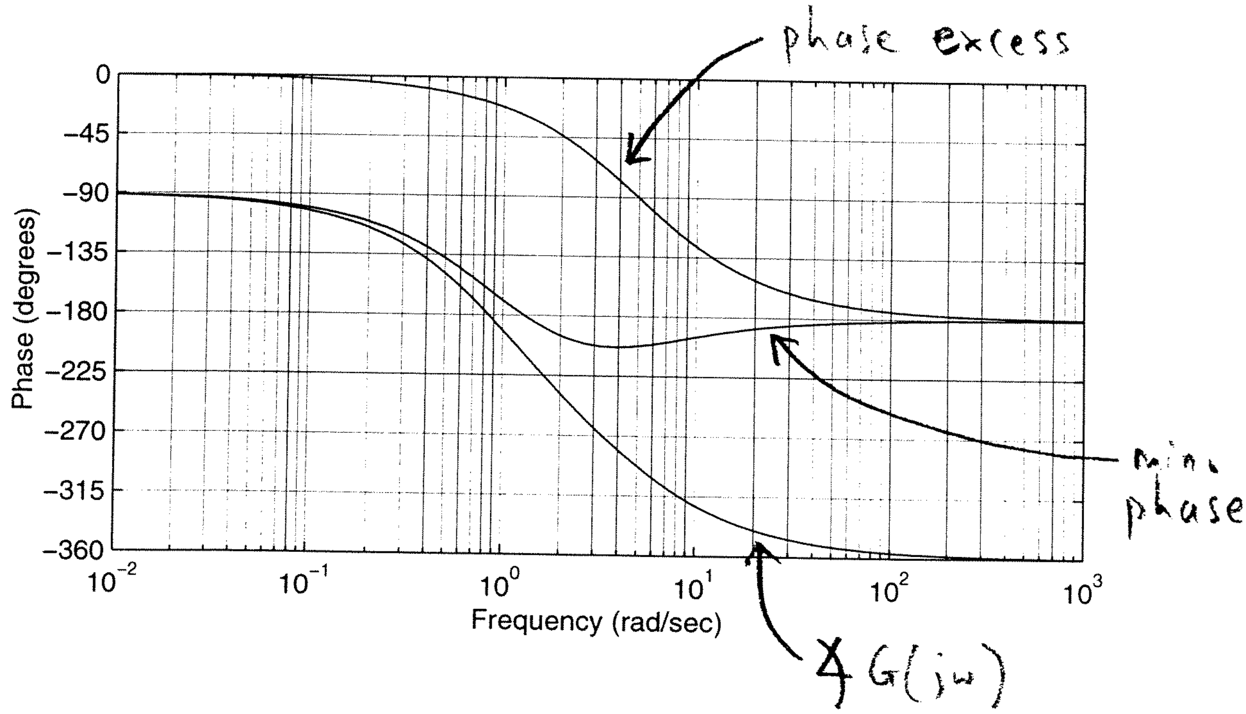
$$T_{\bar{r}\to\bar{y}} = \frac{\frac{s}{(s-1)^2}}{1+\frac{4s}{(s-1)^2}} = \frac{s}{(s+1)^2} \quad \text{as desired.}$$

This particular choice of $K(s)$ and $F(s)$ removed the need for a pre-filter.

3(a)(i)



-20dB/dec
≈ -50 dB dec
-40dB dec
60
40
20
0
−20
−40
−60
−80
−100
−120
−140
Gain (dB)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)



phase excess
min. phase
∡G(jω)
0
−45
−90
−135
−180
−225
−270
−315
−360
Phase (degrees)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

3(a) (ii) Excess phase goes from 0° to −180°. This suggests a single RHP zero at around 5 rad/sec.

(iii) Difficult to achieve a crossover frequency much greater than 5 rad/sec.

(Actual transfer function of $G(s) = \dfrac{-0.2s+1}{s(s+1)^2}$.

Not required.)

(b) (i) At $w=1$, $\measuredangle G(jw) < -180°$. A lag compensator can only reduce the phase further. Therefore spec. D is impossible.

