ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

Saturday 26th April 2003 2.30-4.00

Module 4A6

FLOW INDUCED SOUND AND VIBRATION

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(TURN OVER

1 a) At $x = 0$, the acoustic pressure and velocity of a one-dimensional sound field are given by the real parts of $P\exp(i\omega t)$ and $V\exp(i\omega t)$, respectively, where both P and V are real. Determine the acoustic pressure and the Sound Pressure Level everywhere. Comment briefly on the special case $P = \rho_0 c_0 V$, where c_0, ρ_0 are the ambient sound speed and density. [30%]

b) Low-frequency sound propagates along a tube of cross-sectional area A_1 in the positive x direction. At $x = 0$ the area abruptly changes to A_2, and at $x = L$ the tube is completely blocked by a rigid end wall. Determine the amplitude of the wave reflected back into $x < 0$. [70%]

2 a) Consider a discontinuous medium with uniform mean sound speed and density c_0, ρ_0 in $x < 0$ and c_1, ρ_1 in $x > 0$. A plane sound wave is obliquely incident on the interface from $x < 0$. Deduce Snell's Law relating the angles of the incident and transmitted waves, and calculate the amplitude of the transmitted wave. Explain briefly what might happen if $c_1 > c_0$. [50%]

b) In two dimensions, a single-frequency point source is placed on the rigid plane $x = 0$. The sound speed in $x \geq 0$ is given by

$$c_0(x) = \alpha x + \beta \qquad 0 \leq x \leq H$$

$$= \alpha H + \beta \qquad x \geq H,$$

where α, β, H are positive constants. Sketch the ray paths in $x \geq 0$, being careful to note what happens to any ray which returns to $x = 0$. Find the angle at which a ray which ends up propagating along $x = H$ was launched from the source. What proportion of the acoustic power emitted by the source will never return to $x = 0$? State briefly what happens if $\alpha < 0$ instead. [50%]

3 a) Sound is generated by the action of mass creation at a rate $m(\mathbf{x},t)$ per unit volume. By combining the equations of conservation of mass and momentum derive the equation

$$\frac{\partial}{\partial t}\left(e_p + e_k\right) + \nabla \cdot \mathbf{I} = \frac{p'm}{\rho_0},$$

explaining clearly the meaning of the various terms in the equation. [50%]

b) A concentrated point source $m(t) = m_0 \delta(\mathbf{x}) \cos \omega t$ is added to the sound field at a point where the pressure would otherwise be $p_i \sin \omega(t-\tau)$. Show that the mean sound power output from the point source is changed by the presence of the other field by an amount $-\frac{1}{2} m_0 p_i \sin(\omega\tau) / \rho_0$. [15%]

c) When acting alone, the point source would radiate a mean acoustic power

$$\frac{\omega^2 m_0^2}{8\pi\rho_0 c}$$

Use the result in b) to determine power emitted by the source, when placed a distance ℓ from an 'anti-source' which generates a pressure $\omega m_0 / (4\pi r') \sin \omega(t - r'/c)$ at a distance r' away. [30%]

d) Why is the presence of the neighbouring source able to prevent the radiation of sound power and is the question of 'where has that power gone to' a sensible one? [5%]

4 Derive Lighthill's eighth-power law for the sound generated by a low Mach number jet. [40%]

How would the far-field sound be modified if the jet blows

a) near and parallel to a very large rigid plane surface; [20%]

b) over a small, fixed and rigid body? [20%]

How would the far-field sound be modified if the surface in a) had a small hole in it? [20%]

Note: formulae on the data card may be used without proof.

END OF PAPER