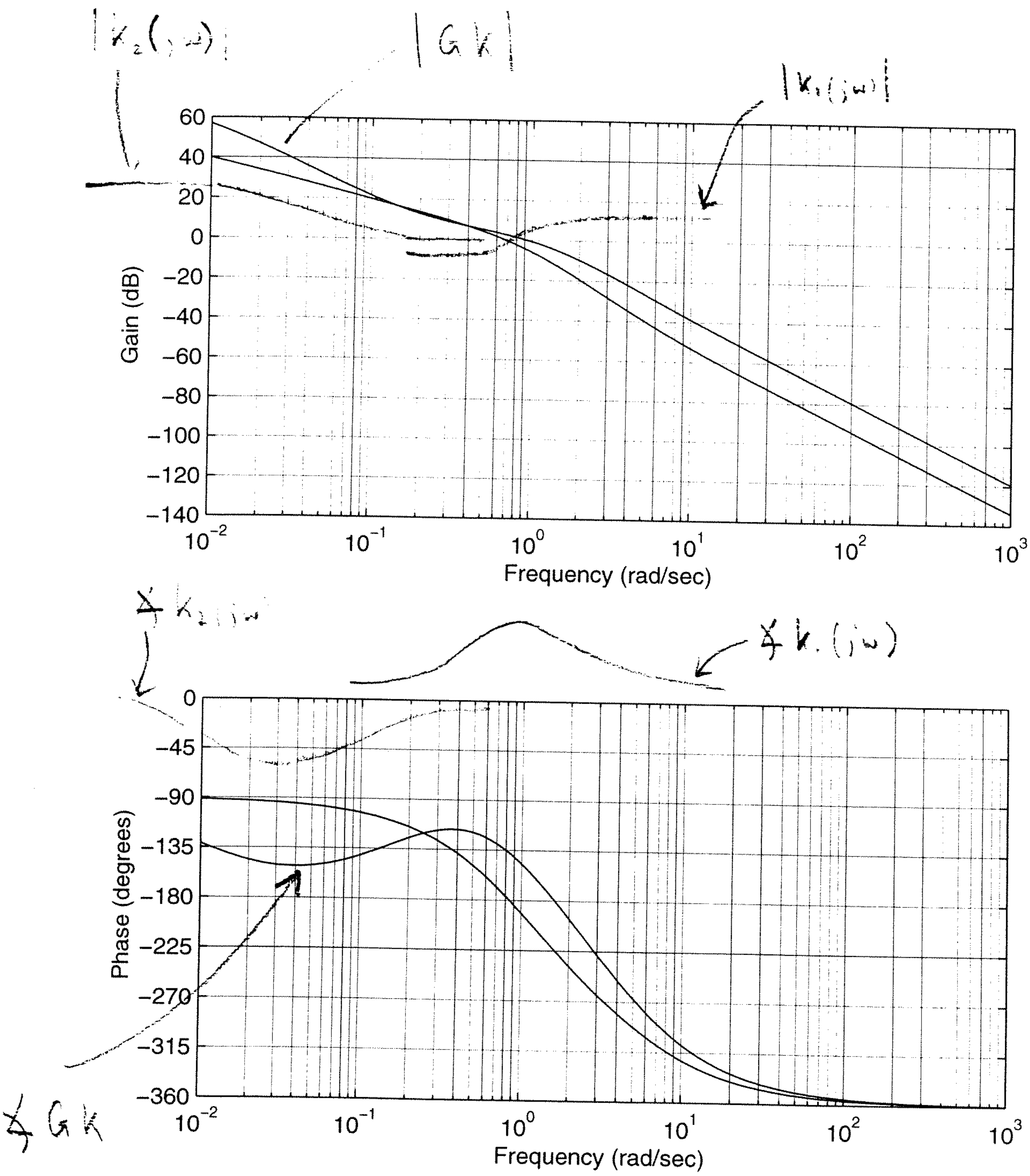
(ii) To achieve B the magnitude characteristic needs to be raised by 20dB at low frequency. To achieve C the magnitude characteristic needs to be raised by about 6dB at $w=1$. This is impossible with a lead compensator, which always increases the gain at a higher frequency more than at a lower one.

(iii) To achieve D about 45° extra phase lead is needed at $w=1$. Take

$$K_1(s) = 2\times 3\ \frac{s+1/3}{s+3} = 6\ \frac{s+\frac{1}{3}}{s+3}$$

to give an extra 53.1° of phase at $w=1$. A little extra phase is taken since a small amount of phase lag might be introduced at the next stage. The extra gain of 2 in $K_1(s)$ is to achieve C, approximately.

3 (b) (iii)


|K₂(jω)|
|GK|
|K₁(jω)|
Gain (dB)
60
40
20
0
−20
−40
−60
−80
−100
−120
−140
Frequency (rad/sec)
∡K₂(jω)
∡K₁(jω)
Phase (degrees)
0
-45
−90
−135
−180
−225
−270
−315
−360
∡GK
Frequency (rad/sec)

We now introduce a lag compensator to satisfy B.
$k_1(s)$ has reduced the low frequency gain by $\frac{2}{3}$, so it now needs to be increased by $10 \times \frac{3}{2} = 15$.

Set

$$k_2(s) = \frac{s + 0.15}{s + 0.01}$$

so that the gain and phase are not affected much near $w = 1$. Final compensator

$$k(s) = k_1(s)\, k_2(s) = 6 \, \frac{s + 1/3}{s + 3} \, \frac{s + 0.15}{s + 0.01}$$

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19 May 03